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A QUASIOPTICAL CASCADE MODEL OF THE ^{4}He+p INTERACTION AT INTERMEDIATE ENERGIES

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The subject of the paper presented is a description of the quasioptical cascade model of the collision between the lightest nuclei and hadrons at intermediate momenta ($1.5 \div 6$ GeV/c per nucleon) with special attention devoted to the ^{4}He-proton interaction.

The production of a $\Delta(1232)$ isobar and its absorption in the collision with a proton as well as the Fraunhofer diffractive scattering on a semi-transparent nucleus are included into the model. The described model makes it possible to determine the total cross section of the interaction, the elastic scattering cross section as well as details of pionless fragmentation channels.

КАСКАДНАЯ КВАЗИОПТИЧЕСКАЯ МОДЕЛЬ ВЗАИМОДЕЙСТВИЯ ^{4}He+p ПРИ ПРОМЕЖУТОЧНЫХ ЭНЕРГИЯХ

В работе описана каскадная квазиоптическая модель столкновений между легчайшими ядрами и адронами для области промежуточных импульсов ($1,5 \div 6,0$ Гэв/с на нуклон) с особым вниманием к взаимодействию ^{4}He+p.

В модель включены продукция изобары $\Delta(1232)$ и ее поглощение в столкновении с протоном, а также дифракционное рассеяние Фраунгофера на полупрозрачном ядре. Указанная модель позволяет определить полное поперечное сечение взаимодействия, поперечное сечение упрогого рассеяния, а также детали каналов безпионной фрагментации.

I. INTRODUCTION

Cascade models are often used to describe collisions between hadrons and nuclei [6—9]. They are usually in good agreement with experimental data for intermediate and heavy nuclei interactions ($A \geq 12$), where the mean free path of the

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hadron in the nuclear matter is less than or at least comparable to the nucleus dimensions and where evaporation of particles from the nucleus is dominant in the last phase of interaction. In such a description these nucleons, which are not directly participating in the mentioned processes, are considered only as a part of the nucleus matter continuum — without specified individual characteristics. This kind of interaction is not directly applicable to the collision with the lightest nuclei. In this case the individual nucleons are correlated and one must take into consideration the role of those nucleons which do not directly participate in the collision cascade. Further, because of the small dimensions of nuclei, one must take into consideration also the impact parameter of the N—N interaction as it is done, e.g., in the Glauber model. The main difference between the Glauber model and that presented here is that in the Monte Carlo procedure the cascade process is calculated according to the rules of geometrical and not those of wave optics.

Such a classical approach is applicable when the de Broglie wave length of the primary hadron is much smaller than the distance between nucleons in the nucleus. It enables us to follow easily and in details the individual nucleons and gives a possibility to determine first of all the characteristics of fragmentation channels. For these channels the application of the Glauber model would be rather clumsy, especially if it is necessary to take into consideration phenomena like production and absorption of particles, influence of the binding energy, etc. The Fraunhofer diffraction has been added to the cascade process to describe the elastic channel of the reaction.

Due to the flexibility the model is advantageously applicable also to background calculations for those effects which were not included into the model (for example: cumulative processes, absorption of pions, etc.). Finally, the usefulness of the Monte Carlo model method with a real distribution of particles in phase-space for testing the quality of data procesing system is not negligible.

II. CASCADE GENERATION

The first step in cascade generation is the determination of the initial state of the nucleons in the nucleus and the colliding hadron before the interaction, i.e. the determination of their momenta and space coordinates. **Part II.a describes the** procedure which generates parameters of the nucleons in the nucleus. The incoming hadron is generated with a constant density in the plane perpendicular to the direction of its motion. Its initial coordinates are chosen in a region, the radius R_{max} of which is equal to the maximum possible impact parameter. A hadron generated in this way interacts with nucleons of the nucleus if the impact parameter b is smaller than $B=\sqrt{\sigma_{tot}(s)/(\pi 10)}$ [fm], where $\sigma_{tot}(s)$ is the total cross section of

their interactions in mb. The cross section for one event (the so-called milibarn equivalent) is then

$$\varepsilon=\frac{10\,\pi R^2_{max}}{N_{ev}} \tag{2.1}$$

where R_{max} is equal to 7 fermi and N_{ev} is the total number of all generated initial situations. It is necessary to remark that only in a small part of the generated initial situations does a collision actually take place — mostly the hadron passes through the nucleus without interaction (Fig. 1).

