

## DYNAMICALLY REDUNDANT PARTICLE COMPONENTS IN MIXTURES

B. LUKÁCS<sup>1)</sup>, K. MARTINÁSZ<sup>2)</sup>, Budapest

Examples are shown where the number of different kinds of particles in a system is not necessarily the number of particle degrees of freedom in the thermodynamical sense, and at the same time the observed dynamics of the evolution of the system does not indicate a definite number of degrees of freedom. The possibility for introducing dynamically redundant particles is discussed, and the relevance of this problem is demonstrated in the thermodynamical description of energetic heavy ion collisions and the early Universe.

### ДИНАМИЧЕСКИ ЛИШНИЕ КОМПОНЕНТЫ ЧАСТИЦ В СМЕСЯХ

В работе приводятся примеры, когда число разных видов частиц в системе обязательно равно числу степеней свободы в термодинамическом смысле и в то же время наблюдаемая динамика развития системы не указывает на определенное число степеней свободы. Обсуждается возможность введения динамически лишние частицы в системе и демонстрируется отношение этой проблемы к термодинамическому описанию высокоэнергетических столкновений тяжелых ионов, а также в стадии ранней Вселенной.

### 1. INTRODUCTION

Continuum mechanics (or, rather especially, hydromechanics) is very useful for describing various physical situations when the number of the manifested macroscopic degrees of freedom is limited. As extreme examples one can mention the treatment of supertova detonations [1], the evolution of the Universe [2], [3], and the expansion of excited nuclei [4]. The main advantage of the hydrodynamical description is its simplicity; the whole complexity of the matter is condensed into several data as the velocity field, energy density, pressure and such. It is rather surprising that such a simplified model can work under so differing circumstances; in fact, Table 1 of Ref. [5] is an excellent display of the increasing degree of simplifications leading to hydrodynamics. Nevertheless, this approach is generally

<sup>1)</sup> Central Research Institute for Physics, Br. 114, Pt. 49, 1525 BUDAPEST, Hungary.  
<sup>2)</sup> Dept. of Low Temperature Physics, Roland Eötvös University, Puskin u. 5—7, 1088 BUDAPEST, Hungary.

quite satisfactory. However, it cannot work alone, some relations are needed even among the fundamental quantities. When the time evolution of the system is not extremely fast, such relations are yielded by equilibrium or near equilibrium thermodynamics. The usual way is to assign the independent degrees of freedom of a simple system as some particle number densities  $n_a$ , one thermal quantity (the entropy density  $s$  or temperature  $T$ ), and the velocity field  $u^i$ , and to express a proper thermodynamic potential by means of them. Then the other hydrodynamic quantities can be expressed by these variables [6]. Of course, some hypotheses, as e.g. the local equilibrium, have to be used, because the continuum is generally inhomogeneous; it is not a priori obvious, e.g. in the cases when the homogeneity is not sufficient enough, but this problem will not be discussed in this paper.

Now, let us assume that the hydrodynamics and the thermodynamics can be applied in their usual forms to the system. Then the question is, which is the actual specific description fit for the actual physical system. Since the values of the independent thermodynamic parameters fix the actual state, while the form of the actual thermodynamic potential fixes the thermodynamic behaviour of the system [7], and the velocity field describes the motion of the matter, it seems that the main problem is the determination of the functional form of the potential, which needs the knowledge of the interactions between the particles of the matter.

Nevertheless, there are cases when the interactions are quite well known, but the thermodynamic predictions seem to be incoherent or unclear. One can mention at least three such cases: the "entropy excess" in heavy ion collisions [8], the absence of a peak in energetic heavy ion collisions [9], and the tendency for monopole dominance in the Universe [10]. In all these cases the approximations for monopole interactions seem to be quite satisfactory, but there are doubts (in the second case some evidence) that the determination of the actual number of freedom needs some careful analysis.

One may believe that the number of particle degrees of freedom can be directly determined, because the different particles can easily and clearly be distinguished. Nevertheless, the situation is not so simple as we shall see. Now, let us leave for a moment the details, which are the characteristics of the independent particle components, and consider some possible cases.

Obviously, if one miscalculates the number of independent particle degrees, and chooses a too low number, then the full richness of the system cannot be described. Moreover, any quantitative prediction connected with equipartition will be necessarily wrong (See, e.g., the discussion in Sect. 2 of Ref. [11] about the specific entropy of a nucleon-deuteron mixture.) Maybe it is surprising, but serious errors can be caused also by the overestimation of the number of particle degrees. By introducing a  $\Delta$  component in a hot nuclear matter (which is, in fact, there from the particle physicists' viewpoint), assuming that the particle gas is dilute (which is

true), calculating the hadro-chemical reactions in near-equilibrium formulation, and supposing thermal equilibrium (some reactions are rapid compared to the expansion), a very expressed peak of the  $\Delta$  resonances is predicted in the detected  $\pi$  spectrum, surviving any reasonable correction, while the measured  $\pi$  spectrum is quite thermal. The explanation is that the  $\Delta$  "particles" do not exist as asymptotic states, and so their masses are undefined; in fact the  $\Delta$  momentum distribution is governed by the  $p$  and  $\pi$  distributions. Introducing only two particle degrees, the predicted spectrum is far better [9].

