acta phys. slov. 35 (1985), No. 6

BLEUSTEIN GULYAEV WAVES IN A PIEZOELECTRIC HALF SPACE WITH A TIME DEPENDENT PIEZOELECTRIC CONSTANT

M. GANGULY[1]), Calcutta

The present paper investigates the propagation of Bleustein-Gulyaev waves in a 6 mm hexagonal piezoelectric half space assuming the piezoelectric constant to be time dependent as in Voight's model of viscoelasticity. Expressions for the various surface wave characteristics have been determined when there exists a shorting plane at a certain height above the free surface of the medium.

ВОЛНЫ БЛЕЙСТЕЙНА-ГУЛЯЕВА В ПЬЕЗОЭЛЕКТРИЧЕСКОМ ПОЛУПРОСТРАНСТВЕ С ПЬЕЗОЭЛЕКТРИЧЕСКОЙ ПОСТОЯННОЙ, ЗАВИСЯЩЕЙ ОТ ВРЕМЕНИ

В работе исследовано распространение волн Блейстейна-Гуляева в 6-миллиметровом гексагональном пьезоэлектрическом полупространстве в предположении, что пьезоэлектрическая постоянная зависит от времени аналогично модели Фойгта для упруговязких свойств среды. Выражения для различных характеристик плоских волн были определены для случая, когда на определенной высоте над свободной поверхностью среды существует плоскость замыкания.

I. INTRODUCTION

The existence of transverse surface waves in piezoelectric crystals of 6 mm hexagonal symmetry with no counterpart in purely elastic homogeneous material was first established independently by Bleustein and Gulyaev in two of their papers, [1] and [2], respectively. After the discovery of Bleustein and Gulyaev several other researches have worked on these waves (Ludvik and Quate [3], Cambon [4], Fischler [5], Pajewski [6], etc., to name only a few).

The object of the present paper is to extend the works of the above named researchers to a new situation where the piezoelectric constant is assumed to depend on time t as in Voight's model of viscoelasticity. The expressions for the displacement components, group velocity, maximum penetration depth, power flow components, the deviation of group velocity from the direction of phase velocity, amplification or attenuation coefficient, effective surface permittivity, etc., have been determined considering the existence of a shorting plane at a certain height above the free surface of the medium.

[1]) Department of Mathematics Jadavpur University, CALCUTTA — 700032, India.

II. PROBLEM AND FUNDAMENTAL EQUATIONS

The fundamental equations of the problem are the equations of mechanical motion and the equations of state of the piezoelectric material.

The vibration equation and Gauss's divergence equations are given by

$$\varphi \ddot{u}_i = T_{ij,j} \tag{1}$$

and

$$D_{i,i} = 0. \tag{2}$$

The constitutive equations of the piezoelectric half space on which the surface waves are assumed to propagate are

$$T_{ij} = c_{ijkl} S_{kl} - e_{mij} E_m \tag{3}$$

and

$$D_j = e_{jkl} S_{kl} + \epsilon_{ij} E_i$$

(see Tiersten [7]).

Where T_{ij} are the stress components, S_{kl} are the strain components, D_i are the electric displacement components, E_i are the electric field components and u_i the displacement components. c_{ijkl}, e_{mij}, ϵ_{ij} and φ are the elastic stiffness constants, piezoelectric constants, dielectric constants and density of the material. Here the summation convention for repeated tensor indices is employed and an index preceded by a comma denotes differentiation with respect to some space coordinates. Dot notation signifies time derivative. In addition to the equation presented above the following two are also important for the problem.

The strain component

$$S_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k}) \tag{4}$$

and if φ be the potential function then the electric field components are given by

$$E_i = -\varphi_{,i}. \tag{5}$$

The elastic stiffness constant c_{ijkl} and the piezoelectric constants e_{mij} appearing in the constitutive equations (5) are with four and three indices, respectively. These constants can be expressed in two index notations (see, Mason [8]). After reducing all the constants to double index notations and making use of the matrices for elastic stiffness, piezoelectric and dielectric constants for piezocrystals with 6 mm hexagonal symmetry, (see, Tiersten [7]) we find the equation (3) in the following form

