acta phys. slov. 36 (1986), No. 5

# MEASUREMENT OF RELATIVE PERMITTIVITY OF WATER BY THE NODE-DISPLACEMENT METHOD IN THE CASE OF AN OPEN CIRCUIT

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A simple method for measuring the complex permittivity is described. The values of $\varepsilon'$ and tg $\delta$ for dielectrics with $1 \leq \varepsilon' \leq 95$ and $0 \leq \text{tg}\ \delta \leq 1$ can be calculated by means of the expression — $\text{tg}(\beta_0 l_2)/\beta_0 d$, where the shifts are measured for two dielectric thicknesses in an open circuit. Water is investigated at $(2.5 \leq \lambda_0 \leq 3.5$ cm). The experimental graph $\varepsilon'(\lambda_0)$ in the $X$-band can be used to determine the relaxation time when the values of $\varepsilon$ and $\varepsilon_\infty$ are known.

## ИЗМЕРЕНИЕ ОТНОСИТЕЛЬНОЙ КОМПЛЕКСНОЙ ДИЭЛЕКТРИЧЕСКОЙ ПРОНИЦАЕМОСТИ ВОДЫ ПРИ ПОМОЩИ МЕТОДА СМЕЩЕНИЯ УЗЛА В СЛУЧАЕ НЕЗАМКНУТОГО КОНТУРА

В работе описан простой метод измерения комплексной диэлектрической проницаемости воды. Показано, что значения $\varepsilon'$ $(1 \leq \varepsilon' \leq 95)$ и tg $\delta \leq 1$ можно определить на основе экспериментально измеренных величин сдвигов $\text{tg}(\beta_0 l_2)/\beta_0 d$ для двух толщин диэлектрика в незамкнутом контуре. Исследования проводились для волн с длиной из интервала $2,5 \leq \lambda_0 \leq 3,5$ cm. График экспериментальных данных для $\varepsilon'(\lambda_0)$ в диапазоне $X$ позволяет определить время релаксации при известных значениях величин $\varepsilon_0$ и $\varepsilon_\infty$.

## I. INTRODUCTION

It is known [1] that the magnitude of the complex reflection coefficient $\dot{\Gamma}_2$ in the plane $z = 0$ (Fig. 1) is approximately equal to unity ($|\dot{\Gamma}_2| \simeq 1$) when the dielectric of an comparatively large value of $\varepsilon'$ $(\varepsilon' \geq 50)$ is investigated. Obviously in this case it is not convenient to use the voltage standing wave ratio (VSWR $= K_2$) as a source of information about the values of the phase constant $\beta$ and the attenuation constant $\alpha$ and hence for the values of $\varepsilon'$ and $\varepsilon''$. This means that some difficulties exist in the precise selection of a waveguide method, which uses values of VSWR to calculate $\varepsilon'$ and $\varepsilon''$ [1, 9]. Of course, such a problem appears mostly in the analysis of dielectric dispersion data, when it is necessary to obtain values of $\varepsilon'$ and $\varepsilon''$ as a function of frequency.

It is also worth noting that the existing waveguide methods generally fall into two categories. One of them deals with VSWR and the minimum shift in order to obtain values of $\varepsilon'$ and $\varepsilon''$ [1, 2] and the other deals only with the standing wave minimum shift [5, 10—12]. The methods in [10—12] deal with low-loss dielectric and that described in [5] has a simple application in the case of investigating a fixed wavelength and the dielectric thickness $d$.

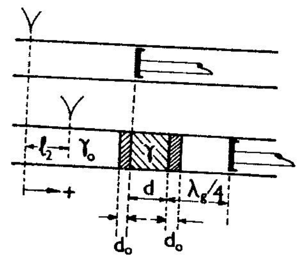

Fig. 1. Measurement of the shift $l_2$ for a dielectric sample in an open circuit, and $d_0 = 0.003$ cm.

According to the previous consideration it is useful to investigate the possibility of determining the values of $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$ for a wide frequency band by measuring the shifts of the standing wave minimum. Indeed, this possibility stimulated the work reported here.

In the present paper a method for determining $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$ in the $X$-band is described, using values of standing wave minimum shifts for two dielectric thicknesses measured in an open circuit. The short circuit measurement and the combination with the open circuit measurement are given briefly in the Appendix.