Another interesting system is a hot nucleon-deuteron mixture. Since the equilibrium  $d/p$  ratio is expected to decrease with temperature, in spite of the great size of the deuteron at double normal nuclear density and several dozen MeV temperature such a mixture is actually dilute in the sense that  $n_n V_n + n_d V_d \ll 1$  (where  $V$  is some characteristic volume parameter). According to this philosophy, the mixture was treated as a two-component Boltzmann gas in chemical equilibrium [8] resulting a predicted deuteron number quite high compared to the experimental results. The calculations which explicitly incorporate the deuteron volume can reproduce the measured  $d/p$  ratio [12], [13], [14], thus one may interpret the facts in such a way that the thermodynamic potential had to be corrected. On the other hand, the mixture is actually dilute, and so the interactions neglected in the Boltzmann approximation are not too strong. Hence the picture is awkward, the average interparticle distance are not too strong. Hence the picture is, when the primary existence of deuterons is not obvious.

There is an extreme situation when it is not clear if the number of particle degrees is 0 or 1. In usual GUT cosmologies the early stages of the evolution of the Universe are charge-symmetric. There is a phase transition during this charge-symmetric evolution, producing the magnetic monopoles of the theory [15], [16]. Before the phase transition thermal equilibrium is generally assumed [15], [16]. The fluid is of zero component, having only one independent thermodynamic variable,  $s$ . The monopoles are heavy objects, and their annihilation is not efficient below  $n_m/s \sim 10^{-10}$ . Present cosmological observations impose an upper limit of cca  $10^{-22}$  for  $n_m/s$ .

Now, if one does not introduce the monopoles as an independent particle degree of freedom, the initial  $n_m/s$  ratio can be calculated by counting  $m\bar{m}$  pairs in a thermal bath, it contains a Boltzmannian suppression  $e^{-M/T}$  where  $M$  is the monopole mass, and can easily be as low as  $10^{-22}$  [17], [18]. On the other hand, introducing an independent particle degree of freedom for them, and calculating the creation rate in decent quantum field approximation, the result is  $n_m/s \sim 10^{-8}$  [15], which is reduced to  $10^{-10}$  by annihilation [19], in complete disagreement with the observations. Sophisticated processes can be manufactured to reduce this number after the phase transition [16], [20], [21], [22], but there are serious problems, and the situation is definitely not happy.

These three examples show that there are cases when it is not clear which the relevant particle degrees of freedom are. Of course, from a higher viewpoint this is a technical problem, redundant particle degrees of freedom may be used, with a correct equation of state. (As a trivial example, one may distinguish "red" and "blue" protons, with properly chosen multiplying factors in the Fermi distributions.) Nevertheless, for practical use such a viewpoint is not constructive, e.g. in a Boltzmann approximation, where the reactions are fast enough, one is tempted to start with a dilute mixture, where the reactions are fast enough, one is tempted to start with depend on the chosen number of particle degrees of freedom. Since the existence of the problem has been demonstrated, in this paper we investigate which are the way to introduce redundant particle degrees of freedom without influencing the dynamical predictions of a model.

## II. THE SELECTION OF THE THERMODYNAMIC PARAMETERS

Obviously, the selection of the proper set of thermodynamic variables is the task of thermodynamics. Nevertheless, usually the necessary thermodynamic parameters are well known from other sources [23—27] (gas, e.g., mechanics). Since in the present case this way obviously cannot be followed, let us recapitulate the constraints of the thermodynamical formalism on the parameter set. The real systems are, of course, of infinite degrees of freedom. In order to get a practical description, the real system must be substituted by an approximate one, possessing only a finite number of characteristic macroscopic data. Following Landsberg [28], this set can be selected in four steps.

First, generally there is some set of macroscopic variables, supported by common sense, for an actual physical system in a given group of equilibrium states. Secondly, using these variables one must determine the domain of permissible changes. Thirdly, at least in principle, one has to manufacture copies of the original system in the sense that all the above selected variables have the same values. Then, if an observer can see macroscopic differences between any pairs of copies, means of any possible measurement, then the set is not complete; the procedure has to be repeated with further variables representing the observed difference. If the system can indeed be approximated by a finite number of degrees of freedom, then the procedure comes to its end in finite steps. Fourthly, the so obtained description will depend on the definition of real equilibrium states and on the permissible changes in the system.

Now, when realizing this scheme, one has to specify the possible measurements, in the optimal case any equilibrium state of the evolving system can be substituted for investigation; then by producing arbitrary small changes in the parameters chosen previously, and observing the answers of the system, the entropy matrix

$$g_{\mu\nu} = -\frac{\partial^2 S}{\partial q^{\mu} \partial q^{\nu}} \quad (2.1)$$

can be measured (here  $s$  stands for the entropy density, while  $q^i$  denote the extensive densities). This matrix has to be positive definite; if it is semidefinite, we have superfluous parameters. On the other hand, by measuring the changes of intensive  $Y_i$  as answers for the prepared changes of the extensives  $X^r$  one can check the validity of the Gibbs-Duhem relation

$$\sum_r X^r dY_r = 0. \quad (2.2)$$

This relation is a consequence of the homogeneous linearity of the entropy as a function of the extensives; if the right-hand side is found to be empirically nonzero, then the set of variables is not complete. Therefore one can determine the sufficient and necessary set by combining these two kinds of measurements. Nevertheless, there are systems for which the observer is not in the position to sustain the equilibrium states of the evolution. In heavy ion collisions the evolution of the system happens on a time scale  $10^{-22}$  s, thus only the initial and final states can be compared. Similarly, in cosmology we cannot go back to the past to perform observations. Then it is not possible to induce small arbitrary changes of the parameters in the intermediate steps of evolution. The system will follow its own dynamic laws, the energy, momentum and particle balance equations completely determine the development of consecutive states. Even then, it is possible that the dynamical quantities of the system, such as, e.g., its energy-momentum tensor, may be measurable (or deducible from the final detected state), since they are not derivatives but state functions. Nevertheless, then the total entropy matrix obviously cannot be measured.

