acta phys. slov. 37 (1987), No. 2

LIMITING VALUES OF THE LONGITUDINAL DIFFUSION COEFFICIENT AND THE TEMPERATURE-RELAXATION TIME FOR A PLASMA IN A MAGNETIC FIELD[1])

ŠESTÁK, B.,[2]) FOREJT, L.,[2]) Plzeň

Explicit expressions for the longitudinal diffusion coefficient D, and for the temperature-relaxation time τ are derived for a weakly-coupled plasma in an external magnetic field. It is shown that $D(B=0)/D(B=\infty)=\frac{3}{2}$, and $\tau(B=0)/\tau(B=\infty)=\frac{1}{2}$. These results are valid for both one- and two-component plasmas.

I. INTRODUCTION

Lately a number of papers have dealt with the longitudinal diffusion and the temperature relaxation of particles in a plasma in an external magnetic field. Nevertheless, so far only little is known of the quantitative dependence of these phenomena upon the magnetic field.

In a recent paper [1], Cohen and Suttorp studied diffusion processes in a one-component plasma in an external magnetic field. On the basis of the first Chapman-Enskog approximation, they determined the longitudinal diffusion coefficient $D_{\|}$ for the case when collisions between identical-type particles played the dominant role, and they showed that $D_{\|}(B=0)/D_{\|}(B=\infty)=\frac{3}{2}$. In our earlier papers [2, 3, 4], we suggested that the longitudinal diffusion coefficient, as well as the temperature-relaxation time, exhibited a similar limiting behaviour in both one- and two-component plasmas. But the limiting values obtained hitherto are not sufficietly precise because the approximation used is no longer valid as $B\to\infty$.

[1]) Contribution presented at the 6th Symposium on Elementary Processes and Chemical Reactions in Low Temperature Plasma, JELŠAVA-HRÁDOK, June 9—13, 1986

[2]) Institute of Mechanical and Electrical Engineering, Nejedlého sady 14, 306 14 PLZEŇ, Czechoslovakia

In the present paper, we shall derive more precise expressions for the longitudinal diffusion coefficient and the temperature-relaxation time of charged particles in a weakly-coupled plasma in an external magnetic field. These expressions will be legitimate for arbitrary values of $\boldsymbol{B}$, and on the basis of their asymptotic behaviour we shall determine the limiting values of those quantities as $B\to 0$ or $B\to\infty$.

II. TEMPERATURE RELAXATION OF PARTICLES IN AN ISOTROPIC PLASMA

In accordance with our earlier paper [3], the relaxation time $\tau_{e,i}$ for the electron and ion temperatures to equilibrate with each other is given by the relation

$$\tau_{e,i}^{-1}=\frac{4}{3}\pi(T_eT_i)^{-1}(m_ek_B)^{-2}(2\pi m_ek_BT_e)^{-3/2}A \tag{1}$$

where

$$A=\frac{1}{2}m_e^5v_{T_i}^3\{(\hat{\Omega}_e+\hat{\sigma})^2-7(\hat{\Omega}_e+\hat{\sigma})+12\}\tilde{I}, \tag{2}$$

$$\tilde{I}=\int_0^{\infty}\mathrm{d}u_{e\perp}u_{e\perp}\int_{-\infty}^{+\infty}\mathrm{d}u_{e\|}\exp(-u_e^{2'}/\sigma^2)g^{B(M)}(u_{e\perp},u_{e\|}), \tag{3}$$

$$\hat{\Omega}_e=\Omega_e\partial/\partial\Omega_e,\quad \hat{\sigma}=\sigma\partial/\partial\sigma,\quad \sigma=v_{T_e}/v_{T_i},\quad v_{T_j}=(2k_BT_j/m_j)^{1/2}$$

is the thermal velocity of the particles of the type $j=e,i$, $u_{e\perp}=p_{e\perp}/m_ev_{T_i}$, $u_{e\|}=p_{e\|}/m_ev_{T_i}$, and $g^{B(M)}$ is the magnetic scalar potential for a Maxwellian distribution of background particles [5]

$$g^{B(M)}(p_{e\perp},p_{e\|})=N_i(2\pi m_ik_BT_i)^{-3/2}\int\mathrm{d}P_i\exp(-(p_{i\perp}^2+p_{i\|}^2)/2m_ik_BT_i)\,G^B. \tag{4}$$

