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MODELS OF PARTICLE COUNTERS WITH PROLONGING DEAD TIME[1])

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The models of modified particle counters with prolonging dead time are treated, and the following problems are solved: 1. The number of emitted particles arriving at the counter during the dead time. 2. The distribution of the time interval between two successive registered particles. 3. The joint distribution of the dead time and the successive idle period. 4. The approximative probability formulae.

МОДЕЛИ СЧЕТЧИКОВ ЧАСТИЦ С МЕРТВЫМ ВРЕМЕНЕМ ПРОДЛЕВАЮЩЕГОСЯ ТИПА

В работе изучаются модели модифицированных счетчиков частиц с удлиненным мертвым временем. При этом решаются следующие задачи: 1. число испускаемых частиц, попадающих на счетчик за период мертвого времени; 2. распределение временного интервала между двумя следующим друг за другом частицами; 3. совместное распределение мертвого времени и последовательного времени простоя; приближенные вероятностные формулы.

I. INTRODUCTION

One of the basic tools of physicists-experimenters in high energy physics are particle counters, i. e. counting devices designed to detect and record particles due to radioactive substances, and placed within the range of radioactive material.

There are no ideal particle counters. Due to inertia of the counting device it is possible that not all emitted particles will be counted. The time during which the device is unable to record (within the measuring process) is called the dead time.

The particle registration process has a stochastic character. Consider a sequence of random events consisting of the arrival of emitted particles. This sequence is

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called the primary process. We suppose that any arriving particle generates an impulse of a random length X (may be constant, too) which starts after arrival of the particle at the counter. The impulse is a reaction of the counter to a particle action, and the counter may register only if no particle impulse is present, that is when it is idle (i. e., able to record). The sequence of the registered particles forms a secondary process selected from the primary one, according to the type of the counter used.

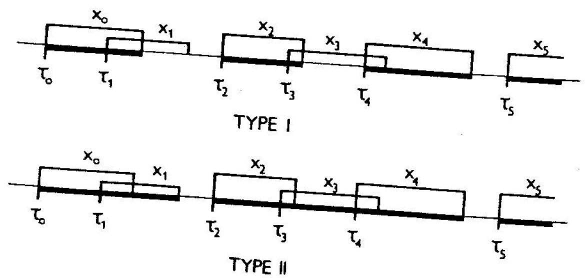

Fig. 1.

The mathematical [1—5] and physical [6—7] literature deals mainly with two types of particle counter models. A type I counter (counter with the non-prolonging dead time) is one in which the dead time is produced only after the impulses of particles have been registered. A type II counter (counter with prolonging dead time) is one in which the dead time is produced after the registration of all impulses of the emitted particles. Examples of type I and type II counters are the Geiger-Müller counters, and the electron multipliers or scintillation counters.

Let us suppose that the particles arrive at the counter at the instances $0=\tau_0<\tau_1<\ldots<\infty$ and let us the registration process start from $t=0$. Denote by X_n the duration of impulse starting at τ_n $(n=0, 1, 2, \ldots)$. If the nth particle is registered, then the dead time starting at τ_n is equal to (*i*) X_n, for the type I counter; to (*ii*) $\max\{X_n,\ \tau_{n+1}-\tau_n+X_{n+1}\tau_{n+2}-\tau_n+X_{n+2}, \ldots\ \tau_{k(n)-1}-\tau_n+X_{k(n)-1}\}$, where $k(n)$ is the subscript of the successive registered particle, so that $k(n)=\min\{k\colon k>n,\ \tau_k>\tau_r+X_r,\ r=n, \ldots, k-1\}$, for the type II counter. The diagrams below illustrate the situation.

Here the time intervals with full lines denote the dead times for the corresponding types of counters.

In the following we shall deal exclusively with the counters with the prolonging dead time.

In many important cases the primary process of the emitted particles is a Poisson process, the secondary process of registered particles, however, is not a Poisson one [1, 2]. In physical practice the repeated handling of particles by several counters

causes the initial process inputing at each successive counter to form a more general process than a Poisson one. Therefore, one of the important problems of the counter theory is to determine the characteristics of the secondary process when the primary one is known. In other words, it is necessary to determine the distribution function of the time interval between two successive registered particles.