In such cases, following Landsberg's prescriptions as closely as still possible, one has to regard two copies identical if their evolutions go through the same dynamical data, and they produce the same detected final states. This is just the way conform to the methods of heavy ion physics and cosmology. Then all the parameters unnecessary to reproduce the measurable data become redundant in the description. Of course, these superfluous parameters might or might not be necessary as thermodynamic parameters to describe the behaviour of the system if one were able to prepare arbitrary intermediate stages of the evolution. However, if this cannot be done for a specific system, one has to be contented with the above mentioned definition of nonredundant degrees of freedom.

## III. THE EQUATIONS

As a consequence of the Einstein equations of General Relativity [29] the energy-momentum tensor  $T^{\mu\nu}$  of the matter fulfils conservation laws:

$$T_{\nu}^{\nu} = 0$$

$T^{\mu\nu}$  can be decomposed with respect to any timelike unit vector field  $u^i$  as

$$\begin{aligned} T^{\mu\nu} &= \rho u^{\mu} u^{\nu} + q^{\mu} u^{\nu} + u^{\mu} q^{\nu} + p^{\mu\nu} \\ q^{\mu} u_{\mu} &= p^{\mu} u_{\mu} = 0. \end{aligned} \quad (3.1)$$

For an observer of velocity  $u^i$   $\rho$  is an energy density,  $q^i$  is a (heat) energy flux and  $p^{\mu\nu}$  is a stress tensor [6]. If the matter possesses a unique macroscopic velocity field, which is assumed in hydrodynamics, then it is to be used for  $u^i$ , and, for perfect fluids, one gets [6]

$$\begin{aligned} q^i &= 0 \\ p^{\mu\nu} &= p(g^{\mu\nu} + u^{\mu} u^{\nu}) \end{aligned} \quad (3.3)$$

where  $p$  is the pressure. Heat or momentum transfer processes lead to extra terms in  $T^{\mu\nu}$ , here such irreversibilities are ignored, which is a merely technical simplification.

Eq. (3.1) can be decomposed too as

$$\begin{aligned} (\rho + p)u^i u^{\nu} + p_{\nu}(g^{\mu\nu} + u^{\mu} u^{\nu}) &= 0 \\ \rho_{\nu} u^{\nu} + (\rho + p)u^{\nu} &= 0. \end{aligned} \quad (3.4)$$

The first group fully determines the time evolution of  $u^i$ , thus the second is not an equation of motion. In fact, it is the differential version of the First Law of Thermodynamics [30], governing the evolution of the thermal variable [29], [31], [32], [33].

Now, if the independent variables are  $n_A$  and  $s$ , then  $\rho$  is the proper potential density, there are evolution equations for  $u^i$  and  $s$ , but the conservation laws (3.1) do not give anything for  $n_A$ . The system of evolution equations has to be completed by evolution equations for  $n_A$  of the form

$$(n_A u^{\nu})_{;\nu} = \Psi_A. \quad (3.5)$$

The  $\Psi_A, s$  are the source terms, but these source terms are not determined by the energy and momentum conservation [6], [11], [29], [31], [32], [33]; (as we shall see, the Second Law yields an inequality for them); they are to be calculated according to the information about individual particle reactions (cf. e.g. Ref. [34]). There is no entropy production from the source terms if they vanish, or if the reactions occur in complete chemical equilibrium [11], [30].

Using  $n_A$  and  $s$ , a combination of eqs. (3.4) and (3.5) yields

$$\rho_A(s, u^i + s u^i)_{;\nu} = -\sum_A \rho_A \Psi_A + (\rho s + \sum_A \rho_A n_A - \rho - p)u^{\nu}; \quad (3.6)$$

where the subscripts of  $\rho$  denote derivation. The left-hand side has to be positive semidefinite for normal systems of positive temperature [35], and in near equilibrium approximation  $\Psi_A, \rho$  and  $p$  are independent of  $u^i$ ; hence

$$\begin{aligned} \sum_A \rho_A \Psi_A &\leq 0 \\ p &= s\rho + \sum_A n_A \rho_A - \rho. \end{aligned} \quad (3.7)$$

Let us assume that the particular system under investigation can be described by hydrodynamics and near equilibrium thermodynamics. Then the necessary hypotheses specifying the system and leading to definite predictions are as follows: 1. An assumption for the number of independent particle degrees of freedom. 2. An expression for the energy density  $\rho$  as a function of the entropy density and the densities of the chosen (true or quasi) particles, according to the possible information about the interactions. 3. Some expressions for the source terms of the chosen particles, based again on the information about the particle interactions; these expressions have to be compatible with the inequality (3.7). 4. If there are momentum and heat transfer processes, expressions for viscosity and heat conduction coefficients satisfying positivity conditions coming from the Second Law [6], [31].

The formalism used up to now is the energy convention; the entropy is an extensive parameter, and the energy is a potential. If useful, these two roles can be reversed, arriving at the entropy convention. Consider  $\rho$  as the thermal variable and  $s = s(\rho, n_A)$  as a potential. Then a combination of eqs. (3.4, 3.5) leads to the entropy production

$$s_{\nu} u^{\nu} + s u^{\nu};_{\nu} = s_A \Psi_A + \left( s - s_0 \rho - s_0 p - \sum_A s_A n_A \right) u^{\nu}; \quad (3.8)$$

whence

$$\begin{aligned} s_A \Psi_A &\geq 0 \\ p &= -\rho + \frac{1}{s_0} \left( s - \sum_A n_A s_A \right). \end{aligned} \quad (3.9)$$

Of course, the function  $s(\rho, n_A)$  is not in as direct a connection with the particle interactions as the inverse function  $\rho(s, n_A)$ . Nevertheless, they can be calculated from each other.