I should be explicitly noted that the method described below can be used in two general cases:

a) The investigation of dielectric dispersion data ($\lambda_0 \neq$ const.) when $\varepsilon'$ and $\varepsilon''$ are functions of $\omega$.

b) The investigation at a given wavelength ($\lambda_0 =$ const.) when $1 \leq \varepsilon' \leq 90$ and $0 \leq \text{tg}\ \delta \leq 1$.

## II. METHOD

The method is based on the investigation of two dielectric thicknesses $d'$ and $d''$ in an open circuit and on the measurement of the standing wave minimum shifts $l_2'$ and $l_2''$ (Fig. 1). The values of $\varepsilon'$ and $\varepsilon''$ are calculated by the equations [2]

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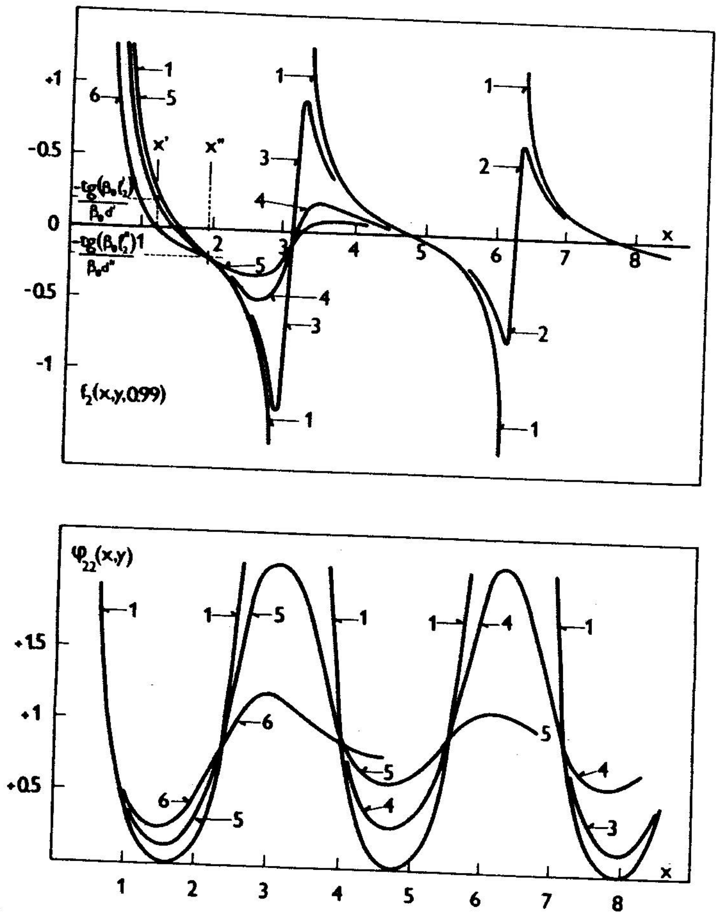


Fig. 2. Function $\varphi_{22}(x, y)$ and $f_2(x, y, 0.09)$ for $0 \leqslant x \leqslant 8$ and $1-y=0$; $2-y=0.02$; $3-y=0.05$; $4-y=0.13$; $5-y=0.23$; $6-y=0.4$.

$$\varepsilon' = [1 + (\beta^2 - \alpha^2)(\lambda_c/2\pi)^2]/[1 + (\lambda_c/\lambda_g)^2], \tag{1}$$

$$\varepsilon'' = [2\alpha\beta(\lambda_c/2\pi)^2]/[1 + (\lambda_c/\lambda_g)^2], \tag{2}$$

where $\lambda_c$ is the cut-off wavelength, $\lambda_g$ is the waveguide wavelength $\gamma = \alpha + j\beta$ is the propagation constant in the liquid, $\gamma_0 = j\beta_0 = j2\pi/\lambda_g$ is the propagation constant in the empty waveguide (Fig. 1).

In eqs. (1) and (2) the phase constant $\beta$ and the attenuation constant $\alpha$ are calculated with the help of the quantities $x(x = \beta d)$ and $y(y = \alpha/\beta)$.