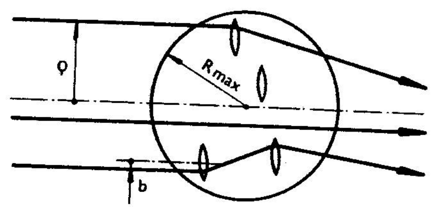

Fig. 1.

We assume that the hadron-nucleon collision is central and centripetal. The four-momentum transfer is determined by analogy with geometrical optics as a function of the impact parameter $t=t(b)$. The function $t(b)$ must be defined in such a way as to ensure the correct behaviour of the differential cross section $d\sigma/dt$. To do this we determine t by solving the differential equation

$$Q(t)\frac{dt}{db}=2\,\pi b \tag{2.2}$$

where $Q(t)$ is the sum of differential cross sections of the processes taken into consideration — for example for the $N-N$ collision in the $p+{}^4He$ interaction we put

$$Q(t)=\frac{d\sigma_{el}}{dt}+\frac{d\sigma_{\Delta}}{dt}\,. \tag{2.3}$$

For the boundary condition $t(B)=0$ we can write the solution of the equation (2.3) in the form

$$|t|=\Phi^{-1}(\pi(B^2-b^2)) \tag{2.4}$$

where

$$\Phi(|t|)=\int_t^0 Q(t)\,dt\,. \tag{2.5}$$

After the collision the momenta of particles which took part in the interaction will change in such a way that for a given t the energy and momentum conservation laws hold and the effective mass of the nucleon does not change (see II.a).

New trajectories after the collision are determined so as to conserve the angular momentum of the interacting particles. In the case of the $p-{}^4He$ interaction we consider, as mentioned above, the following elementary processes

$$N+N \to N+N \tag{2.6}$$

$$N+N \to \Delta+N \tag{2.7}$$

$$\Delta+N \to \Delta+N \tag{2.8}$$

$$\Delta+N \to N+N \tag{2.9}$$

$$N+N \to \pi+X \tag{2.10}$$

where (2.10) represents inelastic collision without Δ (1232) production. From (2.3) and from the condition $t(B)=0$ it follows that these inelastic events are in the central region. The parametrization of processes (2.6)$\div$(2.10) is described in parts II.b and II.c.

II. a Generation of the ^{4}He nucleus

The distribution of nucleon density in the CMS of the ^{4}He nucleus is well known from the exact measurements of the form-factor [2]. Because of technical problems connected with the generation of four nucleons directly in the CMS we used the Bassel-Wilkin wave function and generated four independent vectors according the distribution function

$$|\Phi(r_i)|^2 = N \exp(-\alpha^2 r_i^2)\left[1-\delta \exp\left(-\frac{\alpha^2}{\gamma^2} r_i^2\right)\right] \tag{2.11}$$

where $\alpha^2=0.579$ fm^{-2}, $\gamma^2=0.308$, $\delta=0.858$.
Nucleons, generated in this way, are then tranformed to their CMS. Fig. 2 shows the radial distribution of these nucleons (histogram) together with the distribution according to Sick [2] (smooth curve). From the picture can be seen that there is a significant difference only for the $r>2.2$ fm region which contains a negligible number of nucleons. Momenta of nucleons (Fig. 3) are generated independently of their space coordinates, analogously as above, in accordance with the normal distribution

$$|\Phi(p_i)|^2 = N \exp(-\varkappa p_i^2) \tag{2.12}$$

where the consant $\varkappa=51.5$ (GeV/c)$^{-2}$ is chosen in such a way as to obtain agreement with the momentum distribution by Sick's data [3] (smooth curve). Effects connected wtih the fact that the nucleons are off a mas shell, were included

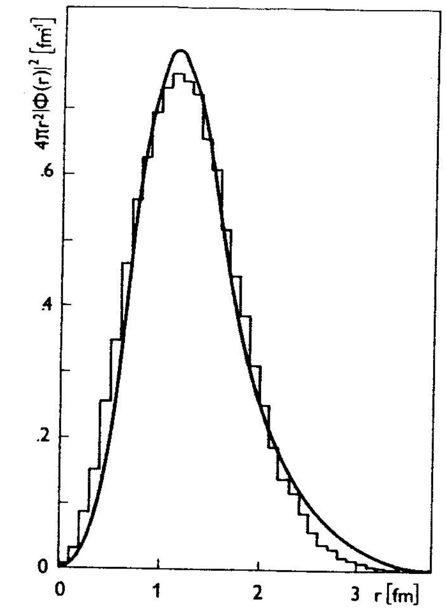

Fig. 2.