Wherefrom we introduce a convenient shorthand notation

$$\dot{a} = a_{,\nu} u^{\nu} \quad (3.10)$$

which is, in fact, the change rate of a scalar along a world line.

#### IV. THE INTRODUCTION OF REDUNDANT COMPONENTS

Let us first recapitulate the steps of a hydrodynamic description. Consider a physical system for which there exist some arguments so that it can be suitably described by (for simplicity's sake) a perfect fluid. Then

$$T^{ik} = \rho u^i u^k + p(\rho^{\#k} + u^i u^k). \quad (4.1)$$

Since the components of the energy-momentum tensor can be directly measured, the energy density  $\rho$ , the pressure  $p$  and the velocity  $u^i$  possess a direct physical meaning and their values are uniquely fixed. Hence one gets the equations of motion for  $u^i$  (the first of eq. (4.4), not repeated here), and an evolution equation for the energy density  $\rho$  regarded here as the thermal variable:

$$\dot{\rho} + (\rho + p)u^i_{;i} = 0. \quad (4.2)$$

Now, assume (from some information) that  $N$  particle degrees are necessary for a satisfactory description. Then there exist an entropy density function  $s$

$$\begin{aligned} s &= s(\rho, n_A) \\ A &= 1, \dots, N. \end{aligned} \quad (4.3)$$

Nevertheless, from our viewpoint, these particles are auxiliary quantities; they are not necessarily identified with the observable entities of particle physics (in the Introduction some examples supporting this viewpoint were mentioned). For these particles evolution equations of the form

$$\dot{n}_A + n_A u^i_{;i} = \Psi_A(\rho, n_A) \quad (4.4)$$

are assumed. The pressure can be expressed as

$$p = -\rho + \frac{1}{s_0} \left( s - \sum_{R=1}^N s_R n_R \right) \quad (4.5)$$

$$s_A = \frac{\partial s}{\partial n_A}$$

Then, for the entropy production, one gets

$$\dot{s} + s u^i_{;i} = \sum_{R=1}^N \Psi_R s_R. \quad (4.6)$$

Thus, there exists a compatibility condition between  $\Psi_A(\rho, n_A)$  and  $s(\rho, n_A)$ , according to the Second Law

$$\sum_{R=1}^N \Psi_R s_R \geq 0 \quad (4.7)$$

for any set of variables  $\rho, n_A$ . This reflects the fact that both  $\Psi_A$  and  $s$  represent something about the interactions of the (hypothetic) particles.

Now, assume that the chosen model has given a satisfactory description of the physical situation, but, from some reason, one wants to introduce one more component, which is a combination of the original ones

$$n_e = \varphi(\rho, n_A). \quad (4.8)$$

When can it be done and how?

One has to go back before eq. (4.3). Introduce first the extra particle density in the original formalism in such a way that there nothing depends on  $n_e$ . Then assume that there exists another entropy density  $r$  for the system explicitly depending on  $n_e$ , which lead to the same observable behaviour of the system. That is,  $\rho, p$  and  $u^i$  remain the same, and the first  $N$  particle components are the same by construction. Then

$$\begin{aligned} r &= r(\bar{\rho}, \bar{n}_A, n_e); \quad \bar{\rho} = \rho; \quad \bar{n}_A = n_A; \quad n_e = \rho \\ A &= 1, \dots, n \end{aligned} \quad (4.3')$$

$$\begin{aligned} \dot{\bar{n}} + \bar{n}_A u^i_{;i} &= \bar{\Psi}_A(\bar{\rho}, \bar{n}_A, n_e) = \Psi_A(\rho, n_A) \\ \dot{n}_e + n_e u^i_{;i} &= \Psi_e = \dot{\varphi} + \varphi u^i_{;i} \end{aligned} \quad (4.4')$$

$$p = -\rho + \frac{1}{r_0} \left( r - \sum_{R=1}^N r_R \bar{n}_R - r_e n_e \right); \quad \bar{p} = p \quad (4.5')$$

$$\dot{r} + r u^i_{;i} = \sum_{R=1}^N \bar{\Psi}_R r_R + \Psi_e n_e \geq 0. \quad (4.6')$$

Now, it is necessary to formally introduce a new function of the old variables

$$\bar{r}(\rho, n_A, \varphi) = r(\bar{\rho}, \bar{n}_A, n_e). \quad (4.9)$$

Then the in variance of the pressure yields an equation between  $s$  and  $r$ :

$$s_0(\bar{r} - \bar{r}_0 \varphi) + \sum_{R=1}^N (\bar{r}_0 \varphi_R - \bar{r}_R) n_R - (\bar{r}_0 - \bar{r}_0 \varphi) \left( s - \sum_{R=1}^N s_R n_R \right) = 0. \quad (4.10)$$

The entropy production equation can be rewritten as

$$\begin{aligned} \bar{r}_0 \dot{\varphi} + \sum_{R=1}^N \bar{r}_R \dot{n}_R + \bar{r} u^i_{;i} &= \sum_{R=1}^N \Psi_R (\bar{r}_R - \bar{r}_0 \varphi_R) + \\ &+ \bar{r}_0 \left\{ \varphi_0 \dot{\varphi} + \sum_{R=1}^N \varphi_R \dot{n}_R + \varphi u^i_{;i} \right\} \geq 0. \end{aligned} \quad (4.11)$$