$$\begin{aligned} T_{11} &= c_{11}u_{1,1} + c_{12}u_{1,2} + u_{3,3} + e_{31}\varphi_{,3} \\ T_{22} &= c_{12}u_{1,2} + c_{11}u_{2,2} + c_{13}u_{3,3} + e_{31}\varphi_{,3} \\ T_{33} &= c_{13}u_{1,1} + c_{13}u_{2,2} + c_{33}u_{3,3} + e_{33}\varphi_{,3} \\ T_{23} &= c_{44}(u_{3,2} + u_{2,3}) + e_{15}\varphi_{,2} \\ T_{31} &= c_{44}(u_{3,1} + u_{1,3}) + e_{15}\varphi_{,1} \\ T_{12} &= c_{66}(u_{1,2} + u_{2,1}) \\ D_1 &= e_{15}u_{3,1} + e_{15}u_{1,3} - \epsilon_{11}\varphi_{,1} \\ D_2 &= e_{15}(u_{2,3} + u_{3,2}) - \epsilon_{11}\varphi_{,2} \\ D_3 &= e_{31}u_{1,1} + e_{31}u_{2,2} + e_{33}u_{3,3} - \epsilon_{33}\varphi_{,3}. \end{aligned} \tag{6}$$

Since we limit ourselves to a discussion of the propagation of transverse Bleustein Gulyaev waves, the displacement components u_1 and u_2 can be taken to be zero and the remaining unknowns u_3 and φ are independent of the x_3 coordinate. The above simplifying conditions reduce the equations (6) to the following form

$$\begin{aligned} T_{11} &= T_{22} = T_{33} = T_{12} = 0 \\ T_{23} &= c_{44}u_{3,2} + e_{15}\varphi_{,2} \\ T_{31} &= c_{44}u_{3,1} + e_{15}\varphi_{,1} \\ D_1 &= e_{15}u_{3,1} - \epsilon_{11}\varphi_{,1} \\ D_2 &= e_{15}u_{3,2} - \epsilon_{11}\varphi_{,2} \\ D_3 &= 0. \end{aligned} \tag{7}$$

In the present problem the piezoelectric constant e_{15} is assumed to be time dependent similar to Voight's viscoelastic model. So we can write

$$e_{15} = e_{15}^{(1)} + e_{15}^{(2)} \frac{\partial}{\partial t}.$$

Now using equations (6) and (7) and remembering the assumption mentioned above we find from the vibration equation (1) and Gauss's divergence equation (2) the following two fundamental equations of the problem:

$$\varphi \ddot{u}_3 = c_{44}\nabla^2 u_3 + e_{15}^{(1)}\nabla^2\varphi + e_{15}^{(2)}\nabla^2\varphi \tag{8}$$

$$e_{15}^{(1)}\nabla^2 u_3 + e_{15}^{(2)}\nabla^2 u_3 - \epsilon_{11}\nabla^2\varphi = 0. \tag{9}$$

Where ∇^2 is the Laplacian operator

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}.$$

Introducing the potential function

$$\psi = \varphi - \frac{1}{\epsilon_{11}}[e_{15}^{(1)}u_3 + e_{15}^{(2)}\dot{u}_3]$$

the above two equations further reduce to the following

$$\varphi \ddot{u}_3 = \left[c_{44} + \frac{(e_{15}^{(1)})^2}{\epsilon_{11}}\right]\nabla^2 u_3 + \frac{2e_{15}^{(1)}e_{15}^{(2)}}{\epsilon_{11}}\nabla^2\dot{u}_3 + \frac{(e_{15}^{(2)})^2}{\epsilon_{11}}\nabla^2\ddot{u}_3 \tag{10}$$

and

$$\nabla^2\psi = 0. \tag{11}$$

III. BOUNDARY CONDITIONS OF THE PROBLEM

Since the disturbance is propagating over a hexagonal piezoelectric half space $x_2 \geqslant 0$ there must be some prescribed conditions to be satisfied on the surface $u_2 = 0$.

In the present problem we consider the propagating surface to be stress-free and we also assume the existence of a massless electrical shorting plane at a distance h above the surface. The region between the piezoelectric substrate and the shorting plane being vacuum. Here we have to consider the equation of the electric field in vacuum between the half space and the shorting plane.

(i) If $\hat{E}_i$ is the electric field in vacuum then $\hat{E}_{i,i} = 0$ and hence using the potential $\hat{E}_i = -\hat{\varphi}_{,i}$ we find

$$\nabla^2\hat{\varphi} = 0. \tag{12}$$

Where $\hat{\varphi}$ is the electric potential in vacuum.

Since we assume the existence of a shorting plane at a distance h above the free surface of the piezoelectric half-space

$$\hat{\varphi}\big]_{x_2=-h} = 0. \tag{12'}$$

(ii) The boundary condition at the free surface can be taken as stress component

$$T_{23} = 0 \quad \text{at} \quad u_2 = 0. \tag{13}$$

(iii) The tangential components of the electric potential are continuous at the free surface $u_2 = 0$. Hence

$$\varphi_{11} = \hat{\varphi}_{,1} \quad \text{at} \quad u_2 = 0 \tag{14}$$

where φ and $\hat{\varphi}$ are electric potentials in the substrate and in the vacuum.