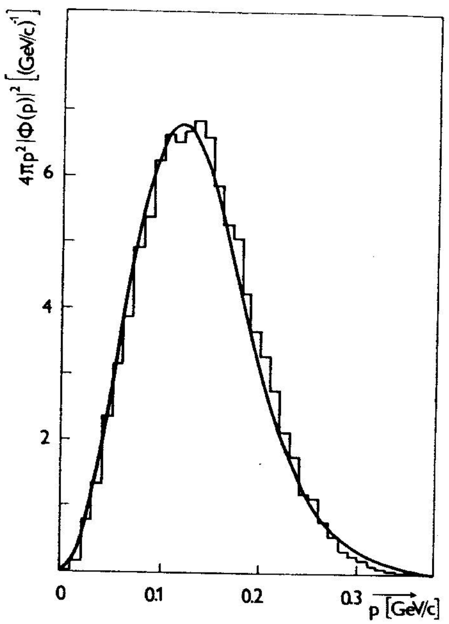

Fig. 3.

into the model by considering quasi-free nucleons with a mass smaller than the mass of free particles. The effective mass of quasi-free nucleons was determined from the assumption that one nucleon carries out 1/4 of the total nucleus energy, i.e.

$$\sqrt{m_{i,ef}^2+p_i^2}=\frac{m_{{}^4He}}{4} \tag{2.13}$$

II.b Parametrization of the elastic scattering of nucleons

In the introduction to part II. it was mentioned that the parametrization of the differential cross sections of the reactions (2.6—2.9) must be analytically integrable. To describe the experimental data on elastic scattering with a sufficiently high precision we used the function

$$\frac{d\sigma_{el}}{dt}=a+b\exp(-c|t|)+d\exp(-e|t|) \tag{2.14}$$

where the parameters $a \div e$ are functions of the CMS energy as follows:

$$a = a_0 + a_1/s$$
$$b = b_0 + b_1/s \qquad (2.15)$$
$$\vdots$$
$$e = e_0 + e_1/s\,.$$

The following values of constants were determined for $s = 6 \div 12.5$ (GeV)² using the data from [4]: $a_0 = -0.92$, $a_1 = 11.41$, $b_0 = 80.09$, $b_1 = -143.57$, $c_0 = 9.66$, $c_1 = -33.91$, $d_0 = -64.95$, $d_1 = 1101.2$, $e_0 = 32.04$, $e_1 = -100.06$, where s is given