Substituting  $\dot{\varphi}$  and  $\dot{n}_A$  from the evolution equations (4.2) and (4.4), and separating the terms with and without  $u^i_{;i}$ , one gets an equation

$$(\varrho + p)\bar{r}_e + \sum_{R=1}^N \bar{r}_R n_R - \bar{r} = \left[ (\varrho + p)\varphi_e + \sum_{R=1}^N \varphi_R n_R - \varphi \right] \bar{r}_e \quad (4.12)$$

and an inequality

$$\sum_{R=1}^N \Psi_R \bar{r}_R + \bar{r}_e \left[ (\varrho + p)\varphi_e + \sum_{R=1}^N n_R \varphi_R - \varphi \right] u_r' \geq 0 \quad (4.13)$$

But, again, the first term does not contain the velocity divergence, and the sign of  $u_r'$  is indefinite, so the inequality can hold only if

$$(\varrho + p)\varphi_e + \sum_{R=1}^N n_R \varphi_R - \varphi = 0 \quad (4.14)$$

$$\sum_{R=1}^N \Psi_R r_R \geq 0. \quad (4.15)$$

Finally,  $\Psi_r$  in the second of eqs. (4.4') should depend only on the thermodynamical variables, but not on  $u_r'$ . Substituting the dot derivatives from the evolution equations, this condition leads again to eq. (4.14), and there remains

$$\Psi_r = \sum_{R=1}^N \Psi_R \varphi_R. \quad (4.16)$$

Now, there are three equations to be solved, namely eqs. (4.10), (4.12) and (4.14), together with a constraint of inequality form, i.e. eq. (4.15). The solution of the equations has the form

$$\bar{r} = s w(n_A/s, \varphi/s)$$

$$\varphi = s \chi(n_A/s). \quad (4.17)$$

Substituting this into eq. (3.15), the inequality gets the form

$$\sum_{R=1}^N \Psi_R \left\{ w_{SR} + s \chi_{SR} w_r + w_R - \sum_{r=1}^N (w_r n_r) s r \frac{1}{s} \right\} \geq 0$$

$$x_A = \frac{n_A}{s} \quad y = \frac{\varphi}{s}. \quad (4.18)$$

We will not discuss this inequality here; it can trivially hold if all the old components are conserved, otherwise it is a constraint for the functions  $\chi(x_A)$  and  $w(x_A, y)$ , whose form explicitly depends on the forms of the source terms, not specified here.

Observe that we have not required that the value of the new entropy, or that of the entropy production, be equal with the old one. If the entropy could be directly measured, it should be required too, leading to

$$\Psi_r r_e = \sum_{R=1}^N \Psi_R \varphi_R w_r = 0. \quad (4.19)$$

Since  $w_r$  must not vanish (otherwise it has been even the formal introduction of the new component unsuccessful), this means that the new component should be conserved, which is a very special case. Nevertheless,  $s$  is not directly observable, unless the system has a statistical description, when, of course, the number of independent components is well defined, and the whole problem investigated here is nonexistent. Furthermore, there are familiar situations when  $s$  is not fully defined, e.g. Callen's postulate system leaves a freedom in it [35]

$$s' = s + s_0 + c_1 \varrho + \sum_{R=1}^N c_R n_R. \quad (4.20)$$

Then, summarizing the results of this Section: for a perfect fluid system well described by means of  $N$  particle components, a new and redundant particle component can be introduced without influencing the observables if the density of the extra component  $n_e$  is chosen as

$$n_e = \varphi \quad (4.21)$$

with a new entropy  $r = \bar{r}$ . The functions  $\varphi$  and  $\bar{r}$  are given by eq. (4.17), and  $r$  has to satisfy the constraint, (4.18) too. Since the new particles are redundant, this component is not necessary, its use is quite optional. On the other hand, if the density of the new component differs from the choice in eq. (4.21), then the two model systems possessing  $N$  and  $N+1$  components, respectively, do dynamically differ, their predictions cannot be identical, and then the proper model has to be carefully selected by physical arguments.

Now we are going to demonstrate the consequence of this general abstract statement on special but important cases.

#### V. PERFECT FLUIDS WITH ONE CONSERVED COMPONENT

Consider a perfect fluid with a single (independent) particle component:

$$s = s(\varrho, n) \quad (5.1)$$

$$p = -\varrho + \frac{1}{s_e} (s - n s_e).$$

Now, in physical systems, if there is only one particle component it is generally conserved, because there are no possibilities for chemical reactions (an exception is Dirac's steady-state cosmology with continuous creation, cf. Ref. [36]; this case is not discussed here). Then

$$\dot{n} + n u_r' = 0 \rightarrow \Psi = 0. \quad (5.2)$$

Then the inequality (4.18) automatically holds. For  $n_e$  and  $r$  one obtains

$$\begin{aligned} n_e &= s(\varrho, n)\chi(n/s) \\ r &= s(\varrho, n)w(n/s, \chi). \end{aligned} \quad (5.3)$$

The functions  $\chi$  and  $w$  are now completely arbitrary, but this does not mean too much. Namely, because of  $\Psi = 0$ , both  $s$  and  $n$  are conserved densities, so  $n/s$  is conserved along world lines, thus  $n_e/s$  and  $r/s$  are constant along world lines too, independently of the functional forms chosen in eq. (5.3). Nevertheless, the introduction of the new component is not a fully trivial process; the ratio  $n_e/s$  and  $r/s$  can differ between different world lines.