Usually it is supposed that interarrival times $T_n=\tau_n-\tau_{n-1}$, $n=1, 2, \ldots$, are assumed to be independent, identically distributed random variables with the distribution function $F(x)=P(T_n<x)$, $n\geq 1$, and independent of the impulse lengths, $\{X_n\}_{n=0}^{\infty}$, which are assumed to be independent, identically distributed random variables with the distribution function $H(x)=P(X_n<x)$, $n\geq 0$.

For the modified counter with the prolonging dead time we shall assume that any registered particle has the distribution function of an impulse which is different, in general, from the distribution functions of the impulses of nonregistered particles assumed to be identically distributed. This situation corresponds to the natural physical observation that actions of the registered and the nonregistered particles at the counter may be different. Some basic properties of more general types of counters with the prolonging dead time are studied in [8].

In the present contribution we deal with the following problems arising in practice where counters with the prolonging dead time are used:

1. The number of the emitted particles arriving at the counter during the dead time.

2. The distribution of the time interval between two successive registered particles.

3. The joint distributon of the time interval between two successive registered particles.

3. The joint distribution of the dead time and the successive idle period (i. e. the time interval when the counter is able to record) and the exact solution of this problem for the discrete case of distribution.

4. The approximative probability formulae for the cases 1 and 2.

II. EXAMPLES OF COUNTER THEORY APPLICATIONS IN HIGH-ENERGY PHYSICS

Here we show that the theory of counters with the prolonging dead time may be applied to some other actual problems of high-energy physics.

II.1. Grain counting in photoemulsions

Along the ionizing particle trajectory in an emulsion the δ-electrons are distributed according to a Poisson process. The grains form blobs (a simple grain is a blob, too). The measurement of the ionization density caused by particles reduces to the measurement of the number of blobs and gaps between them along the particle trajectory. The diameter of the grain plays the role of the impulse, the left-hand sides of the grain is the arrival time, then the blob is the dead time and the gap the ideal, time. We note that the final measurement problem is not the determination of the blob number but one of the grain number in the trajectory [6, p. 173].

II.2. Streamer track density

This problem arises when we wish to describe blob-length measurement in streamer chambers in high-energy physics. In the known models [9—11] the streamers are described as circles having centres on the trajectory, and the number of centres is distributed according to a Poisson process. Interpreting the left-hand intersections of the circles with the trajectory as the arrival times, and the diameters as the impulse lengths, we obtain the counter with the prolonging dead time. The same models, but with constant diameters, arise in the bubble chambers [9—12].

II.3. Automatic ionization measurement

Due to a scanning apparatus, the experimental data on the blob-length measurements have discrete values [9, 10]. "The particle arrivals" are distributed according to the geometric law and the main problem is the determination of the discretized blob-length (= discrete dead time). This task has been solved in [13].

We note that similar problems (from the mathematical point of view) arise in many fields of science and techniques activity, the number of molecules in a fixed region of gas under conditions of low temperature, communication channels, queueing theory, etc.

III. Number of particles

Our assumption is that the distribution functions of the impulses of registered and nonregistered particles may be, in general, different in dependence whether the counter is busy or not. We assume that the modified counter with the prolonging dead time (shortly modified counter) is a triple $\eta=(F;\ H,\ H^*)$, where F is the distribution function of the interarrival times, H and H^* are distribution functions of impulses of registered and nonregistered particles, respectively. When $H=H^*$, then η is usual (non-modified) counter.

As it has been shown in II.1 the humber of grains along the trajectory is an important physical quantity. Therefore it is interesting to know the number

distribution of the grains in a blob. So let ν be the number of the particles arrived at the counter $\eta=(F;\ H,\ H^*)$ during the dead time.

Let us suppose that the dead time, say B, is formed by the sequences $\{X_n\}_{n=0}^{\infty}$ and $\{T_m\}_{n=1}^{\infty}$, where the sequence of impulse lengths is a sequence of independent random variables with $H(x)=P(X_0<x)$ and $H^*(x)=P(X_n<x)$, $n\geq 1$, and independent of the interarrival times $\{T_n\}_{n=1}^{\infty}$. If we put $A_n=\{X_0<T_1+\ldots+T_n,\ X_1<T_2+\ldots+T_n,\ \ldots,\ X_{n-1}<T_n\}$, $n\geq 1$, then, for $P_n=P(\nu=n)$, we have

$$P_n=P(\bar{A}_1\ \ldots\ \bar{A}_{n-1}A_n),\quad n\geq 1, \tag{III.1}$$

where $\bar{A}$ denotes the negation of A.