(iv) Normal components of the electric displacement are continuous at the free surface $u_2=0$. Hence

$$D_2=\hat{D}_2 \quad \text{at} \quad u_2=0 \tag{15}$$

where D_2 and $\hat{D}_2$ are the normal components of electric displacement in the substrate and vacuum.

IV. SOLUTION OF THE PROBLEM

Let us assume the solution of the problem (10) and (11) in the following form

$$u_3=A_1 \exp(-y_2u_2)\exp\{i(y_1u_1-\omega t)\} \tag{16}$$
$$\psi=A_2 \exp(-y_1u_2)\exp\{i(y_1u_1-\omega t)\}.$$

This assumption satisfies equation (11) identically and the other equation (10) requires

$$\left\{\left[c_{44}+\frac{(e_{15}^{(1)})^2}{\epsilon_{11}}\right]-\frac{2i\omega e_{15}^{(1)}e_{15}^{(2)}}{\epsilon_{11}}-\frac{(e_{15}^{(2)})^2\omega^2}{\epsilon_{11}}\right\}(y_2^2-y_1^2)+\varphi\omega^2=0. \tag{17}$$

Introducing the decay parameter α defined by $\alpha=y_2/y_1$ the above equation becomes

$$\left\{c_{44}+\frac{(e_{15}^{(1)})^2}{\epsilon_{11}}-\frac{2i\omega e_{15}^{(1)}e_{15}^{(2)}}{\epsilon_{11}}-\frac{(e_{15}^{(2)})^2\omega^2}{\epsilon_{11}}\right\}(\alpha^2-1)+\varrho v_s^2=0 \tag{18}$$

where $v_s=\omega/y_1$ is the phase velocity of the surface wave. These decay constant α and velocity v_s are to satisfy the boundary conditions of the problem also.

Let us now assume the electric field outside the piezoelectric substrate in the following form

$$\hat{\varphi}=A_3 \sin h\{y_1(x_2+h)\}\exp\{i(y_1x_1-\omega t)\}, \tag{19}$$

which satisfies the necessary boundary condition $\varphi]_{x_2=-h}=0$ and $\nabla^2\varphi=0$. Now substituting the expressions for the displacement component u_3 and ψ given by equations (16) in the boundary conditions (13), (14) and (15) and remembering the relation

$$\psi=\varphi-\frac{1}{\epsilon_{11}}\{e_{15}^{(1)}u_3+e_{15}^{(2)}\dot{u}_3\}$$

we find the following three equations

$$A_1y_2\left[\frac{2e_{15}^{(1)}e_{15}^{(2)}i\omega}{\epsilon_{11}}+\frac{(e_{15}^{(2)})^2\omega^2}{\epsilon_{11}}-\left(c_{44}+\frac{(e_{15}^{(1)})^2}{\epsilon_{11}}\right)\right]-A_2y_1(e_{15}^{(1)}-e_{15}^{(2)}i\omega)=0$$

$$\frac{A_1}{\epsilon_{11}}(e_{15}^{(1)}-e_{15}^{(2)}i\omega)+A_2-A_3\sinh(y_1h)=0 \tag{20}$$
$$A_2\epsilon_{11}+A_3\epsilon_{11}^0\cosh(y_1h)=0.$$

For a non-trivial solution of the above system of equations we find

$$\begin{vmatrix} y_2\left[\frac{2e_{15}^{(1)}e_{15}^{(2)}i\omega}{\epsilon_{11}}+\frac{(e_{15}^{(2)})^2\omega^2}{\epsilon_{11}}-\frac{c_{44}\epsilon_{11}+(e_{15}^{(1)})^2}{\epsilon_{11}}\right] & -y_1(e_{15}^{(1)}-e_{15}^{(2)}i\omega) & 0 \\ \frac{1}{\epsilon_{11}}[e_{15}^{(1)}-e_{15}^{(2)}i\omega] & 1 & -\sinh(y_1h) \\ 0 & \epsilon_{11}^0 & \cosh(y_1h) \end{vmatrix}=0 \tag{21}$$

where ϵ_{11}^0 is the vacuum permittivity. Expanding the above determinantal equation we get

$$\alpha=\frac{[e_{15}^{(1)}-e_{15}^{(2)}i\omega]^2}{\left[1+\frac{\epsilon_{11}}{\epsilon_{11}^0}\tanh(y_1h)\right][c_{44}\epsilon_{11}+(e_{15}^{(1)})^2-2i\omega e_{15}^{(1)}e_{15}^{(2)}-(e_{15}^{(2)})^2\omega^2]}. \tag{22}$$