The necessary quantities $x$ and $y$ are obtained by the experimentally determined values of $d$ and $l_2$, using the expression

$$\frac{-\operatorname{tg}(\beta_0 l_2)}{\beta_0 d} = f_2(x, y, \beta_0/\beta). \tag{3}$$

In Eq. (3) $l_2(x, y, \beta_0/\beta)$ is given by [5]

$$l_2 = \frac{1}{2\beta_0} \operatorname{arctg} \frac{2(\beta_0/\beta)\varphi_2(x, y)}{1 - (\beta_0/\beta)^2 \varphi_{22}(x, y)}, \tag{4}$$

where

$$\varphi_2(x, y) = \frac{(y \operatorname{sh} 2xy - \sin 2x)}{(1 + y^2)(\operatorname{ch} 2xy - \cos 2x)}, \tag{5}$$

$$\varphi_{22}(x, y) = \frac{(\operatorname{ch} 2xy + \cos 2x)}{(1 + y^2)(\operatorname{ch} 2xy - \cos 2x)}. \tag{6}$$

The theoretical values of the function $f_2(x, y, \beta_0/\beta)$ in Eq. (3), (Fig. 2) are calculated by a computer.

The quantity $\beta_0/\beta$ in Eq. (4) and in the function $f_2(x, y, \beta_0/\beta)$ Eq. (3) is taken as a constant and can be determined from the expression

$$(\beta/\beta_0)^4 + (\beta/\beta_0)^2[(\lambda_g/\lambda_c)^2 - (\lambda_g/\lambda_0)^2\varepsilon'] - [(\lambda_g/\lambda_0)^2(\varepsilon''/2)]^2 = 0, \tag{7}$$

where $\lambda_0$ is the free-space wavelength and $(\lambda_g/\lambda_0)^2 = 1/[1 - (\lambda_0/\lambda_c)^2]$.

The relation (7) is obtained easily from Eqs. (1) and (2).

The solution of (7) is

$$(\beta_0/\beta) = A(\lambda_g/\lambda_0)(\varepsilon')^{1/2}, \tag{8}$$

when $\varepsilon' \geqslant 50$ and $(\lambda_g/\lambda_c)^2 \ll (\lambda_g/\lambda_0)^2 \varepsilon'$.

Note that for $\operatorname{tg}\delta = 0$, $A = [(1 + \sqrt{1 + \operatorname{tg}^2\delta})/2]^{1/2} = 1$, and for $\operatorname{tg}\delta = 1$, $A = 1.098$, which shows little dependence of the quantity $(\beta/\beta_0)$ on $\operatorname{tg}\delta (0 \leqslant tg\ \delta \leqslant 1)$.

In Table 1 the values of $\beta_0/\beta$ appropriate in the case of $0 \leqslant \operatorname{tg}\delta \leqslant 1$ are given. The results in Table 1 are derived from Eq. (7) for the $X$-band ($2.5 \leqslant \lambda_0 \leqslant 3.5$ cm), $\lambda_c = 4.57$ cm and $1 \leqslant \varepsilon' \leqslant 95$. The information in Table 1 can be used when the appropriate value of $\varepsilon'$ is known [2—4]. For example, in the analysis of dielectric dispersion data of water in the $X$-band, it is convenient to use $\beta_0/\beta = 0.09$ in the function $f_2(x, y, \beta_0/\beta)$. Note that in fact $\beta_0/\beta$ is not a constant and its value varies in conformity with Eq. (7), because the values of $\lambda_g/\lambda_0$, $\varepsilon'$, and $\operatorname{tg}\delta = \varepsilon''/\varepsilon'$ are changing. Numerical examples show that the systematical error of the value of $\varepsilon'(1 \leqslant \varepsilon' \leqslant 95)$ and $\operatorname{tg}\delta (0 \leqslant \operatorname{tg}\delta \leqslant 1)$ is less than 0.2 % provided $\beta_0/\beta$ in Eqs. (3) and (4) is taken from Table 1 and

$$|\operatorname{tg}(\beta_0 l_2)/\beta_0 d| \leqslant 1. \tag{9}$$

The systematical error of 0.2 % is due to little dependence of the function $f_2(x, y, \beta_0/\beta)$ on the quantity $\beta_0/\beta$. For example, when tg $\delta = 0(y=0)$ it is easily derived from Eqs. (3—6)

$$f_2(x, 0, \beta_0/\beta) = \mathrm{cotg}\, x/x \,, \tag{10}$$

which shows the independence of $f_2(x, 0, \beta_0/\beta)$ from the values of $\beta_0/\beta$, regardless of condition (9).

