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THE USE OF VERTICAL CUT-OFF RIGIDITIES IN COMPUTATIONS OF COSMIC RAY SPECTRA ON THE EARTH'S ORBITS

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Simplified assumptions for computing spectral transmission probabilities of cosmic rays into the geomagnetic field are used to estimate their flux and energy spectra on low altitude orbits. For this purpose three types of planetary tables of vertical cut-off rigidities are used and two approaches for their extrapolations from the Earth's surface to the altitude of satellite. Comparison of the results leads to errors not sufficient to estimate the radiation exposure by cosmic rays influenced by the geomagnetic field at low altitudes.

ИСПОЛЬЗОВАНИЕ ВЕРТИКАЛЬНЫХ ЖЕСТКОСТЕЙ ОБРЕЗАНИЯ В РАСЧЕТАХ СПЕКТРОВ КОСМИЧЕСКИХ ЛУЧЕЙ НА ОКОЛОЗЕМНЫХ ОРБИТАХ

Для расчета спектральных функций пропускания космических лучей в геомагнитное поле использованы упрошаюшие предположения с целью вычислить их поток и энергетические спектры на малых высотах. Для этой цели были использованы три типа планетарных таблиц вертикальных жесткостей обрезания и два подхода для их экстраполяции от поверхности Земли до высоты спутника. Сравнение результатов приводит к ошибкам, которые несущественны для цели вычисления радиационной экспозиции космическими лучами с учетом влияния влиянием геомагнитного поля на малых высотах.

I. INTRODUCTION

In computations of radiation conditions on the orbits of space vehicles in the near-Earth environment it is necessary to account for the influence of the geomagnetic field on the motion of energetic particles coming from the interplanet-

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ary space to the surface of the Earth [1]. From the given direction, the charged particle is not able to impact on the Earth from outer space if its magnetic rigidity R for the direction mentioned is lower than the geomagnetic cut-off rigidity for that point.

Because of the complicated structure of allowed and not allowed rigidities, as for instance in [2—5], it is not simple to obtain the net flux of the cosmic ray particles impacting on the vehicle in flight on a given orbit. For practical purposes, however, for instance in estimations of the integral flux of cosmic rays causing defects in electronic circuits in space experiments or in manned flights, the need for the computation of an expected flux and energy spectra of particles is evident.

In the paper presented here we adopt a simplified picture of such computations. The change of the external source with given energy spectra could be characterized by the spectral function of transmittance $\tau(R, \mathbf{r})$, where R is the rigidity and $\mathbf{r}$ the radius vector of the point of measurement. The significance of this quantity is the probability to find the particle with the magnetic rigidity R in the point if the particles with such rigidity are impacting isotropically onto the magnetospheric boundary. Further only vertical cut-off rigidity is used as the characterizing rigidity, the dipolar model for simplicity is incorporated and temporal integration of the flux along the orbit is performed.

II. BASIC CONSIDERATIONS

During the time interval T, the energy spectra of particles impacting on the orbit lying in the geomagnetic field, we have

$$\frac{dN}{dR_{orb}} = \frac{1}{T} \int_0^T \frac{dN}{dR} \tau(R, \mathbf{r})\, dt \tag{1}$$

where dN/dR is the energy spectrum of the source (particles in interplanetary space); $\tau(R, \mathbf{r})$ is the spectral function of transmittance for the moment t giving the $\mathbf{r} = \mathbf{r}(t)$ and $\mathbf{r}$ is the radius vector of the vehicle.

If the spectra impacting onto the magnetosphere could be taken as constant during the interval T, which is fulfilled for instance for galactic cosmic rays, then (1) has the form

$$\frac{dN}{dR_{orb}} = \frac{dN}{dR} \tau'_{orb}(R) \tag{2}$$

where $\tau'_{orb}(R)$ is the spectral function of transmittance averaged during the time T of the space vehicle motion along its orbit. It characterizes the transmittance of the charged particles onto the orbit. In a general case $\tau(R, \mathbf{r})$ should be computed on the basis of complicated trajectory tracing of particles in the geomagnetic field. However, for practical puposes it is possible to determine the estimated values of τ

on the basis of tables of planetary distributions of vertical cut-off rigidities [6—8]. In such case τ has the form

$$\tau(R, \mathbf{r}) = \begin{cases} 1 & \text{for } R > R_{vert} \\ 0 & \text{for } R \leq R_{vert} \end{cases} \tag{3}$$

where $R_{vert}(\mathbf{r})$ is the vertical cut-off rigidity for the point where the space vehicle is situated at the given moment.