Eliminating α between equations (18) and (22) we find

$$\frac{(e_{15}^{(1)}-e_{15}^{(2)}i\omega)^4}{\left[1+\frac{\epsilon_{11}}{\epsilon_{11}^0}\tanh(y_1h)\right]^2}=\epsilon_{11}[c_{44}\epsilon_{11}+(e_{15}^{(1)})^2-2i\omega e_{15}^{(1)}e_{15}^{(2)}- \tag{23}$$
$$-(e_{15}^{(2)})^2\omega^2]\left[\left\{c_{44}+\frac{(e_{15}^{(1)})^2}{\epsilon_{11}}-\frac{2i\omega e_{15}^{(1)}e_{15}^{(2)}}{\epsilon_{11}}-\frac{(e_{15}^{(2)})^2\omega^2}{\epsilon_{11}}\right\}-\varrho v_s^2\right].$$

The solution of the problem thus reduces to the following

$$u_3=A_1\exp(-\alpha y_1u_2)\exp[i(y_1u_1-\omega t)] \tag{24}$$
$$\varphi=\psi+\frac{1}{\epsilon_{11}}(e_{15}^{(1)}u_3+e_{15}^{(2)}\dot{u}_3)=A_2\exp(-y_1u_2)\exp[i(y_1u_1-\omega t)]+$$
$$+\frac{e_{15}^{(1)}}{\epsilon_{11}}[A_1\exp(-\alpha y_1u_2)\exp\{i(y_1u_1-\omega t)\}]+$$
$$+\frac{e_{15}^{(2)}}{\epsilon_{11}}[-A_1\exp(-\alpha y_1u_2)\exp\{i(y_1u_1-\omega t)\}i\omega].$$

Introducing an amplitude ratio $A_2/A_1=p$ the expressions for the displacement component and piezoelectric potential can be written as follows

$$u_3=A_1\exp(-\alpha y_1u_2)\exp[i(y_1u_1-\omega t)] \tag{25}$$

$$\varphi = A_1\left[p\exp(-y_1u_2) + \frac{e_{15}^{(1)}}{\epsilon_{11}}\exp(-\alpha y_1u_2) - \right.$$
$$\left. - \frac{e_{15}^{(2)}}{\epsilon_{11}}\exp(-\alpha y_1 u_2)\, \mathrm{i}\omega\right]\exp[\mathrm{i}(y_1u_1-\omega t)].$$

The amplitude ratio ϱ can be found from the boundary condition (13) and is found to be the following

$$p = \alpha\left[\frac{2e_{15}^{(1)}e_{15}^{(2)}\,\mathrm{i}\omega + (e_{15}^{(2)})^2\omega^2 - (c_{44}\epsilon_{11} + (e_{15}^{(1)})^2)}{\epsilon_{11}\{e_{15}^{(1)} - e_{15}^{(2)}\,\mathrm{i}\omega\}}\right]. \qquad (26)$$

V. GROUP VELOCITY OF THE SURFACE WAVE

Replacing V_s by ω/y_1 in the equation (23) we get the following equation for ω.

$$\omega^4\left[(e_{15}^{(2)})^4 + \frac{p\epsilon_{11}(e_{15}^{(2)})^2}{y_1^2}\right] + \omega^3\left[4\mathrm{i}e_{15}^{(1)}(e_{15}^{(2)})^3 + \frac{2\mathrm{i}p\epsilon_{11}}{y_1^2}e_{15}^{(1)}e_{15}^{(2)}\right] -$$
$$- \omega^2\left\{4(e_{15}^{(1)}e_{15}^{(2)})^2 + 2[c_{44}\epsilon_{11} + (e_{15}^{(1)})^2]\,(e_{15}^{(2)})^2 + \right.$$
$$\left. + \frac{p\epsilon_{11}}{y_1^2}(e_{15}^{(1)})^2 + \frac{p\epsilon_{11}^2 c_{44}}{y_1^2}\right\} - 4\mathrm{i}\omega e_{15}^{(1)}e_{15}^{(2)}[c_{44}\epsilon_{11} + \qquad (27)$$
$$+ (e_{15}^{(1)})^2] + [c_{44}\epsilon_{11} + (e_{15}^{(1)})^2] = \frac{(e_{15}^{(1)} - e_{15}^{(2)}\,\mathrm{i}\omega)^4}{\left[1 + \frac{\epsilon_{11}}{\epsilon_{11}^0}\tanh(y_1h)\right]^2}.$$