Table 1

The values of $\beta_0/\beta$ in the case of $0 \leq \mathrm{tg}\, \delta \leq 1$

| $\beta_0/\beta$ | $\varepsilon'$ |
|---|---|
| 0.8 | $1 \leq \varepsilon' \leq 3$ |
| 0.55 | $2 \leq \varepsilon' \leq 4$ |
| 0.4 | $3 \leq \varepsilon' \leq 6$ |
| 0.3 | $5 \leq \varepsilon' \leq 12$ |
| 0.2 | $10 \leq \varepsilon' \leq 20$ |
| 0.15 | $20 \leq \varepsilon' \leq 40$ |
| 0.1 | $40 \leq \varepsilon' \leq 60$ |
| 0.09 | $60 \leq \varepsilon' \leq 75$ |
| 0.08 | $75 \leq \varepsilon' \leq 95$ |

Suppose that in Eqs. (3) and (4) two different constants $a_1$ and $a_2$ are used as values of $(\beta_0/\beta)$. In this case the theoretical function $f_2(x, y, \beta_0/\beta)$ is written under the condition (9) and Eq. (3), in the following forms

$$f_2'(x, y, a_1) = \frac{1}{\beta_0 d}\, \mathrm{tg}\left[\frac{1}{2}\, \mathrm{arctg}\, \frac{2a_1\varphi_2(x, y)}{1 - a_1^2\varphi_{22}(x, y)}\right] \leq 1 \,, \tag{11}$$

$$f_2''(x, y, a_2) = \frac{1}{\beta_0 d}\, \mathrm{tg}\left[\frac{1}{2}\, \mathrm{arctg}\, \frac{2a_2\varphi_2(x, y)}{1 - a_2^2\varphi_{22}(x, y)}\right] \leq 1 \,. \tag{12}$$

The independence of $f_2(x, y, \beta_0/\beta)$ from the values of $\beta_0/\beta$ can be proved from Eqs. (11) and (12). Dividing Eq. (11) by Eq. (12) under the condition (9), excluding the case $(\beta_0/\beta)^2\varphi_{22}(x, y) \simeq 1$, the following approximate equation is valid

$$\frac{f_2'(x, y, a_1)}{f_2''(x, y, a_2)} \simeq \frac{1 - a_2^2\varphi_{22}(x, y)}{1 - a_1^2\varphi_{22}(x, y)} \,. \tag{13}$$

When $\varepsilon' \geq 50$, Eq. (8) gives $(\beta_0/\beta)^2 \ll 1$ and hence the expression (13) is approximately equal to unity, excluding the case $\varphi_{22}(x, y) \simeq 1/a_1^2$ (Fig. 2).

Note that the region of $x$ for which $f_2(x, y, \beta_0/\beta)$ depends on the $\beta_0/\beta$ is very small (Fig. 2).