For our aims it is useful to introduce an integer-valued random variable, ν^*, defined as the number of the particles arrived at the counter $\eta^*=(F;\ H^*,\ H^*)$. Put $A_n^*=\{X_1<T_2+\ldots+T_{n+1},\ X_2<T_3+\ldots+T_{n+1},\ \ldots,\ X_n<T_{n+1}\}$, $n\geq 1$. Then, for $\{A_n\}_{n=1}^{\infty}$ and $\{A_n^*\}_{n=1}^{\infty}$, we have the following: if $1\leq i_0<i_1<\ldots<i_j$, $j\geq 1$, then

$$P(A_{i_1}\ \ldots\ A_{i_n}\mid A_{i_0})=P(A^*_{i_1-i_0}\cdot A^*_{i_n-i_0}) \tag{III.2}$$

and, for $P_n^*=P(\nu^*=n)$, we have $P_n^*=P(\bar{A}_1^*\ \ldots\ \bar{A}_{n-1}^*A_n^*)$, $n\geq 1$. Hence

$$P_1=P(A_1),$$

$$P_n=P(A_n)-\sum_{j=1}^{n-1}P(A_j)P_{n-j}^*,\quad n\geq 2 \tag{III.3}$$

where

$$P(A_n)=\int_0^{\infty}\ldots\int_0^{\infty}H(t_1+\ldots+t_n)\ H^*(t_1)\ldots H^*(t_1+\ldots t_{n-1})$$

$$\mathrm{d}F(t_1)\ \ldots\ \mathrm{d}F(t_n),\quad n\geq 1. \tag{III.4}$$

The probabilities P_n^* and $P(A_n^*)$ may be easily computed from (III.3) and (III.4) changing A_n to A_n^* and P_n to P_n^*, that is, we consider the counter for which $H=H^*$.

It may be shown that there are the limits $\lim_n P(A_n)$ and $\lim_n P(A_n^*)$, and they are identically equal to some $p\geq 0$. If we put

$$\Psi(z)=P(A_1)z+\sum_{n=2}^{\infty}(P(A_n)-P(A_{n-1}))z^n$$

$$\Psi^*(z)=P(A_1^*)z+\sum_{n=2}^{\infty}(P(A_n^*)-P(A_{n-1}^*))z^n,$$

then

$$M(\nu)=(\Psi'(1)-\Psi^{*\prime}(1)+1)/p$$

$$M(\nu^*)=1/p, \tag{III.5}$$

where $M(.)$ denotes the mean value of a random variable.

In particular, if $F(x)=1-e^{-\lambda x}$, $x\geq 0$, i. e., the primary process is a Poisson one, and $D=\int_0^\infty t\,dH^*(t)<\infty$, then

$$p=e^{-\lambda D}. \tag{III.6}$$

IV. THE SECONDARY PROCESS

It has been noted that if the half-decay time is sufficiently large, that is, when the primary process is a homogeneous Poisson process, then the process of the registered particles due to the counter is not a Poisson one. However, these particles may be handled by the successive counters. Therefore it is important to know the secondary process stochastic properties.

Here we determine the characteristics of the output process for the general modified counter $\eta=(F;\ H,\ H^*)$.

This problem has been solved by several authors. A particular case (as the primary process is a Poisson one) has been solved in [15]. As it has been mentioned in [1, 16, 17] the determination of the secondary process is an extremely difficult problem. However, there are the integral equations [5, 17] which formally, but not always in practice, determine it. Pollaczek [18] has solved the general case of the non-modified counter only in the form of complicated counter integrals. This problem in the explicit form has been solved by authors in [19] for the counter $(F;\ H,\ H^*)$. Define $a(s)=\int_0^\infty e^{-sx}\,dF(x)$, $s\geq 0$ ($a(s)$ is the Laplace transform of F), and determine, for any $s\geq 0$, a new distribution function $F_s(x)=a(s)^{-1}\int_0^x e^{-st}\,dF(t)$.