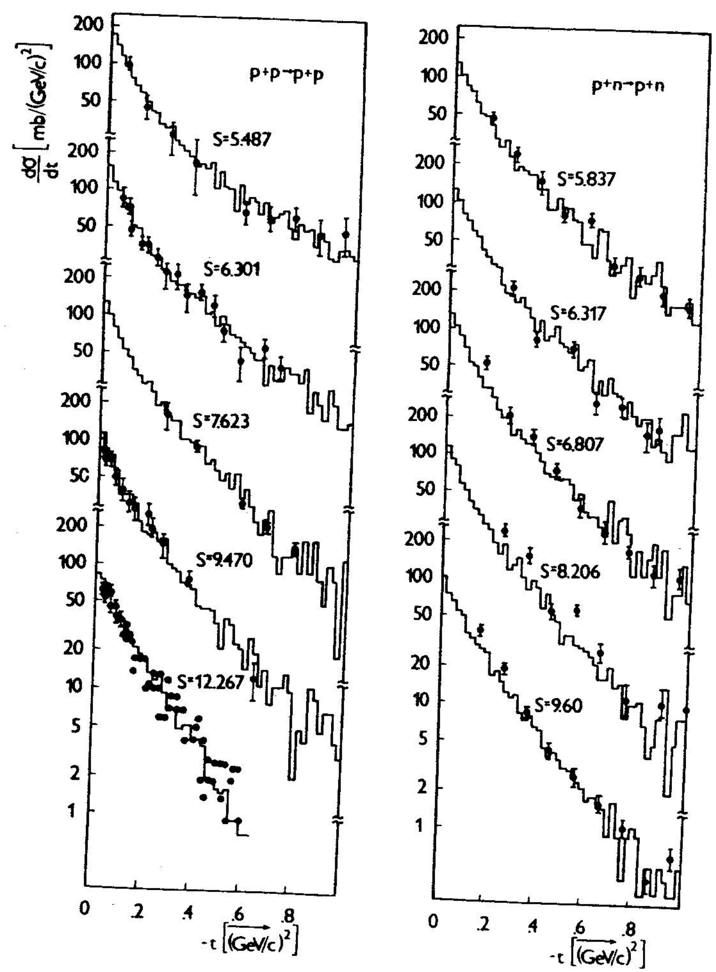

Fig. 4.

in (GeV)², t in (GeV/c)² and the cross sections in mb. For the elastic $N-N$ scattering cross section one can then use, with a relative error less than 0.1 %, this formula

$$\sigma_{el} = a|t|_{max} + \frac{b}{c} + \frac{d}{e}\,. \qquad (2.16)$$

Comparison of the differential cross section of elastic scattering generated in this way (histogram) with experimental data is shown in Fig. 4.

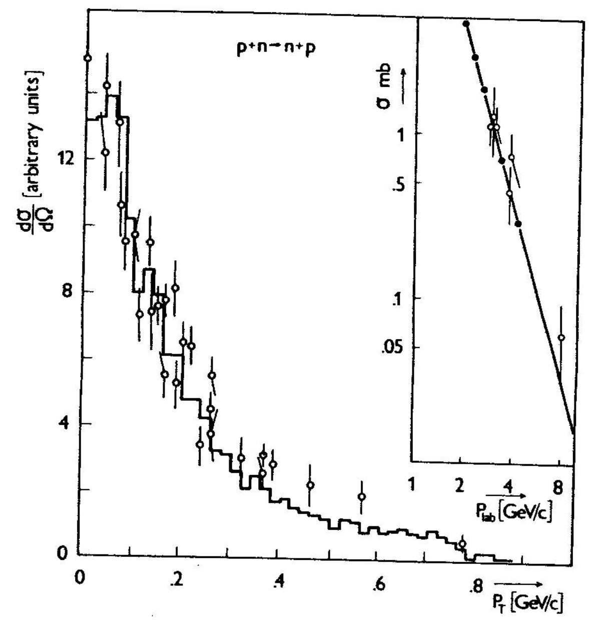

Fig. 5.

For the proton-neutron collision we take into consideration the charge exchange with the probalility

$$w_{ch.ex} = (A + B(P_T - C)^2)/P^3 \qquad (2.17)$$

where P is the proton momentum in the neutron rest system and P_T is the transverse momentum after collision determined by (2.14) and (2.15). The values of constants were determined by experimental data as follows: $A = 1.15$, $B = 11.2$, $C = 0.35$; when momenta are in GeV/c.

Comparison of the differential cross section of the charge exchange generated in this way with experimental data (points with errors) is shown in Fig. 5.

Scattered nucleons or slow Δ isobars from a primary collision can further collide with the other nucleons from the nucleus. Since the energies concerned are low, we assumed for elastic scatterings a uniform distribution in phase-space; the cross section was parametrized for unequal isospins of colliding particles in the form

$$\sigma = 10P^{-2.285} \tag{2.18}$$

and for equal isospins in the form

$$\sigma = 3.113P^{-2.285}\,. \tag{2.19}$$

Values of constants were determined on the basis of [4], momenta are in GeV/c and the cross section in mb.

II.c Parametrization of collisions with Δ (1232) production

The cross section of the reaction (2.7) was parametrized by the formula

$$\frac{d\sigma_\Delta}{dt} = f + g \exp(-h|t|) \tag{2.20}$$

where the dependence of the parameters f, g, h on the reaction energy was determined analogously to that in (2.15)

$$f = f_0 + f_1/S \tag{2.21}$$

$$g = g_0 + g_1/S$$

$$h = h_0 + h_1/S\,.$$

The constants of parametrization were determined on the basis of data concerning the reaction $pp \rightarrow \Delta^{++}n$ [4] as follows: $f_0 = -2.13$, $f_1 = 49.24$, $g_0 = -84.64$, $g_1 = 1550.1$, $h_0 = 16.6$, $h_1 = -41.39$, for t in $(\text{GeV/c})^2$ and σ in mb.