Now, consider a two component system manufactured in this way from a one component one. Perhaps the most characteristic feature is that the ratio of the two particle number densities is constant along world lines if the evolution is adiabatic (which is automatically true now, the momentum and heat transfer processes being ignored). In fact, a combination of eqs. (5.3) yields

$$\frac{n_e}{n} = \Phi\left(\frac{r}{n}\right) \quad (5.4)$$

where  $r$  is the actual entropy and  $n$  is the conserved density. If this functional dependence holds, one component can explicitly be eliminated.

Now, eq. (5.4) holds for the double Boltzmannian model of the nucleon-deuteron-mixture [8]; the  $d/p$  ratio is conserved during adiabatic expansion. In more general models eq. (5.4) does not hold (for an explicit example see Ref. 13). This is not obvious, since one can be sure that after all a nucleon-deuteron mixture consists of nucleons only, in principle it could be described only by means of nucleons, and it is usual to use a chemical equilibrium condition

$$\mu_d = 2\mu_n \quad (5.5)$$

so there are only two independent thermodynamical parameters anyway. Let us briefly discuss this question.

Consider a two component system, and use the intensives  $T$ ,  $\mu_1$  and  $\mu_2$  as independent parameters; then  $p$  is the proper potential, and

$$\begin{aligned} s &= pT \\ n_1 &= p_1 \\ n_2 &= p_2. \end{aligned} \quad (5.6)$$

Assume that  $n_1 + 2n_2$  is conserved, and that there is a chemical equilibrium between the components, then the condition for it is

$$\mu_2 = 2\mu_1 \quad (5.7)$$

Now, obviously, both  $n_2/n_1$  and  $(n_1 + 2n_2)/s$  depend on two variables,  $T$  and  $\mu_1$ , so there is no reason to expect that there be a functional dependence between these quantities. We emphasize that our purpose is not the discussion of the possible number of elementary particles; we required that the forms of the thermodynamic and hydrodynamic equations be the same for two components and for one. For example, we required that the source term of the redundant component (eq. (4.4')) depend only on  $\varrho$  and  $n_A$ , which is not necessarily true if the extra component is some two-particle correlation.

Thus, eq. (5.4) is not a consequence of the chemical equilibrium. Rather, it directly follows from the Boltzmannian hypothesis; then the mass parameter is simply a multiplying factor in the thermodynamic quantities, and there is a factorization in  $T$  and  $\mu/T$  dependences (there is only one  $\mu$  in equilibrium). Now, then both  $n_2/n_1$  and  $(n_1 + n_2)/s$  can depend only on  $\mu/T$ , so they are functionally dependent, and the deuteron degree of freedom is redundant. Eq. (5.4) cannot hold if the deuteron component has a characteristic dimensional parameter of its own (e.g. a volume  $V$ , cf. Refs. [12—14]). Namely then it is expected to occur in the specific entropy. The specific entropy being dimensionless, one expects a formula of the type

$$\begin{aligned} \frac{s}{n} &= \sigma(c, nV) \\ n &= n_1 + 2n_2 \\ c &= n_2/n \end{aligned} \quad (5.8)$$

for dimensional reasons. As a direct consequence, a suppression may appear for the deuterons, [12], [13], [14], and the ratio  $n_2/n_1$  is not conserved in adiabatic processes.

In this context the measurements for the final  $d/p$  ratio in heavy ion collisions may be regarded as measurements for the number of the independent particle degrees of freedom in the dynamic sense. The model, in which the deuteron degree is redundant [8] predicted a  $d/p$  ratio, which is incompatible with experimental results (cf. e.g. Ref. [37]). Thus one can conclude that the deuteron degree is not redundant, the number of components is 2.

*Mutatis mutandis*, some of the results of this section can be applied to a nucleon-delta-pion mixture too, although there one should start from a two component fluid, one component is conserved, one is not. In Ref. [9] it was shown that an explicit incorporation of  $\Delta$  particles with their own characteristic mass leads to predictions incompatible with observations; if one wants to use  $\Delta$ 's during the calculation, he has to do it with an effective mass equal to the centre of mass energy of the colliding  $p, \pi$  pair. Such a  $\Delta$  component is redundant, so the observations yield the information that in this mixture the number of independent thermodynamic particle degrees of freedom is 2.

## VI. PERFECT FLUIDS WITHOUT PARTICLE COMPONENT

There exist very special systems in which there is one single independent extensive density, the energy density  $\varrho$ . For brevity's sake, such systems will be referred to here as blackbody radiation, although the form of their equation of state is not necessarily  $s \sim \varrho^{3/4}$ . Such systems are generally some equilibrium charge-symmetric mixtures of particles; there are (or partially, were) two important situations when they play some role. The first is multiparticle production in particle-particle collisions at high energies. Although then the initial situation is not charge-symmetric, if sufficient number of pairs is produced, they dominate the incoming two particles. Such systems are more or less described by Walecka's mean field theory [38], yielding the equation of state in the limiting case  $\mu = 0$  for nucleon-antinucleon pairs. The other important scene is the very early Universe. There are GUT cosmologies in which the antiparticles are continuously vanishing be some CP-violating interactions [39]. Then the very early stages were charge-symmetric. In fact, it is generally assumed that before the phase transition giving expectation values to some Higgs bosons the matter of the Universe was similar to a blackbody radiation [15], and for some time it remained similar even after the phase transition except for the monopoles produced in the transition. In fact, the monopole distribution is charge-symmetric too, but it is generally accepted that the density of the  $m\bar{m}$  pairs is not the equilibrium value (they have been produced in nonequilibrium processes), so they should be introduced as an independent component.