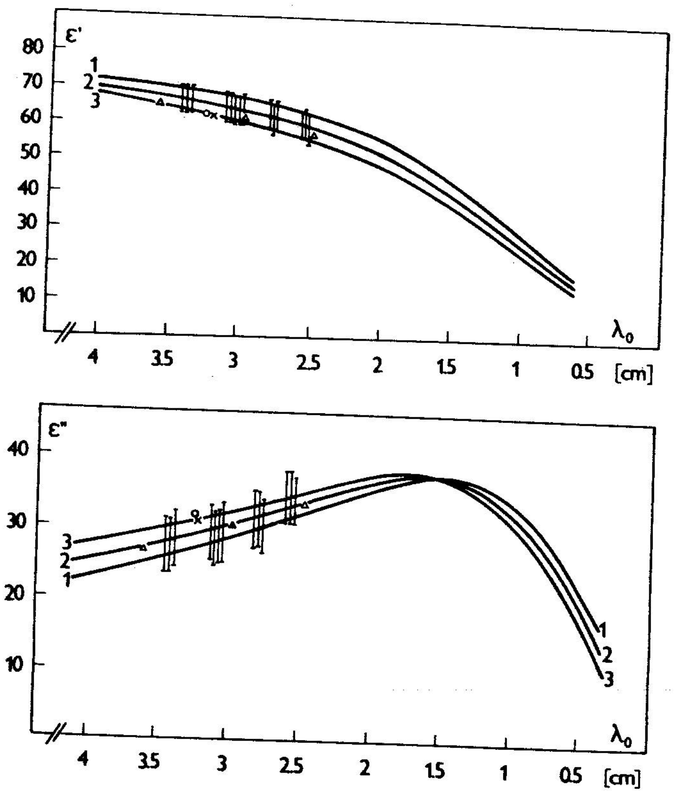


Fig. 3. Values of $\varepsilon'(\lambda_0)$ and $\varepsilon''(\lambda_0)$ for water obtained by: I — author, Δ — [4], × — [6], ○ — [4] and data calculated from the Debye equation, ($\varepsilon = 80.4$ — [6], $\varepsilon_\infty = 6.4$ — [6]; 1 – $\tau = 0.754 \times 10^{-11}$ sec., $\lambda_{max} = 1.43$ cm. 2 — $\tau = 0.823 \times 10^{-11}$ sec., $\lambda_{max} = 1.55$ cm — [7, 8], 3 — $\tau = 0.886 \times 10^{-1}$ sec., $\lambda_{max} =$ 1.67 cm.

Suppose that for a given dielectric thickness $d_1$ the condition (9) is not fulfilled. It means that the thickness must be changed. The same thickness $d_1$ can be used for short circuit measurement given briefly in the Appendix.

## III. RESULTS

An experimental investigation of water with conductivity $\sigma = 5 \times 10^{-5}\Omega^{-1}\mathrm{m}^{-1}$, is performed in the $X$-band with dielectric thicknesses $d' = 0.074$ cm and $d'' =$ 0.114 cm, when $d_0 = 0.003$ cm.

In Fig. 3 are shown the experimental results $\varepsilon'(\lambda_0)$ and $\varepsilon''(\lambda_0)$, the reference data and the data received from the Debye equations

$$\varepsilon'(\omega)=\varepsilon_\infty+\frac{\varepsilon-\varepsilon_\infty}{(1+\omega^2\tau^2)}, \tag{14}$$

$$\varepsilon''(\omega)=\frac{\varepsilon-\varepsilon_\infty}{(1+\omega^2\tau^2)}. \tag{15}$$

In Eqs. (14) and (15) the following values are used: $\varepsilon_\infty=6.4$ [6] (limiting high-frequency permittivity), $\varepsilon=80.4$ [6] (limiting low-frequency permittivity) and $\tau=0.823\times10^{-11}$ s [8, 7] relaxation time.

The measurement procedure is as follows:

a) The shifts $l_2'$ and $l_2''$ for two samples of thicknesses $d'$ and $d''$, are measured in an open circuit, as shown in Fig. 1.

b) The values of the function $f_2(x, y, \beta_0/\beta)$ (Fig. 2) are calculated by a computer using appropriate values of $\beta_0/\beta$, as shown in Table 1.

c) The values of $-\mathrm{tg}\,(\beta_0 l_2')/\beta_0 d'$ and $-\mathrm{tg}\,(\beta_0 l_2'')/\beta_0 d''$ are calculated. The results in Fig. 2 refer to an investigation of water for $d'=0.074$ cm, $d''=0.114$ cm and $\lambda_0=3$ cm.

d) The values of $x'=\beta d'$ and $x''=\beta d''$ are determined for the same values of $y=\alpha/\beta$ (Fig. 2). In this case there holds the relation $x'/x''=d'/d''$.

e) The values of the phase constant $\beta$ are calculated from $\beta=x'/d'$ or $\beta=x''/d''$ and the value of the attenuation constant $\alpha$ is calculated from $\alpha=\beta y$. Values of $\varepsilon'$ and $\varepsilon''$ can be obtained from Eqs (1) and (2), respectively.

f) The error of $\varepsilon'$ and tg $\delta$ can be derived from

$$\frac{\Delta\varepsilon'}{\varepsilon'}=2\,\frac{\Delta\beta}{\beta}=2\left[\frac{\Delta x}{x}+\frac{\Delta d}{d}\right], \tag{16}$$

$$\Delta\,\mathrm{tg}\,\delta=2\Delta y=2\Delta\,\frac{\alpha}{\beta}. \tag{17}$$