Differentiating (27) with respect to y_1 we get the following expression for the group velocity $V_g(=\mathrm{d}\omega/\mathrm{d}y_1)$

$$\mathrm{d}\omega/\mathrm{d}y_1 = \left[\frac{\omega^4 2p\epsilon_{11}(e_{15}^{(2)})^2}{y_1^3} + \frac{4\mathrm{i}pe_{15}^{(1)}e_{15}^{(2)}\epsilon_{11}\omega^3}{y_1^3} - \right.$$
$$\left. - \omega^2\left(\frac{2p\epsilon_{11}^2c_{44}}{y_1^3} + \frac{2p\epsilon_{11}(e_{15}^{(1)})^2}{y_1^3}\right) - \frac{2\left(\frac{\epsilon_{11}}{\epsilon_{11}^0}\operatorname{sech}^2(y_1h)h\right)(e_{15}^{(1)} - e_{15}^{(2)}\,\mathrm{i}\omega)^4}{\left\{1 + \frac{\epsilon_{11}}{\epsilon_{11}^0}\tanh(y_1h)\right\}^3}\right] :$$

$$: \left[4\omega^3\left\{(e_{15}^{(2)})^4 + \frac{p\epsilon_{11}(e_{15}^{(2)})^2}{y_1^2}\right\} + 3\omega^2\left\{4\mathrm{i}e_{15}^{(1)}(e_{15}^{(2)})^3 + \right.\right. \qquad (28)$$

$$\left. + \frac{2\mathrm{i}p\epsilon_{11}e_{15}^{(1)}e_{15}^{(2)}}{y_1^2}\right\} - 2\omega\left\{4(e_{15}^{(1)}e_{15}^{(2)})^2 + 2(c_{44}\epsilon_{11} + (e_{15}^{(1)})^2)\,(e_{15}^{(2)})^2 + \frac{p\epsilon_{11}^2c_{44}}{y_1^2} + \frac{p\epsilon_{11}(e_{15}^{(2)})^2}{\epsilon_{11}}\right\} -$$

$$\left. - 4\mathrm{i}e_{15}^{(1)}e_{15}^{(2)}(c_{44}\epsilon_{11} + (e_{15}^{(1)})^2) + \frac{4\mathrm{i}e_{15}^{(2)}(e_{15}^{(1)} - e_{15}^{(2)}\,\mathrm{i}\omega)^3}{\left[1 + \frac{\epsilon_{11}}{\epsilon_{11}^0}\tanh(y_1h)\right]^2}\right].$$

Deviding both numerator and denominator of equation (28) by y_1 we get

$$V_g = \left[2V_s^4 p\epsilon_{11}(e_{15}^{(2)})^2 + \frac{4\mathrm{i}p\epsilon_{11}e_{15}^{(1)}e_{15}^{(2)}}{y_1}V_s^3 - \right.$$
$$\left. - \frac{2V_s^2 p\epsilon_{11}(c_{44}\epsilon_{11} + (e_{15}^{(1)})^2)}{y_1^2} - \frac{2\left(\frac{\epsilon_{11}}{\epsilon_{11}^0}\operatorname{sech}^2(y_1h)h\right)(e_{15}^{(1)} - e_{15}^{(2)}\,\mathrm{i}\omega)^4}{y_1\left(1 + \frac{\epsilon_{11}}{\epsilon_{11}^0}\tanh(y_1h)\right)^3}\right] :$$

$$: \left[4V_s^3\{(e_{15}^{(2)})^4y_1^2 + p\epsilon_{11}(e_{15}^{(2)})^2\} + \frac{3V_s^2}{y_1}\{4\mathrm{i}e_{15}^{(1)}(e_{15}^{(2)})^3y_1^2 + \right.$$
$$+ 2\mathrm{i}p\epsilon_{11}e_{15}^{(1)}e_{15}^{(2)}\} - \frac{2V_s}{y_1^2}\{4y_1^2(e_{15}^{(1)}e_{15}^{(2)})^2 + 2y_1^2(e_{15}^{(2)})^2(c_{44}\epsilon_{11} + \qquad (29)$$
$$+ (e_{15}^{(1)})^2) + p\epsilon_{11}^2c_{44} + p\epsilon_{11}(e_{15}^{(2)})^2\} - \frac{4\mathrm{i}e_{15}^{(1)}e_{15}^{(2)}}{y_1}(c_{44}\epsilon_{11} + (e_{15}^{(1)})^2) +$$
$$\left. + \frac{4\mathrm{i}e_{15}^{(2)}(e_{15}^{(1)} - e_{15}^{(2)}\,\mathrm{i}\omega)^3}{y_1\left(1 + \frac{\epsilon_{11}}{\epsilon_{11}^0}\tanh(y_1h)\right)^2}\right].$$

VI. MAXIMUM PENETRATION DEPTH OF THE SURFACE WAVE

An important parameter of the surface wave is the penetration depth into the medium. This depth is usually derived in multiples of wave length λ.