We can determine the generating function G^B in the same way as in our paper [6],. Using the modified Debye potential [6] we get G^B in the form

$$G^B=\frac{z^{e,i}}{4\pi}\int_0^{2\pi}\mathrm{d}\mathcal{V}\{|\boldsymbol{V}_{ei}|[\ln\lambda+I(2k_D|\boldsymbol{V}_{ei}|/\Omega_e)]+ \\ +|V_{ei}'|[I(2k_D|V_{ei}'|/\Omega_i)-I(2k_D|V_{ei}'|/\Omega_e)]-|V_{ei\|}|\,I(2k_D|V_{ei\|}|/\Omega_i)\} \tag{5}$$

where $|\boldsymbol{V}_{ei}|=|\boldsymbol{p}_e/m_e-\boldsymbol{p}_i/m_i|$, $|V_{ei}'|=(p_{i\perp}^2/m_i^2+V_{ei\|}^2)^{1/2}$, $V_{ei\|}=p_{e\|}/m_e-p_{i\|}/m_i$, $\lambda=k_O/k_D$, k_O^{-1} and k_D^{-1} being the Landau and Debye lengths respectively, $k_O\gg k_D$, $z^{e,i}=e^4/\varepsilon_0^2$, and the function $I(x)$ has now the form

$$I(x)=(\lambda^2+1)(\lambda^2-1)^{-1}[\mathrm{Ei}(-x)-\mathrm{Ei}(-\lambda x)+\mathrm{e}^{-x}/x-\mathrm{e}^{-\lambda x}/\lambda x]- \\ -2(\lambda^2-1)^{-1}x^{-2}(\mathrm{e}^{-x}-\mathrm{e}^{-\lambda x})-(\lambda-1)^2(\lambda+1)^{-1}(\lambda x)^{-1}. \tag{6}$$

Substituting Eqs. (4) to (6) into Eq. (3) we get

$$\tau_{e,i}^{-1} = \tau_*^{-1}\Big\{\sigma^3(1+\sigma^2)^{-3/2}\ln\lambda + \sigma(1+\sigma^2)^{-3/2}\times$$
$$\times\Big[\int_0^\infty dz\, e^{-z^2} I(\varrho_e z)[6z^{-3}(2-4\sigma^2-\sigma^4) - 4z^{-5}(7-4\sigma^2-\sigma^4) + 8z^{-7}] +$$
$$+\int_0^\infty dz\, e^{-z^2} I(\varrho_i z)[z(\sigma^2-2\sigma^4) - 4z^{-3}(1+4\sigma^2) + 4z^{-5}\sigma^2]\Big] +$$
$$+4\int_0^\sigma dx(1+x^2)^{1/2}\int_0^\infty dz\, e^{-z^2}[z^{-3} I(\tilde\varrho_i z) - (2z^{-7} - 7z^{-5} + 3z^{-3})\, I(\tilde\varrho_e z)]\Big\} \tag{7}$$

where

$$\tau_*^{-1} = \frac{2}{3}\pi^{-3/2} N_i e^4/\varepsilon_0^2 m_i m_e v_{T_e}^3, \tag{8}$$

$$\varrho_j = 2k_D v_{T_j}\Omega_j^{-1}(1+\sigma^2)^{1/2}, \quad \tilde\varrho_j = 2k_D v_{T_j}\Omega_j^{-1}(1+x^2)^{1/2} \quad (j = e, i). \tag{9}$$

The limiting values of $\tau_{e,i}$ can be obtained easily by means of the relations $\lim_{B\to\infty} I(x) \approx -\ln\lambda$, $\lim_{B\to 0} I(x) = 0$. Thus we get

$$\tau_{e,i}^{-1}(B=\infty) = \tau_*^{-1}\sigma^3(1+\sigma^2)^{-3/2}\frac{1}{2}\ln\lambda, \tag{10}$$

$$\tau_{e,i}^{-1}(B=0) = \tau_*^{-1}\sigma^3(1+\sigma^2)^{-3/2}\ln\lambda. \tag{11}$$

From Eqs. (10) and (11), it is clear that

$$\tau_{e,i}(B=0)/\tau_{e,i}(B=\infty) = \frac{1}{2}.$$

In the case of a plasma of identical-type particles, we can proceed in an analogous way. On the basis of the collision integral derived in [7], and the expressions for the temperature-relaxation time for a one-component plasma presented in [4], we again get the ratio $\tau_{e,e}(B=0)/\tau_{e,e}(B=\infty) = \frac{1}{2}$.

III. LONGITUDINAL DIFFUSION OF PARTICLES IN A PLASMA

As our starating point for the following calculation, we take the expression for the longitudinal diffusion coefficient of electrons in an electron-ion plasma. It has the form [2]

$$D_\parallel^{e,i} = -N_e(k_B T)^2/2\pi\int d\boldsymbol{P}_e f_e^M \int d\boldsymbol{P}_i f_i^M\, Q_{\parallel\parallel}^e \tag{13}$$

where $Q_{\parallel\parallel}^e$ is the corresponding element of the Q-dyadic [5], and f_j^M denotes the Maxwellian distribution of the particles of the type j. Taking into account that [5] $Q_{\parallel\parallel}^e = -\frac{1}{2} m_e^2\ ^2G^B/\ p_{e\parallel}^2$ we can rewrite $D_\parallel^{e,i}$ in the form

$$(D_\parallel^{e,i})^{-1} = -\pi m_e N_e^{-1}(k_B T)^{-3}\int d\boldsymbol{P}_e f_e^M \int d\boldsymbol{P}_i f_i^M (1 - p_{e\parallel}^2/m_e k_B T)\, G^B. \tag{14}$$