Eqs. (16) and (17) are obtained from eqs. (1), (2) and

$$\mathrm{tg}\,\delta=\frac{\varepsilon''}{\varepsilon'}=\frac{2y}{1-y^2+(\lambda_\varepsilon/\lambda_c)^2}\simeq 2y \tag{18}$$

when $y^2\ll1$, where $\lambda_\varepsilon=\lambda_g\beta/\beta_0$, where $\lambda_\varepsilon=\lambda_g\beta/\beta_0$. The condition $y^2\ll1$ in Eq. (18) is fulfilled for $\varepsilon'\geqslant50$ and $0\leqslant\mathrm{tg}\,\delta\leqslant1$. In Eqs. (16), (17) the values of $\Delta x$ and $\Delta y$ are obtained in the same way as $x$ and $y$, using the function $f_2(x, y, \beta_0/\beta)$ and the experimental errors of $\Delta l_2$ and $\Delta d$ by the expressions

$$-\mathrm{tg}\,[\beta_0(l_2'\pm\Delta l_2')]/\beta_0(d'\pm\Delta d') \tag{19}$$

$$-\mathrm{tg}\,[\beta_0(l_2''\pm\Delta l_2'')]/\beta_0(d''\pm\Delta d''). \tag{20}$$



## IV. DISCUSSION

The analysis of experimental data for the values of $\varepsilon'$ and tg $\delta$ of water shows an error of less than $\pm7$ %. The experimental curve $\varepsilon'(\lambda_0)$ in the $X$-band (Fig. 3) can be used to calculate the relaxation time with an uncertainty of $\pm7$ % if the values of $\varepsilon_\infty$ and $\varepsilon$ are known [4].

## CONCLUSIONS

The possibility of determining $\varepsilon'(1\leqslant\varepsilon'\neq95)$ and tg $\delta$ $(0\leqslant\mathrm{tg}\,\delta\leqslant1)$ by the comparison of the experimental value $-\mathrm{tg}\,(\beta_0 l_2)/\beta_0 d$ with the theoretical function $f_2(x, y, \beta_0/\beta)$ is described. The standing wave minimum shifts are measured for two dielectric thicknesses, investigated in an open circuit. The method is used to obtain $\varepsilon'$ and tg $\delta$ of water in the $X$-band. The experimental data for $\varepsilon'(\lambda_0)$ can be used to calculate the relaxation time when the values of limiting high-frequency permittivity and low-frequency permittivity are known.

## APPENDIX

The method described in the paper can be applied to two dielectric thicknesses measured in a short circuit, or for one dielectric thickness measured in a short circuit, or for one dielectric thickness measured in an open and a short circuit.

For a short circuit measurement the necessary quantities $x$ and $y$ are obtained using the theoretical function $f_1(x, y, \beta_0/\beta)$,

$$\frac{\mathrm{tg}\,(\beta_0 l_1)}{\beta_0 d}=f_1(x, y, \beta_0/\beta), \tag{21}$$

where $l_1(x, y, \beta_0/\beta)$ is given by [5],

$$l_1=\frac{1}{2\beta_0}\,\mathrm{arctg}\,\frac{2(\beta_0/\beta)\varphi_1(x, y)}{1-(\beta_0/\beta)^2\varphi_{11}(x, y)}, \tag{22}$$

$$\varphi_1(x, y)=\frac{y\,\mathrm{sh}\,2xy+\sin 2x}{(1+y^2)\,(\mathrm{ch}\,2xy+\cos 2x)}, \tag{23}$$

$$\varphi_{11}(x, y)=\frac{\mathrm{ch}\,2xy-\cos 2x}{(1+y^2)\,(\mathrm{ch}\,2xy+\cos 2x)}. \tag{24}$$

The theoretical function $f_1(x, y, \beta_0/\beta)$ is calculated by computer for values of $\beta_0/\beta$ taken from Table 1.

When tg $\delta=0$ $(y=0)$, the independence of $f_1(x, y, \beta_0/\beta)$ from the $\beta_0/\beta$ can be proved from Eqs. (21—24). The independence of $f_1(x, y, \beta_0/\beta)$ from the values of

$\beta_0/\beta$ can be proved with the help of the function $\varphi_{11}(x, y)$ Eq. (24), in the same way as in the case of the open circuit. In this case the following condition is valid

$$|f_1(x, y, \beta_0/\beta)| \leq 1 . \qquad (25)$$

It is also worth noting that the values of $x$ and $y$ can be obtained using the theoretical function $f_1(x, y, \beta_0/\beta)$ and $f_2(x, y, \beta_0/\beta)$ when only one thickness is measured in a short and an open circuit.

Note that the conditions (9) and (25) are satisfied for different values of $x$ because the functions $f_1(x, y, \beta_0/\beta)$ and $f_2(x, y, \beta_0/\beta)$ are shifted by $x \simeq \pi/2$.

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Received December 21st, 1984
Revised version received December 18th, 1985