The expressions for amplitude of the disturbance u_3 and potential φ given by equations (25) are the following

$$\hat{u}_3 = A_1\exp(-\alpha y_1u_2) \qquad (30)$$

$$\hat{\varphi} = A_1\left\{p\exp(-y_1u_2) + \frac{e_{15}^{(1)}}{\epsilon_{11}}\exp(-\alpha y_1u_2) - \frac{e_{15}^{(2)}}{\epsilon_{11}}\exp(-\alpha y_1u_2)\,\mathrm{i}\omega\right\}. \qquad (31)$$

The above two equations can be written in the following form

$$\hat{u}_3/A_1 = \exp(-\alpha y_1\lambda\eta) = L(\eta) \quad \text{(say)} \qquad (32)$$

$$\frac{\hat{\varphi}}{A_1} = p\exp(-y_1\lambda\eta) + \frac{e_{15}^{(1)}}{\epsilon_{11}}\exp(-\alpha y_1\lambda\eta) - \quad \text{(say)}$$
$$- \frac{e_{15}^{(2)}}{\epsilon_{11}}\exp(-\alpha y_1\lambda\eta)\,\mathrm{i}\omega = M(\eta) \qquad (33)$$

where $\eta = u_2/\lambda$.

The value of η for which $\hat{u}_3/A_1(=L(\eta))$ or $\hat{\varphi}/A_1(=M(\eta))$ is minimum gives the maximum penetration depth of the wave. Now since $L(\eta)=0$ does not give us any positive information about the penetration depth we consider the equation (33).

Using the necessary condition of minimization we find

$$\exp(-\alpha y_1\lambda\eta)\left[-p\exp(-y_1\lambda\eta(1-\alpha))y_1\lambda-\frac{e_{15}^{(1)}}{\epsilon_{11}}\alpha y_1\lambda+\frac{e_{15}^{(2)}}{\epsilon_{11}}i\omega\alpha y_1\lambda\right]=0.$$

Now $\exp(-\alpha y_1\lambda\eta)=0\rightarrow\eta\rightarrow\infty$ and thus no new information about the penetration depth of the surface wave is obtained

$$\eta=\frac{1}{y_1(1-\alpha)\lambda}\log\left[\frac{\alpha(e_{15}^{(2)}i\omega-e_{15}^{(1)})}{\epsilon_{11}p}\right]. \tag{34}$$

Hence the expression for the maximum penetration depth of the surface wave can be written in the following form using equations (34) and (26)

$$u_2=\lambda\eta=\frac{1}{y_1(\alpha-1)}\log\left[\frac{(e_{15}^{(1)}-e_{15}^{(2)}i\omega)^2}{\{c_{44}\epsilon_{11}+(e_{15}^{(1)})^2\}-2i\omega e_{15}^{(1)}e_{15}^{(2)}-(e_{15}^{(2)})^2\omega^2}\right]. \tag{35}$$

VII. PIEZOELECTRIC POYINTING VECTOR AND POWER FLOW COMPONENTS

Using Auld's [8] notation the expression for the piezoelectric Poyinting vector $\boldsymbol{P}$ is given by

$$\boldsymbol{P}=\frac{\boldsymbol{V}^*.T}{2}+\frac{\boldsymbol{E}\times\boldsymbol{H}}{2} \tag{36}$$

where $\boldsymbol{E}$ is the electric field, $\boldsymbol{H}$ the magnetic field, $\boldsymbol{V}$ the particle velocity and T the stress tensor. * denotes the complex conjugate of the corresponding quantity. The first and second term on the right-hand side of (36) represent the average acoustic and electromagnetic power flow, respectively.

Neglecting the electromagnetic part $(\boldsymbol{E}\times\boldsymbol{H})/2$ of the Poyinting vector we get

$$\boldsymbol{P}=\frac{\boldsymbol{V}^*.T}{2}. \tag{37}$$

Here the power flow of the surface wave under consideration is given by the vector $\boldsymbol{P}_T$ with components

$$\boldsymbol{P}_{T,i}=\frac{1}{2}\operatorname{Real}\left\{\int_0^\infty\left(T_{ij}\frac{\partial u_j^*}{\partial t}\right)du_2\right\} \tag{38}$$

(see Auld [8]).