The cross section of the Δ production is

$$\sigma_\Delta \doteq f|t_{max}| + \frac{g}{h}\,. \tag{2.22}$$

The charge of produced Δ was determined from the isospin relations

$$\sigma_{pp \rightarrow \Delta^{++}n} = 3\sigma_{pp \rightarrow \Delta^{+}p} \tag{2.23}$$

$$\sigma_{nn \rightarrow \Delta^{-}p} = 3\sigma_{nn \rightarrow n\Delta^{0}} \tag{2.24}$$

$$\sigma_{pn \rightarrow \Delta^{+}n} = \sigma_{pn \rightarrow p\Delta^{0}}\,. \tag{2.25}$$

The probability of the charge exchange in the case of a pn collision is taken also in accordance with (2.14).

The cross section of the reverse reaction (2.9), i.e. absorption of Δ in collision with a nucleon, was determined from the principle of detailed balance

$$\sigma_{NN \rightarrow \Delta N} = 4\sigma_{\Delta N \rightarrow NN}\frac{p^2_{\Delta N}}{p^2_{NN}} \tag{2.26}$$

where $p_{\Delta N}$ is the Δ or N momentum in CMS and p_{NN} is the momentum of nucleons in CMS [22].

We use the same formulae for the $\Delta - N$ elastic scattering as those used for the $N-N$ scattering. Before the collision of Δ with a nucleon a check was made to see whether the isobar had not decayed; the mean life-time was determined from the width of the resonance $\Gamma = 0.115$ GeV and the relativistic time dilatation was also taken into consideration.

III. FRAGMENTATION OF THE NUCLEUS

In nucleus fragmentation — similarly as in the case of cascades, we follow in detail the fate of each nucleon. Because of the low number of the nucleons it is possible to check all their possible combinations in the final state. Each combination called a hypothesis is verified to see whether it is energetically permitted, whether the fragments have the requested spin, charge, and if they are not in contradiction with Pauli's principle — we assume that all the fragments are in the s-state. If after such a selection there does not remain any correct hypothesis (about 5 % of events), the cross section for this events will be equally distributed among all the correct events. If more than one hypothesis remains, we select the one with maximal binding energy of nucleons in fragments. After selecting the hypothesis, the passing of nucleons to the mass shell is solved (the energy of fragments becomes equal to the free particles energy). We assume that the fastest fragment in the rest system of the excited nucleus leaves the nucleus as the first. Its direction in this system and the total energy of the system are conserved. After this step, the whole procedure is repeated iteratively for the remaining system — until the whole fragmentation process is finished. If in some iteration the fragment cannot leave the remaining system because of energy shortage, the hypothesis is rejected. Using this procedure for describing a fragmentation of an excited nucleus guarantees total energy and momentum conservation in each step and therefore also in the whole process.

IV. ELASTIC SCATTERING

In the framework of the model described in parts II. and III. we obtain also the non-diffractive part of the elastic scattering. This category comprise first of all those events in which nucleons in collisions with a primary proton do not obtain energy sufficient for the fragmentation of the nucleus. In this case the energy conservation law does not hold in the model because the mass of ^{4}He nucleus is destroyed by the hadron-nucleon interaction.

As a supplement of the Monte Carlo procedure it is possible to determine within the first approximation the diffractive scattering. With the help of this supplement we can determine the total cross section of the reaction. This problem was already solved in [10] by means of the quasiclassical formula for elastic scattering. The disadvantage of this method lies in the underestimation of the differential cross section for elastic scattering in the region of the optical point and thus also the underestimation of the total cross section of the reaction. For the diffractive scattering description the formula for Fraunhofer's amplitude of the scattering on a semitransparent spherical nucleus

$$f(\vartheta)=ik\frac{(1+\cos\vartheta}{2}\int_0^{\varrho_{max}} d\varrho\varrho\Gamma(\varrho)J_0(k\varrho\sin\vartheta) \tag{4.1}$$