From using Eq. (5), we get

$$(D_\parallel^{e,i})^{-1} = D_*^{-1}\Big\{\frac{1}{3}\sigma^3(1+\sigma^2)^{-1/2}\ln\lambda - 2\sigma(1+\sigma^2)^{-1/2}\int_0^\infty dz\, e^{-z^2} I(\varrho_e z)\times$$
$$\times\Big[z^3(1+2\sigma^2) - \frac{2}{3}\sigma^2 z^5\Big] + \sigma(1+\sigma^2)^{-1/2}\int_0^\infty dz\, e^{-z^2} I(\varrho_i z)[2z^3 + \sigma^2 z] - \tag{15}$$
$$-2\int_0^\sigma dx(1+x^2)^{1/2}\int_0^\infty dz\, e^{-z^2} z^3 [I(\tilde\varrho_i z) - I(\tilde\varrho_e z)]\Big\}$$

where

$$D_*^{-1} = 2\pi^{-3/2} N_i e^4/\varepsilon_0^2 m_i m_e v_{T_e}^5 \tag{16}$$

$\sigma = (m_i/m_e)^{1/2}$, the function $I(x)$ and the expressions $\tilde\varrho_i$, $\tilde\varrho_e$ being given by Eqs. (6) and (9). Eq. (15) gives the longitudinal diffusion coefficient for arbitrary values of the magnetic field $\boldsymbol{B}$. If $\sigma = 1$, i.e. $e = i$, Eq. (15) gives the self-diffusion coefficient of a plasma of identical-type paraticles in a magnetic field.

From the limiting values of the function $I(x)$, we can find the limiting values of $D_\parallel^{e,i}$ (as well as $D_\parallel^{e,e}$). Thus we get the following results

$$1/D_\parallel^{e,i}(B=0) = \frac{1}{3} D_*^{-1}\sigma^3(1+\sigma^2)^{-1/2}\ln\lambda, \tag{17}$$

$$1/D_\parallel^{e,i}(B=\infty) = \frac{1}{2} D_*^{-1}\sigma^3(1+\sigma^2)^{-1/2}\ln\lambda. \tag{18}$$

Consequently,

$$D_\parallel^{e,i}(B=0)/D_\parallel^{e,i}(B=\infty) = \frac{3}{2}. \tag{19}$$

This is the same relation as that derived by Cohen and Suttorp for the case of a one-component plasma. Thus Eq. (19) extends the validity of the result of the paper [1] showing it also holds for two-component plasmas.

IV. CONCLUSION

We have shown that the ratio of the limiting temperature-relaxation times, and the ratio of the limiting longitudinal diffusion coefficients taken for a plasma in an external magnetic field in the limits $B \to 0$ and $B \to \infty$ obey the one-half and three-halves laws respectively, and are independent of the species of the particles. In addition, the limiting values are asymptotic ones under the condition $\varrho_i < \lambda^{-1}$, hence they can be used for a guiding centre plasma.

REFERENCES

[1] Cohen, J. S., Suttorp, L. G.: Physica 123*A* (1984), 549.
[2] Šesták, B., Forejt, L.: *XVII ICPIG*, Budapest (1985), 39.
[3] Šesták, B., Forejt, L.: Czech. J. Phys. *B*33 (1983), 1165.
[4] Šesták, B.: *XVI ICPIG* Düsseldorf (1983), 76.
[5] Šesták, B.: Czech. J. Phys. *B*32 (1982), 893.
[6] Šesták, B., Forejt, L.: Czech. J. Phys. *B*33 (1983), 149.
[7] Šesták, B.: *XII SPIG*, Šibenik (1984), 534.

Received August 6th, 1986
Revised version received November 14th, 1986

ПРЕДЕЛЬНЫЕ ЗНАЧЕНИЯ КОЭФФИЦИЕНТА ПРОДОЛЬНОЙ ДИФФУЗИИ И ВРЕМЕНИ ТЕМПЕРАТУРНОЙ РЕЛАКСАЦИИ ПЛАЗМЫ, НАХОДЯЩЕЙСЯ В МАГНИТНОМ ПОЛЕ

В работе приводятся явные выражения для коэффициента продольной дифузии D и времени температурной релаксации τ слабовзаимодействуюцей плазмы, находящейся в магнитном поле. Показано, что отношение $D_{\parallel}(B = 0)/D_{\parallel}(B = \infty)$ равно 1,5 и отношение $\tau(B = 0)/\tau(B = \infty)$ равно 0,5 Эти результаты имеют место как для однокомпонентной, так и для двухкомпонентной плазмы.