The magnitude of the above vector gives the time average power crossing a strip of unit width and infinite depth oriented perpendicular to the vector. In the above equation (38) substituting the expression for the stress components T_{ij} and the time

derivative of the complex conjugate of the displacement component u_j from equation(25) we find on integration the components of the power flow in the following form

$$\boldsymbol{P}_{T,1}=|A_1|^2\omega\left[\frac{1}{2\alpha}\left\{-c_{44}-\frac{(e_{15}^{(1)})^2}{\epsilon_{11}}+\omega^2\frac{(e_{15}^{(2)})^2}{\epsilon_{11}}\right\}-\frac{Pe_{15}^{(1)}}{1+\alpha}\right] \tag{39}$$

$$\boldsymbol{P}_{T,2}=-|A_1|^2\omega^2\left\{\frac{e_{15}^{(1)}e_{15}^{(2)}}{\epsilon_{11}}+\frac{Pe_{15}^{(2)}}{1+\alpha}\right\} \tag{40}$$

$$\boldsymbol{P}_{T,3}=0. \tag{41}$$

VIII. DEVIATION OF THE GROUP VELOCITY FROM THE DIRECTION OF THE PHASE VELOCITY

Another important parameter of the surface wave is the deviation $\Delta\psi$ of the group velocity from the direction of the phase velocity, which in practical use means the deviation of the energy flow from the geometrical axis of an interdigital transducer. Now

$$\Delta\psi=\operatorname{arc\,tg}\frac{\boldsymbol{P}_{T,3}}{\boldsymbol{P}_{T,1}} \tag{42}$$

where $\boldsymbol{P}_{T,1}$ and $\boldsymbol{P}_{T,3}$ are the power flow components as already mentioned.

Using equations (39) and (40) we get from (42)

$$\Delta\psi=0, \tag{43}$$

which indicates that in the present situation the two directions coincide.

IX. AMPLIFICATION OR ATTENUATION COEFFICIENT OF THE SURFACE WAVE

Amplification of the surface wave is an important problem of acoustics.

To determine the amplification or attenuation coefficient of the surface wave we substitute

$$y_1=y_1^*+iy_1^{**} \tag{44}$$

in equation (17) and then separating the imaginary part we get the amplification/or attenuation coefficient in the following form

$$y_1^{**}=\frac{1}{\sqrt{2}}(-A+\sqrt{A^2+B^2}) \tag{45}$$

where

$$A' = y_2^2\left\{\left[\left(c_{44}+\frac{(e_{15}^{(1)})^2}{\epsilon_{11}}\right)-\frac{(e_{15}^{(2)})^2\omega^2}{\epsilon_{11}}\right]+\frac{4\omega^2(e_{15}^{(1)})^2(e_{15}^{(2)})^2}{\epsilon_{11}}\right\}$$

$$A=\frac{A'+P\omega^2\left\{\left(c_{44}+\frac{(e_{15}^{(1)})^2}{\epsilon_{11}}\right)-\frac{(e_{15}^{(2)})^2\omega^2}{\epsilon_{11}}\right\}}{\left[c_{44}+\frac{(e_{15}^{(1)})^2}{\epsilon_{11}}-\frac{(e_{15}^{(2)})^2\omega^2}{\epsilon_{11}}\right]+4\omega^2\frac{(e_{15}^{(1)})^2(e_{15}^{(2)})^2}{\epsilon_{11}^2}} \tag{46}$$

$$B=\frac{\frac{2\omega e_{15}^{(1)}e_{15}^{(2)}}{\epsilon_{11}}}{\left[c_{44}+\frac{(e_{15}^{(1)})^2}{\epsilon_{11}}-\frac{(e_{15}^{(2)})^2\omega^2}{\epsilon_{11}}\right]+\frac{4\omega^2(e_{15}^{(1)})^2(e_{15}^{(2)})^2}{\epsilon_{11}^2}}. \tag{47}$$