In general three types of errors would exist: a) the use of only vertical cut-off rigidities while not all allowed directions of the particle arrival are considered; b) errors originating from the extrapolation of vertical cut-off rigidities to the altitudes of the satellite; c) errors of computation of the vertical cut-off rigidities table and of interpolation from the lattice points to the point of measurement.

In the present paper we are trying to estimate the possible interval of errors of the first two types.

The function $\tau_{orb}(R)$ for the instantaneous orbit depends on the longitude (in the GSE coordinates) of crossing the equator because of the tilt angle of the dipolar moment to the Z axis of this coordinate system. To suppress these effects, computations of the function $\tau_{orb}(R)$ were carried out for the interval T equal to one day. Errors caused by replacing the allowed cones of particle acceptance with vertical cut-off rigidities were estimated for the case of the axially symmetric magnetic field on the Earth, which allows to approximate functions $\tau(R, \mathbf{r})$ analytically.

For the dipolar case the function $\tau(R, \mathbf{r})$ can be described by the expression

$$\tau(R, \mathbf{r}) = \frac{1}{2}(1 + \cos \omega(R, \mathbf{r})) \tag{4}$$

where the angle ω — the angular aperture of Störmer cone characterizing the forbidden directions of particles — is obtained from the expression of cut-off rigidity obtained by Störmer in [9]. According to this approximation the cut-off rigidity for particles at geomagnetic latitude Λ at the point $\mathbf{r}$ is obtained by the formulae

$$R = 59.3 \frac{r_e^2}{r^2} \frac{\cos^4 \Lambda}{(1 + \sqrt{1 - \cos \omega \cos^3 \Lambda})^2} \text{ [GV]} \tag{5}$$

where r_e is the Earth's radius. The angle ω is measured with respect to the direction east-west.

III. RESULTS

Computations of spectral functions of transmittance in dipolar approximation on the basis of vertical cut-off rigidities were performed applying (3) where R was computed according to (5) for $\omega = 90°$. The procedure is similar to that used in [12].

The results of computation of the function $\tau_{orb}(R)$ for the circular orbit at the altitude of 20 km and inclination $i = 52°$ are given in Fig. 1. It can be seen that the use of vertical cutoff rigidities leads in the interval below 12 GV to an over--estimation of the spectral function of transmittance by less than 10 %. For rigidities above 12 GV the errors increase and reach 16 % for 14 GV. In such cases in computations of spectra of galactic as well as solar cosmic rays for practical purposes the use of vertical cutoff rigidity leads to an error of the order of 10 %.

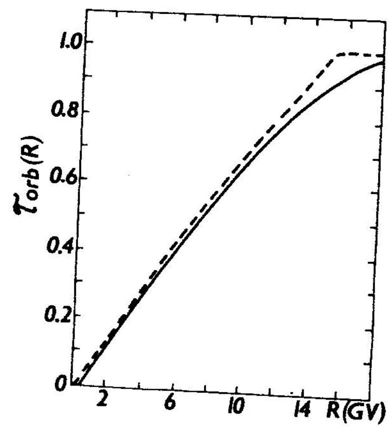

Fig. 1. Spectral functions of transmittance τ_{orb} and their dependence on the rigidity R for the circular orbit with the altitude 200 km and inclination 52°. The full line corresponds to the computations with the assumption of the main cone of acceptance; the dashed line corresponds to the vertical cutoff rigidity. Computations are made for the dipolar approximation of the geomagnetic field.

For the estimation of spectral functions of transmittance an their dependence on the used tables of vertical cut-off rigidities the computations of τ on the basis of tables taken from papers [6—8] were performed together with rigidities given by Störmer's formula (5).

Tables from paper of Quenby and Webber [6] were obtained by analytical methods assuming the corrections for non-dipolar components of the geomagmetic field. In the paper of Quenby and Wenk [7] for the computation of cut-off rigidities the approximate method of trajectory computations of particles in geomagnetic field by numerical methods was used. Shea and Smart [8] used a method determining the cut-off rigidities by numerical integration of the charged particle motion in a real geomagnetic field.