The modified counter $\eta_s=(F_s;\ H,\ H^*)$ determined a v_s, i. e. the number of emitted particles during the dead time of the counter η_s. Let $f_s(z)=\sum_{n=1}^{\infty} P(v_s=n)z^n$, $|z|<1$, be the generating function of v_s. Then, for $\Phi(s,z)=M(e^{-sZ}z^v)$, $s\geq 0$, $|z|<1$, where Z is the time interval between two successive registered particles (we recall, that all Z' s are independent, identically distributed random variables, we have, due to $Z=\tau_v$

$$\Phi(s,z)=\sum_{n=1}^{\infty}\int_{(v=n)} e^{-s\tau_v}z^v\,dP=\sum_{n=1}^{\infty}\int\underset{C_n}{\dots}\int e^{-s(t_1+\dots+t_n)}z^n$$

$$dF(t_1)\dots dF(t_n)\,dH(x_1)\,dH^*(x_2)\dots dH^*(x_n),$$

where the integration area C_n has the following form

$$(x_1<t_1)^c,\quad \begin{pmatrix}x_1<t_1+t_2\\ x_2<t_2\end{pmatrix}^c,\ \dots,\ \begin{pmatrix}x_1<t_1+\dots+t_{n-1}\\ \vdots\\ x_{n-1}<t_{n-1}\end{pmatrix}^c$$

$$\begin{pmatrix}x_1<t_1+\dots+t_n\\ \vdots\\ x_n<t_n\end{pmatrix}.$$

(here the superscript „c“ denotes the complement of the set mentioned in the parentheses). Hence

$$\Phi(s,z)=\sum_{n=1}^{\infty} a(s)^n z^n\,P(v_s=n)=f_s(a(s)z), \tag{IV.1}$$

especially

$$M(e^{-sZ})=f_s(a(s)),\quad s\geq 0. \tag{IV.2}$$

Due to a one-to-one correspondence between the distribution functions and their Laplace transforms, the converse Laplace transform of (IV.2) gives us the distribution function of Z. For the mean value of Z we obtain from (IV.2) as follows

$$M(Z)=\mu M(v), \tag{IV.3}$$

where $\mu=\int_0^\infty t\,dF(t)$ is assumed to be finite.

For example, if $F(x)=\lambda^\alpha/\Gamma(\alpha)\,e^{-\lambda x}x^{\alpha-1}$(i. e., $F(x)$ is the Gamma distribution with the parameters $\alpha\geq 1$ 1 and $\lambda>0$), then $F_s(x)$ is the Gamma distribution with the parameters α and $\lambda+s$.

V. DEAD TIME AND IDLE PERIOD

The dead time distribution is known only in particular cases. Takács [5, 15] has derived it for a Poisson process of emitted particles. In [6] there is the formula for the case when the inpulse lengths do not fluctuate. In these cases the dead time, B, and the successive idle period, I (the time interval when the counter is able to record) are independent random variables, and, moreover, the idle period is distributed according to exponential law with the same parameter λ as a Poisson primary process. In general case they are dependent random variables.

In the first part of this section we derive the integral equation for the joint distribution of the dead time and the successive idle period, so that we shall study $W(z,u)=P(B<z,\ I<u)$ for the modified counter $\eta=(F;\ H,\ H^*)$. In the second part the precise solution to the integral equation (IV.1) will be given for the discrete modified counter (for the definition of that counter see below).

For the counter $\eta^* = (F;\ H^*,\ H^*)$ we define the dead time, B^*, the idle period, I^*, and $W^*(z,\ u) = P(B^* < z,\ I^* < u)$.

The event $\{B < z,\ I < U\}$ is the union of two disjoint events A_1 and A_2, where $A_1 = \{B < z,\ I < u,\ X_0 < T_1\}$ and $A_2 = \{B < z,\ I < u,\ X_0 \geq T_1\}$. Clearly $P(A_1) = \int_0^z (F(y+u) - F(y+))\,dH(y)$.