X. EFFECTIVE SURFACE PERMITTIVITY

In this section it will be shown that for waves propagating along the traction free surface of a piezoelectric material there exists a unique relationship between the voltage and charge density that appears at the surface. This relationship is characterized by a quantity called effective permittivity ϵ_{eff}, which is of great importance in problems of generation and detection of surface waves. Now,

$$\epsilon_{eff}=\epsilon_{eff)_1}+\epsilon_{eff)_2} \tag{48}$$

where

$$\epsilon_{eff)_1}=\frac{D_{2|u_2=0^+}}{|y_1|\varphi|_{u_2=0^+}} \tag{49}$$

$$\epsilon_{eff)_2}=-\frac{D_{2|u_2=0^-}}{|y_1|\varphi|_{u_2=0^-}} \tag{50}$$

$$=\text{permittivity of free surface}$$
$$=\epsilon_{11}^0. \tag{51}$$

Evaluating the above expression for $\epsilon_{eff)_1}$ we get

$$\epsilon_{eff)_1}=\frac{\epsilon_{11}}{1+\frac{(e_{15}^{(1)}-e_{15}^{(2)}\,i\omega)^2}{\alpha[2e_{15}^{(1)}e_{15}^{(2)}\,i\omega+(e_{15}^{(2)})^2\omega^2-(c_{44}\epsilon_{11}+(e_{15}^{(1)})^2)]}}. \tag{52}$$

Hence from (52) and (51) we have from (48)

$$\epsilon_{eff}=\frac{\alpha(\epsilon_{11}+\epsilon_{11}^0)\,[2e_{15}^{(1)}e_{15}^{(2)}\,i\omega+(e_{15}^{(2)})^2\omega^2-(c_{44}\epsilon_{11}+(e_{15}^{(1)})^2)]+\epsilon_{11}^0(e_{15}^{(1)}-e_{15}^{(2)}\,i\omega)^2}{\alpha[2e_{15}^{(1)}e_{15}^{(2)}\,i\omega+(e_{15}^{(2)})^2\omega^2-(c_{44}\epsilon_{11}+(e_{15}^{(1)})^2)]+(e_{15}^{(1)}-e_{15}^{(2)}\,i\omega)^2} \tag{53}$$

XI. DISCUSSION

To summarize the above analysis we recall that the dispersion equation (18) relates the phase velocity V_s, wave number y_1 and decay parameter α. The decay constant α, wave number y_1 and phase velocity V_s also satisfy the equation (22) derived from the boundary conditions. To determine a set of values of V_s, α and y_1, which satisfy both the equations (18) and (22), one should resort to the technique adopted by White and Tseng [9]. The method consists in assigning a value for the phase velocity V_s and then, solving the two equations (18) and (22), the decay constant α and the wave number y_1 can be determined. Thus for a particular choice of the phase velocity V_s it is possible to determine particular values for α and y_1. These values are then substituted in equation (23) to see whether the equation is identically satisfied. If not, a new value is assigned to the phase velocity V_s and the same steps are repeated over and over again until we find a set of values for V_s, α and y_1 for which the three equations (18), (22) and (23) are satisfied.

With the help of these values of V_s, α and y_1 we can determine $\omega(=V_s y_1)$ and $y_2(=\alpha y_1)$ and hence can find the exact values of the displacement component, group velocity, maximum panetration depth of the surface wave, power flow components amplification/or attenuation coefficient, effective surface permittivity from equations (25), (29), (35), (39), (45) and (53), respectively.

Since tanh $(y_1 h)\to 0$ or 1 as $h\to 0$ or $-\infty$, the two extreme cases when the half space is electrically shorted or open to vacuum correspond to the situations for $h\to 0$ and $h\to -\infty$, respectively. Substituting these values for tanh $(y_1 h)$ and putting the time dependent part of the piezoelectric constant $e_{15}^{(2)}$ equal to zero, the characteristic equation and the determinantal equation obtained from the boundary conditions are very much simplified and are found to agree with the corresponding results obtained by Bleustein [1].

I am grateful to Dr. A. K. Pal, Department of Mathematics, Jadavpur University, for his helpful suggestions in the preparation of this paper.

REFERENCES

[1] Bleustein, J. L.: Appl. Phys. Lett. *13* (1969), 412.
[2] Gulyaev, Y. V.: Soviet Phys. JETP *9* (1969), 63.
[3] Ludvik, S., Quate, C. F.: Appl. Phys. *43* (1972), 3619.
[4] Cambon, G.: Electron Lett. (1972), 860.
[5] Fischler, C.: J. Appl. Phys. *42* (1971), 919.
[6] Pajewski, H.: Arch. Acoust. *2* (1977), 3.
[7] Tiersten, H. F.: *Linear Piezoelectric Plate Vibrations.* Plenum Press, New York, 1969.
[8] Auld, B. A.: *Acoustic Fields and Waves in Solids.* Vol. 1., Widey-Interscience Publication, New York, 1973.
[9] White, Q. M., Tseng, C. C.: J. Acoust. Soc. Am. *38* (1967), 4274.

Received May 8th, 1984