The results of computations of τ with the help of the tables mentioned above for the circular orbit at the altitude of 200 km and with an inclination angle 52° are presented in Fig. 2. It is apparent that the spectral functions of transmittance are practically identical whether we use the tables of Shea and Smart [8] or those of Quenby and Wenk [7]. The profiles of $\tau_{orb}(R)$ are also very similar for computations based on the tables of Quenby and Wenk [7] and on the Störmer

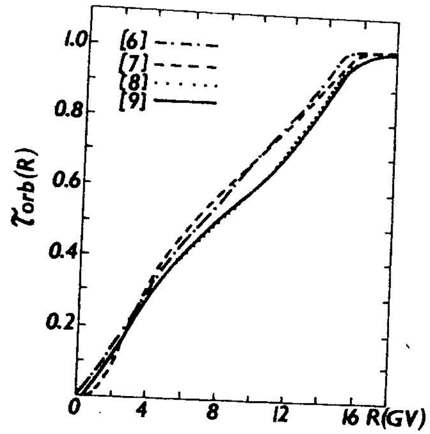

Fig. 2. Comparison of the function $\tau_{orb}(R)$ using different planetary tables of cut-off rigidities [6, 7, 8] and for the vertical cut-off rigidity according to equation (5) [9]. The orbital parameters are the same as those for Fig. 1.

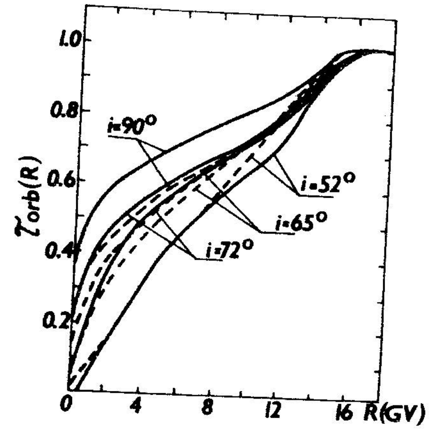

Fig. 3. Comparison of computed functions $\tau_{orb}(R)$ for the circular orbit $h = 200$ km for different angles of inclination. The full line is obtained by the approximation used in the program GKI [10] and the dashed line by the program ORBIT [11], respectively. For explanation see the text.

theory. The difference in the values of τ at worst are not higher than 20 % for inclinations of the orbit up to 60° and for altitudes up to 400 km.

The use of planetary tables of vertical cut-off rigidities, computed by the most accurate methods of numerical integration of charged particle trajectories, requires the re-calculation of cut-off rigidities from the Earth's surface to the altitude h of the orbit of the satellite. We have compared the results of computations of τ based on the use of the dependence $R(h) \sim (a+h)^{-2}$ and $R(h) \sim L^{-2}(h)$ where a is the radius of the Earth and L the McIlwain's parameter. The first method was used in the program called GKI [10] and the second in the program ORBIT [11]. In both programmes we have used the planetary table of cut-off rigidities given by Shea and Smart [8]. In Fig. 3 the spectral functions of transmittance for the circular orbit at the altitude of 200 km and inclination angles $i = 52°$, 65°, 72° and 90° respectively, are given. For the angles 52°, 65° and 72° the difference is not higher than 10 %. For the polar orbit $i = 90°$ the difference caused by the re-calculation of R from the Earth's surface to 200 km above it, reaches 20 %.

IV. CONCLUSIONS

Computations of the spectral functions of transmittance of cosmic rays carried out under simplified assumptions in the present work enable us to make the following main conclusions.

a) The use of tables of vertical cut-off rigidities leads to the over-estimation of the daily averaged function of transmittance for the near-Earth orbits for the assumed energy interval by not more than 10—15 %. Comparison of computations of τ, obtained with the help of different planetary tables of cut-off rigidities shows that the most suitable tables for this application are those of Shea and Smart [8] or those by Quenby and Wenk [7].

b) The application of different methods of re-calculation of cut-off rigidities from the Earth to the altitude of the orbit leads to differences of the daily averaged τ, which are not higher than 20 %.

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