Since even in equilibrium one may need the number of one kind of particles, e.g. to calculate mean free paths [15], [19] let us apply our formalism to this case. Now the results are almost trivial. There is no particle density variable on the right-hand side of eqs. (4.17), so the  $\chi$  is to be taken at 0 argument, and is a constant dimensionless number:

$$n_c = C_s$$

$$r = sw \left( \frac{n_c}{s} \right) = sw(C). \quad (6.1)$$

Since  $w$  is dimensionless,  $w(C)$  is another dimensionless number. But then both  $n_c/s$  and  $n_c/r$  are constant numbers.

This result reflects some physical facts. Consider light particles (whose mass is small compared to the temperature). Then, according to the equipartition of the energy in an equilibrium system, we have for them

$$\frac{n}{s} \sim \frac{1}{N} \quad (6.2)$$

(there is a factor of order of 1, depending on the Bose or the Fermi character, and on degeneracy factors) [15].  $N$  is the number of different types of light particles, so then  $C \sim 1/N$ . Nevertheless, eq. (6.1) possesses this direct meaning only for light particles. For heavy ones equilibrium distributions give an exponential suppression in  $n$

$$\frac{n}{s} \sim e^{-M/r} \quad (6.3)$$

where  $M$  is the particle mass. Such a factor cannot be incorporated into eq. (6.1). Thus, if one is convinced that the actual system is charge-symmetric in equilibrium, then he may introduce the light particles as independent degrees of freedom (although they are redundant), but neither should be done for the heavy ones, or they will be completely meaningless because their number densities will be different from the physical densities.

Now, there is no a priori reason to believe that this result has any connection with monopoles. If one has some particle physical arguments that the monopoles are important and independent components of the matter after the phase transition, then there the matter has one, nonredundant, particle degree of freedom. Nevertheless, in the usual approximation, monopoles surprisingly simulate redundant particles. Namely, most of the field theoretical predictions agree in two points. Just after the phase transition the monopoles appear a) without exponential suppression; and b) in simplest approximation  $n/s \sim 1/(p\lambda^3 N)$ , where  $p \sim 8$  and  $\lambda$  is a dimensionless constant coefficient of the Higgs self-interaction.

For a review see Ref. [15] (This density may later decrease by annihilation.) This is exactly which can be expected for a redundant particle (cf. eq. (6.1)). Here it is worthwhile to note that there exists an approach when both  $n$  and  $s$  are calculated from thermodynamics; its prediction is  $n/s \sim \bar{p}$  [40]. However, similarly to field theoretical estimations, energy and entropy balance of the monopole creation was not investigated there, hence it is quite possible that the result is the density of some monopole precursors only.

Now we can observe the difference between the prediction of Ref. [17] and that of the majority of the literature from a new angle. Ref. [17] does not introduce explicitly the monopole component, and, of course, obtain an exponential suppression. In the other approach this degree of freedom is introduced as an independent one, but it remains dynamically redundant (and, e.g. in Ref. [19], where the annihilation is calculated, it does not appear even in the equation of state), so that from thermodynamical viewpoint it is quite possible that this number density is a ghost quantity.

Therefore it is possible that there is no real contradiction between the different predictions. Nevertheless, if one accepts the predicted  $n/s$  ratios in face values, then the prediction of Ref. [17] is directly compatible with recent cosmologic



observations [41], while the predictions giving  $n/s$  in the order of (more or less) 1 are not; then the present Universe would be monopole-dominated. In these models generally some nonadiabatic stage is assumed after the monopole creation [15], [20], [22]. Such processes are possible but there is so direct evidence for their role in the evolution of the Universe.

## VII. CONCLUSIONS

In the present paper we have investigated the conditions that one particle component of a perfect fluid be hydrodynamically redundant. Then in the description of processes, when the intermediate steps of evolution cannot be sustained for detailed measurements (which is definitely the situation in heavy ion physics and cosmology), such a component may be introduced if needs be, or removed from among the thermodynamic parameters without altering the thermodynamic formalism and the hydrodynamic observables. Therefore if there is no possibility to prepare such states of the actual system when it is in hydrodynamic equilibrium, then this particle component is unnecessary, hence there is no evidence for its existence in the thermodynamic sense. (Of course, a redundant particle may be detected on the one hand, and superfluous for the description on the other.) We have concluded that in the Boltzmannian model of the nucleon-deuteron mixture the deuteron component is redundant, the dynamics of a closed system of such a mixture is identical with that of an appropriately chosen simpler system of one component. Since such a mixture can be prepared and investigated only in heavy ion collisions, when one cannot influence the intermediate steps of the evolution, i.e. the system is dynamically closed (energy and momentum balances are valid), the measurements can indicate only the number of dynamically nonredundant particle degrees of freedom; in the Boltzmannian model it is 1, while the experimental value seems to be rather 2.

When describing a  $p + \pi + \Delta$  mixture at double nuclear density and cca. 100 MeV temperature, a  $\Delta$  component of fixed mass and width would not be redundant; on the contrary it was shown [9] that it is not to be incorporated into the thermodynamical formalism, so the number of the particle degrees of freedom of the system suggested by measurements is again 2.