was used in our model, where ϱ is the impact parameter, ϑ is the scattering angle, $\Gamma(\varrho)$ is the profile function and J_0 — the Bessel function of zero order. It follows from the optical theorem that $\sigma_{tot}=2\int\Gamma(\varrho)\,d^2\varrho$, i.e. that $2\Gamma(\varrho)$ is the probability density of the scattering at the impact parameter ϱ [11]. The use of the formula (4.1) in the Monte Carlo procedure is connected with the following problem. The diffractive scattering as part of the total scattering, is an interaction with the whole nucleus and we cannot ascribe a concrete impact parameter to it. Considering that the ratio of the diffractive to the non-diffractive component of the cross section

$$\beta=\frac{\sigma_{difr.}}{\sigma_{tot}-\sigma_{difr.}} \tag{4.2}$$

is much smaller than 1, we use for obtaining $\Gamma(\varrho)$ the formula

$$2\pi\varrho\Delta\varrho 2\Gamma(\varrho)=(1+\beta)\varepsilon\sum_i w_i \tag{4.3}$$

in which $\sum_i w_i$ is the sum of weights of the generated events with impact parameter $\varrho^*\in(\varrho,\varrho+\Delta\varrho)$, ε is the milibarn equivalent for one event obtained by (2.1) and $(1+\beta)$ expresses the correction of the included diffraction into σ_{tot}. On the basis of (4.1) ÷ (4.3), we get after replacing integration by summing and making arithmetical modifications

$$\frac{d\sigma_{difr}}{dt}=(1+\beta)^2F(t) \tag{4.4}$$

and

$$F(t)=-\frac{\varepsilon^2}{16\,\pi\hbar^2}\left(1-\frac{|t|}{4P^2}\right)^2\left[\sum_j\sum_i w_iJ_0\left(\frac{\varrho_i}{\hbar}\sqrt{|t|-\frac{t^2}{4P^2}}\right)\right]^2 \tag{4.5}$$

where P is the momentum in the rest system of the nucleus and the primary proton and the summation through j represents summation with respect to all circular rings with $\varrho=(\varrho_i,\varrho_i+\Delta\varrho)$. After integrating (4.4) by t and substituting from (4.2) we get an expression for the constant β

$$\beta=\frac{\sigma^*}{2\Phi}-\sqrt{\frac{\sigma^{*2}}{2\Phi^2}-\frac{\sigma^*}{\Phi}-1} \tag{4.6}$$

where σ^* is the non-diffractive part of the total cross section

$$\sigma^*=\sigma_{tot}-\sigma_{difr}=\varepsilon\sum_j\sum_i w_i \tag{4.7}$$

and

$$\Phi=\int_{-\infty}^0 F(t)\,dt\,. \tag{4.8}$$

With the help of (4.4) ÷ (4.8) it is possible to determine the diffractive scattering from the distribution of the impact parameters obtained in the Monte Carlo procedure, in agreement with the optical theorem.

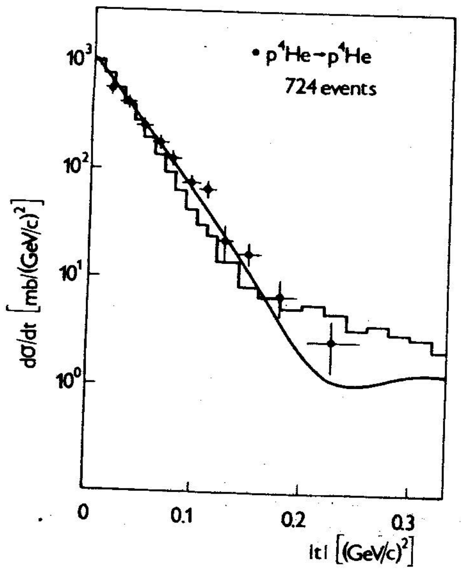

Fig. 6.