Under the condition $\{0 < x < T_1 \leq X_0 = y < z\} = C$ say, $P(A_2|C) = P(y - x \leq B^* < z - x) + \sum_{r=1}^{\infty} P(Z_1^* + \ldots + Z^* < y - x \leq Z_1^* + \ldots + Z_r^* + B_r^* < z - x)$, where Z_k^* is the time interval between the k-1st and kth particles that have been registered, and analogically we define B_k^*. Hence using the probabilistic arguments we may show that

$$W(z,\ u) = \int_0^z (F(y+u) - F(y+))\,dH(y) + \int_0^z \int_x^z \int_0^{y-x} (W^*(z - x - t,\ u) - W^*(y - x - t,\ u))\,dN^*(t)\,dH(y)\,dF(x), \quad z \geq 0,\ u \geq 0, \tag{V.1}$$

where N^* is a renewal function of Z^*, that is, $N^*(t) = \sum_{n=0}^{\infty} G_n^*(t)$, where G^* is the distribution function of Z^* and G_n^* denotes the nth convolution of G^* with itself.

Using the result of [21] we may show that, for the modified counter $\eta = (F;\ H,\ H^*)$ we have

$$P(I < t) = 1 - p_t/P, \tag{V.2}$$

$$M(I) = p^{-1} \int_0^{\infty} p_t\,dt, \tag{V.3}$$

$$M(B) = (\mu - \int_0^{\infty} p_t\,dt)\ M(v), \tag{V.4}$$

where $p_t = \lim_n \int_0^{\infty} \ldots \int_0^{\infty} H(t_1 - t) \ldots H(t_1 + \ldots + t_n - t)\,dF(t_1) \ldots dF(t_n)$.

The solution of (V.1) is known only in special cases, for example, when the input process of the emitted particles is a Poisson one. In example II.3 we have seen that there are „counters" with discrete values only.

We assume that the particles arrive at the counter at the discrete values of time $h,\ 2h, \ldots,$ and impulse lengths may have values $h,\ 2h, \ldots,$ where $h > 0$. This modified counter is said to be a discrete modified counter. For this counter we may give the exact solution to the integral equation (V.1).

Suppose $h = 1$ and $f(n) = P(T_1 = n)$, $h(n) = P(X_0 = n)$, $h^*(n) = P(X_1 = n)$, $n \geq 1$. Let $W(n,\ m) = P(B = n,\ I = m)$, $W^*(n,\ m) = P(B^* = n,\ I^* = m)$ for any $n,\ m \geq 1$. Then

$$W(n,\ m) = \sum_{j=1}^{n} W(n,\ j,\ m), \tag{V.5}$$

where $W(n,\ j,\ m) = P(B = n,\ X_0 = j,\ I = m)$. Using the simple probabilistic arguments we may obtain

$$W(1,\ 1,\ m) = h(1)\ f(1+m), \tag{V.6}$$

$$W(2,\ 1,\ m) = h(1)\ f(1)\ W^*(1,\ m), \tag{V.7}$$

$$W(2,\ 2,\ m) = h(2)\ (f(m+2) + f(1)\ W^*(1,\ m)).$$

If $n \geq 3$, then

$$W(n,\ 1,\ m) = h(1)\ f(1)\ W^*(n-1,\ m). \tag{V.8}$$

recursively we obtain, for $2 \leq j \leq n-1$,

$$W(n,\ j,\ m) = h(j) \sum_{i=1}^{j} f(i)\ A(n,\ j,\ m,\ i), \tag{V.9}$$

where $A(n,\ j,\ m,\ i) = P(B = n,\ I = m | X_0 = j,\ T_1 = i)$. Therefore

$$A(n,\ j,\ m,\ i) = \sum_{r=1}^{\left[\frac{i-j}{2}\right]} B(n,\ j,\ m,\ i,\ r), \tag{V.10}$$

where $B(n,\ j,\ m,\ i,\ r) = P(Z_1^* + \ldots + Z_{r-1}^* \leq j - i,\ \ Z_1^* + \ldots + Z_{r-1}^* + B_r^* = n - i)$, here $[x]$ denotes the integer part of a real x. Then

$$B(n,\ j,\ m,\ i,\ r) = \Sigma\ W^*(j_1,\ t_1) \ldots W^*(t_{r-1},\ j_{r-1})\ W^*(j_r,\ m), \tag{V.11}$$

where the summation is taken over the integers $j_s,\ t_s \geq 1\ (s = 1, \ldots,\ r-1)$, $j_r \geq n - j$, with $j_1 + t_1 + \ldots + j_{r-1} + t_{r-1} + j_r = n - i$.