For the early Universe just after the creation of GUT magnetic monopoles the situation is less clear. The assumption that the number of independent particle components is 0 leads to monopole densities compatible with observation, but this assumption is criticized from particle physical viewpoint. On the other hand, introducing explicitly the monopole component into the formalism the predictions of recent calculations are the same as if this component were redundant. So one can conclude that the determination of the number of particle degrees of freedom in the very early Universe would need a thermodynamically more consequent and

self-consistent treatment of the monopole creation and of the evolution of the Universe.

## ACKNOWLEDGEMENTS

The authors would like to thank Prof. I. Kirschner, Drs. I. Borbély, H. W. Barz, P. Danielewicz, L. Diósi, G. Paál and J. Zimányi for illuminating discussions.

## REFERENCES

- [1] Colgate, S. A., White, R. H.: *Ap. J.* **143** (1966), 626.
- [2] Hawking, S. W., Ellis, G. F. R.: *The Large Scale Structure of Space-Time*. The Univ. Press, Cambridge, 1973.
- [3] Ellis, G. F. R.: *Proc. Int. School "Enrico Fermi"* **47** (1971), 104.
- [4] Bondorf, J., Garpman, S., Zimányi, J.: *Nucl. Phys. A* **296** (1978), 320.
- [5] Amsten, A. A., Harlow, F. H., Nix, J. R.: *Phys. Rev. C* **15** (1977), 2059.
- [6] Ehlers, J.: in: *Relativity, Astrophysics and Cosmology*, ed. W. Israel, D. Reidel Publ. Co., Dordrecht-Boston, 1973.
- [7] Diósi, L., Forgács, G., Lukács, B., Frisch, H. L.: *Phys. Rev. A* **29** (1984), 3343.
- [8] Siemens, P. J., Kapusta, J. I.: *Phys. Rev. Lett.* **43** (1979), 1486.
- [9] Barz, H. W., Lukács, B., Zimányi, J., Fai, G., Jacobson, B.: *Z. Phys. A* **302** (1981), 73.
- [10] Fry, J. N., Schramm, D. N.: *Phys. Rev. Lett.* **44** (1980), 1361.
- [11] Biró, T., Barz, H. W., Lukács, B., Zimányi, J.: *Phys. Rev. C* **27** (1983), 2695.
- [12] Biró, T., Barz, H. W., Lukács, B., Zimányi, J.: *KFKI* — 1981 — 90.
- [13] Lukács, B.: *Proc. 6th Balaton Conf. on Nucl. Phys.* 1983.
- [14] Kibble, T. W. B.: in: *Monopoles in Quantum Field Theories*, Proc. Monopole Meeting, Trieste, eds. M. S. Craigie et al., World Scientific, Singapore 1983.
- [15] Guth, A.: *Phys. Rev. D* **23** (1981), 347.
- [16] Bais, F. A., Rudaz, S.: *Nucl. Phys. B* **170** (1980), 507.
- [17] Diósi, L., Bettina Keszthelyi, Lukács, B., Paál, G.: *KFKI* — 1984 — 10.
- [18] Diósi, L., Bettina Keszthelyi, Lukács, B., Paál, G.: *KFKI* — 1984 — 10.
- [19] Preskill, J.: *Phys. Rev. Lett.* **43** (1979), 1365.
- [20] Albrecht, A., Steinhart, P. J.: *Phys. Rev. Lett.* **48** (1982), 1220.
- [21] Langacker, P., Pi, S. Y.: *Phys. Rev. Lett.* **45** (1980), 1.
- [22] Diósi, L., Bettina Keszthelyi, Lukács, B., Paál, G.: *Acta Phys. Polonica B* **15** (1984), 828.
- [23] Kirschner, I.: *Magy. Fiz. Foly.* **18** (1969), 71.
- [24] Kirschner, I.: *Acta Phys. Hung.* **29** (1970), 209.
- [25] Kirschner, I.: *Acta Phys. Hung.* **30** (1971), 61.
- [26] Tisza, L.: *Ann. Phys.* **13** (1961), 1.
- [27] Fényes, I.: *Z. Phys.* **134** (1952), S 95.
- [28] Landsberg, P. T.: *Thermodynamics*. Interscience Publ. New York, 1961.
- [29] Csernai, L. P., Lukács, B.: *Acta Phys. Polonica B* **15** (1984), 149.
- [30] Lukács, B., Martinás, K.: *KFKI* — 1984 — 25.
- [31] Lukács, B.: *KFKI* — 1978 — 82.
- [32] Lukács, B.: *Acta Phys. Polonica B* **14** (1983), 33.
- [33] Lukács, B., Csernai, L. P.: *Proc. EPS Top. Conf. on Large Ampl. Coll. Nucl. Motion, Keszthely* 1979.

- [34] Montvay, I., Zimányi, J.: *Nucl. Phys. A* 316 (1979), 490.
- [35] Martinás, K.: *Acta Phys. Hung.* 50 (1981), 121.
- [36] Dirac, P. A. M.: *Proc. Roy. Soc. Lond., A* 338 (1974), 439.
- [37] Nagamiya, S., Lemaire, M. C., Moeller, E., Schnetzer, S., Shapiro, G., Steiner, H., Tanihara, I.: *LBL — 12123* (1981).
- [38] Walecka, J. D.: *Phys. Lett.* 59 B (1975), 109.
- [39] Langacker, P.: *Phys. Rep.* 72 (1981), 185.
- [40] Diósi, L., Lukács, B.: *KFKI — 1984 — 102*.
- [41] Faber, S. M., Gallagher, J. S.: *Ann. Rev. Astron. Astroph.* 17 (1979), 135.

Received February 12th, 1985