V. COMPARISON WITH EXPERIMENTAL DATA

The model ideas described above were realized in a FORTRAN program of about 2000 instructions. We want to emphasize that the program does not contain any free parameters which would be determined from the described reaction and that the normalization is absolute. With the help of the model events of $^4He + p$ reaction at a momentum of the primary ^4He nucleus 8.6 GeV/c were generated. For this momentum there are experimental data from the JINR 1 m hydrogen bubble chamber [3], [12] ÷ [21]. In Table 1 the predictions of the cross sections for the pionless reaction channels and of the total cross section are compared with experimental data. Only statistical errors are given. One can see that the model predictions are in relatively good agreement with the experimental data.

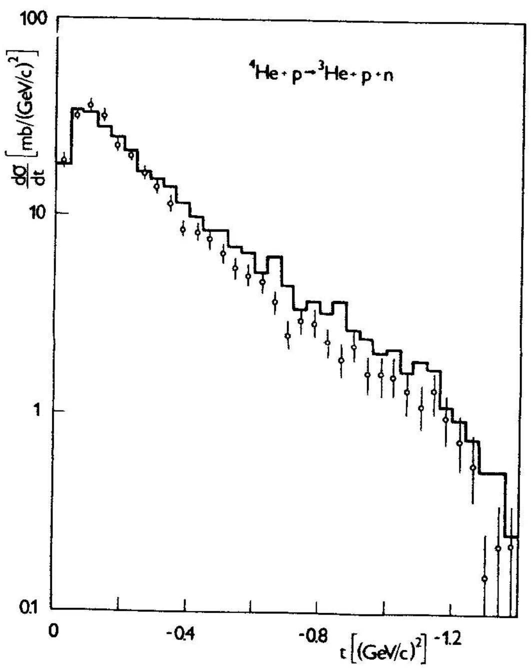

Fig. 7.

Fig. 6 shows the differential cross section of the elastic scattering. The curve shows $d\sigma/dt$ according to the Glauber model, which together with the experimental points was taken from [12]. The histogram shows the prediction of the model. From the comparison of these graphs one can see that the predictions of the model of the region of the optical pount are correct and so they can be used for the estimation of the total cross section of the reaction as well as for its elastic channel. However, in the region of the diffractive minimum the quasi-optical model is considerably less exact than the many time verified Glauber model. This inaccuracy is caused by a significant amount of non-diffractive events in this region, connected probably

with the fact that the energy conservation law was not fulfilled in the model for this channel (see part 4). Much better is the agreement of the quasi-optical model with data on the fragmentation channels for the description of which it was created. As an example we show the differential cross section of the channel $^4He + p \to {}^3He + p + n$. Fig. 7 shows the distribution of the four-momentum transfer squared between the target proton and a slower nucleon in the laboratory system for this channel. The model data, depicted by the histogram, are in very good agreement

Table 1

$^4He + p$ at 8.6 GeV/c			
Channel	σ_{model} [mb]	σ_{exp} [mb]	Citation
$^4He + p$	37.98 ± 0.10	36 ± 3	12
$t + p + p$	12.60 ± 0.14	10.2 ± 0.5	17
$^3He + p + n$	12.24 ± 0.14	11.0 ± 0.5	17
$d + d + p$	1.66 ± 0.05	1.41 ± 0.13	18
$d + p + p + n$	8.39 ± 0.11	9.30 ± 0.32	18
$p + p + p + x^0$		10.29 ± 0.19	19
$p + p + p + n + n$	2.23 ± 0.06		
σ_{total}	151.2 ± 0.4	148 ± 7	12

with the experimental results. The decrease of the differential cross section at small $|t|$ is due to a non-sufficient energy for opening the channel. It is interesting to observe the slope change of the dependence in the region $|t| = 1.1$ (GeV/c)^2 which is very well described by the model, although it does not appear in the parametrization of the $N - N$ interaction. The reason for this is probably rather complex and is connected with a mutual overlapping of single and multiple collisions and with the distribution of the events into various channels. High flexibility of the model in obtaining dependences of various types was advantageously used in [14] where the model was used to determine the background in the study of the role of pion absorption in the channel $^4He + p \to d + p + p + n$.

VI. CONCLUSION

The present paper gives a description of the quasi-optical cascade model of the collision between the lightest nuclei and hadrons. The described model is convenient for the determination of the cross section of the whole reaction, its elastic channel and for the determination of the more detailed characteristics of pionless fragmentation channels. In future we expect an improvement of its characteristics in the description of the elastic scattering and its further extension to the description of channels with pion production.

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