For $j = n$, in an analogous way as above, we have

$$W(n,\ n,\ m) = h(n) \sum_{i=1}^{n-1} f(i)\ A(n,\ n,\ m,\ i), \tag{V.12}$$

where $A(n,\ n,\ m,\ i) = P(B = n,\ I = m,\ X_0 = n,\ T_1 = i)$. Hence

$$A(n,\ n,\ m,\ i) = \sum_{r=1}^{(n-i+1)/2} B(n,\ n,\ m,\ i,\ r), \tag{V.13}$$

where $B(n,\ n,\ m,\ i,\ r)$ has a similar meaning as $B(n,\ j,\ m,\ i,\ r)$ in (V.10). Therefore

$$B(n,\ n,\ m,\ i,\ r) = \sum W^*(j_1,\ t_1) \ldots W^*(j_{r-1},\ t_{r-1})\ W^*(j_r,\ m + t_r), \tag{V.14}$$

here $j_s,\ t_s \geq 1 (s = 1, \ldots,\ r-1)$, $j_r \geq 1$, $t_r \geq 0$, with $j_1 + t_1 + \ldots + j_r + t_r = n - i$.

We see that the formulae (V.5—V.14) give the joint distribution of the dead time and the successive idle period for the discrete modified counter. We recall that the expressions for $W^*(n, m)$ may be evaluated from (V.5—V.14) when the change $W(n, m)$ to $W^*(n, m)$, or, equivalently, when $h(n) = h^*(n)$ for any $n \geq 1$.

The more simple counter corresponding to actual automated measurement described in Example II.3 has been solved in [13] using the method different from that described in (V.5—V.14).

VI. APPROXIMATIVE FORMULAE

In some practical aspects of the application of the above theory of the modified counters it is necessary to know only the numerical estimates of the above formulae. In this section we present some computationally convenient approximative formulae, and some limit estimates under the condition of a large intensivity of the emitted particles will be given.

For the numeber of the emitted particles arriving at the counter during the dead time we have established the formula (III.3). When we have that $H^*(t) \leq H(t)$, $t \geq 0$, $\int_0^\infty H^*(t)\, dF(t) > 0$ and $\sup\{\mu \geq 0 : \int_0^\infty e^{\mu t}\, dH^*(t) < \infty\} = \infty$ (for example, if H^* is the distribution function of the constant impulse length, then this is true), then

$$P_n = \beta_1 \beta_2 \beta^{-n-1} + r_n, \quad n \geq 1 \qquad \text{(VI.1)}$$

where

$$\beta = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} \frac{d^k}{dz^k} [\Psi^{*k}(z)]_{z=1} \qquad \text{(VI.3)}$$

$\beta_2 = \Psi(\beta)$ (if $H = H^*$, then $\beta_2 = \beta - 1$), and $|r_n| \leq CR^{-n}$ (the constant C does not depend on n, and $R > 1$). The proof of (VI.1) may be outlined as follows. Let $P(z) = \sum_{n=1}^{\infty} P_n z^n$, $|z| < 1$. Then $P(z) = \Psi(z)/(1 - z + \Psi^*(z))$ and, due to the Cauchy formula

$$P_n = \frac{1}{2\pi i} \oint_{|z|=1} P(z)/z^{n+1}\, dz.$$

From the condition it follows that there is $R > 1$ such that

$$1 - z + \Psi^*(z) = 0 \qquad \text{(VI.4)}$$

has a unique root β, $R > \beta > 1$. If we put

$$r_n = \frac{1}{2\pi i} \oint_{|z|=R} \frac{\Psi(z)\, dz}{(1 - z + \Psi^*(z))\, z^{n+1}} = \frac{\Psi(\beta)}{(\Psi^{*\prime}(\beta) - 1)\beta^{n+1}} + P_n.$$

The integral on the left-hand side may be estimated by the maximum module $|r_n| \leq CR^{-h}$. Putting $\beta_1 = 1/(1 - \Psi^{*\prime}(\beta))$ and $\beta_2 = \Psi\beta)$ we obtain (VI.1).

To obtain the explicit expression for β and β_1, respectively, we consider a function $w = z - \Psi^*(z)$ which in a conform way transforms some neighbourhood of the point $w = 1$ to another of $z = \beta$. Therefore $w = w(z)$ has its inverse function $z = z(w)$. It is clear that $\beta = z(1)$ and $\beta_1 = z'(1)$. Using the Lagrange expansion formulae [20] we obtain (VI.2) and (VI.3).

Here we note that the root β of the equation (VI.4) may be evaluated more effectively using the Newton approximation method. In fact, if suffices to take into account the form of (VI.4). Then for β_1, we have $\beta_1 = 1(1 - \Psi^{*\prime}(\beta))$.

Example. In the Table 1 we give a numerical example of the aplication of (VI.1) to the counter $\eta = (F; H, H)$, where F is the distribution of the constant equal 1, $H(t) = 1 - e^{-t}$, $t \geq 0$, and β and β_1 are evaluated by the Newton method: $\beta = 2.515773$, $\beta_1 = 2.338680$.

Table 1

n	P_n	$\beta_1\beta_2\beta^{-n-1}$	n	P_n	$\beta_1\beta_2\beta^{-n-1}$
1	6.3212-01	5.6010-01	6	5.5578-03	5.5578-03
2	2.2097-01	2.2263-01	7	2.2092-03	2.2092-03
3	8.8531-02	8.8500-02	8	8.7814-04	8.7814-04
4	3.5175-02	3.5176-02	9	3.4905-04	3.4905-04
5	1.3982-02	1.3982-02	10	1.3875-04	1.3875-04

From the Table 1 we may see that formula (VI.1) yields a very precise estimate for P_n even for small n, so that we have $P_n \approx \beta_2 \beta_1 \beta^{-n-1}$.

In the following we note that if the emitted particles are distributed according to a Poisson process, and the intensivity λ is very large, then

$$P(v/M(v) > t) \approx e^{-t},$$

$$P(Z/M(Z) > t) \approx e^{-t},$$

$$P(B/M(B) > t) \approx e^{-t},$$

for any $t > 0$. The conditions which this guarantee are the following: $\int_0^\infty t^2\, dH^*(t) < \infty$ and $H^*(+0) = 0$. The proof is not present; it is based on the methods developed in [21].

In the figure below we present: a) the dead time density function of the counter $\eta=(F;H,H)$, where $F(t)=1-e^{-\lambda t}$, $t\geq 0$, and H is the distribution function of the constant equal 1; b) the dead time distribution evaluated by (V.5—V.13) for a discretization of a Poisson process, for details see [13] for a discretization of a Poisson process, for details see [13]. In both cases the limit exponential law holds for $\lambda \to \infty$.

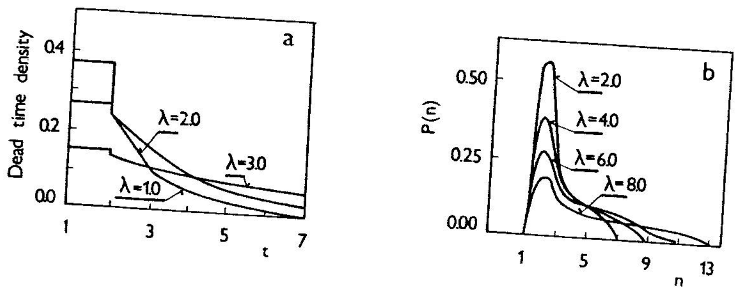

Fig. 2.

VII. CONCLUSION

In the present contribution there has been made a survey of problems which appear in physical practice and which are being solved at the Joint Institute for Nuclear Research, Dubna. These problems are very interesting in both aspects — the physical and the mathematical, and they have a wide variety of applications in divers fields.

In the present work we have treated only ideal cases and we do not take into account a large number of physical effects which, for example, arise in the measurement of the ionization density in streamer chambers (Example II.2): optical distortions, beam track at an angle to the film planes, confusion due to crossing tracks, flares, etc.

From the above survey one may be able to appreciate the significant success that has been achieved so far and the great work that lies ahead before one could say that the problem of the counter theory has been solved completely. As further interesting problems may be mentioned the following: the registration of particles from many sources, the explicit solution to the integral equation (IV.1), etc.

The amount and variety of deep mathematical knowledge that is needed to solve these problems are surprising. Further developments will be conditioned by the progress in mathematics.

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