# MONTE CARLO SIMULATION OF THE MOTION OF ELECTRONS IN $SF_6$ IN UNIFORM ELECTRIC FIELDS[1])

J. DŘÍMAL[2]), D. TRUNEC[2]), Brno

The motion of electrons in $SF_6$ in uniform electric fields is simulated using a Monte-Carlo technique for the range of reduced electric fields $75 \leq E/p \leq 135$ $Vm^{-1}\,Pa^{-1}$. Electrons distribution functions and drift velocity have been obtained.

## СИМУЛИРОВАНИЕ ДВИЖЕНИЯ ЭЛЕКТРОНОВ В ПОМЕЩЕННОМ В ОДНОРОДНОЕ ЭЛЕКТРИЧЕСКОЕ ПОЛЕ ОБРАЗЦЕ $SF_6$ ПРИ ПОМОЩИ МЕТОДА МОНТЕ-КАРЛО

В работе приведены результаты симулирования движения в помещенном в однородное электрическое поле образце $SF_6$ при помощи метода Монте-Карло в интервале приведенных напряженностей электрического поля $75 \leq E/p \leq 135$ ($Вм^{-1}\,Па^{-1}$). Получен явный вид функции распределения электронов и их дрейфовая скорость.

## 1. INTRODUCTION

The current extensive use of $SF_6$ as an insulating medium in high voltage equipment has prompted efforts towards correlating the observed discharge phenomena with the basic processes. The measured values of Townsend's first ionization coefficient $\alpha/p$ and attachment coefficient $\eta/p$ have been reported in literature. Theoretical work to predict the transport and ionizing properties of electrons using the measured collision cross section is based on a numerical analysis of the Boltzmann equation in which various mechanisms by which electrons lose energy are included. The numerical solution of the Boltzmann equation yields the electron energy distribution with the electric field $\boldsymbol{E}$ and the gas number density $N$ as parameters [1]. Appropriate integration of the energy distribution function yields the transport and ionizing properties of the electron swarm [2, 3].

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[2]) Department of Physical Electronics, Faculty of Science, J. E. Purkyně University, Kotlářská 2, 611 37 Brno, Czechoslovakia.

The Monte-Carlo simulation of the electron drift in a uniform electric field has the advantage that the motion of the electron at all stages during its passage in the discharge is traced. Several authors have used Monte-Carlo techniques in a number of gases but only Dincer used this method in $SF_6$ [4].

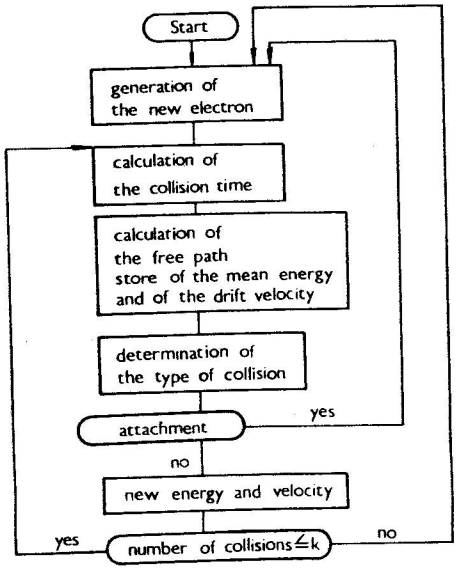


Fig. 1. The block scheme of the model.

## II. COLLISION MODEL AND MECHANICS

In a spherical coordinate system a background gas of $SF_6$ molecules with a number density of $N = 3.29 \times 10^{16}$ cm$^{-3}$ which corresponds to a gas pressure of 1 Torr at 20 °C is considered. The ionizing and transport properties are assumed to be independent of the gas number density. The applied electric field $\boldsymbol{E}$ is antiparallel to the $z$ axis. $N$ electrons with a cosine distribution of an energy in the interval 0.1—8 eV are injected from the origin of the coordinate system assuming a cosine distribution for the angle of entry with respect to the $z$ axis. At $t = 0$ an electron follows a free flight time with a randomly selected angle of entry depending on the distribution. The block scheme of our model is in Fig. 1.

Dincer [4] adopted a mean collision time approach in which the total time between collisions is divided into 40 intervals for the energy range $0 \leqslant W \leqslant 3$ eV and for $W > 3$ eV 10 intervals are chosen. The mean collision time $T_M$ of an electron is inversely dependent upon the total collision cross section $Q_T$, the gas number density $N$ and the electron velocity $|\boldsymbol{v}|$ and accordingly



$$T_M = (NQ_T v)^{-1}. \tag{1}$$

The probability of collision in the time step $\Delta T$ is

$$P = 1 - exp\left(-\frac{\Delta T}{T_M}\right) \tag{2}$$

and the collision is simulated by comparing $P$ with $R_1$ at the end of each step where $R_1$ is a random number uniformly distributed between 0 and 1.

Our preliminary calculations showed that this model collapsed. We thing that the manner of the simulation of the collision time is not sufficiently exact. Therefore we have calculated the collision time $T$ from the equation

$$-\int_0^T \frac{v_e(t)\,dt}{l[\boldsymbol{v}_e(t)]} = \ln R \tag{3}$$

here

$$l[\boldsymbol{v}_e(t)] = (NQ_T(t))^{-1} \tag{4}$$

and $R$ is again a random number uniformly distributed between 0 and 1.

For the coordinate system selected the position and energy of an electron in the time of the collision step $T$ undergo the following variation for an initial velocity $v_0$ and kinetic energy

$$\begin{aligned} W &= mv_0^2/2 \\ \Delta z &= v_{0z}T + aT^2/2 \qquad a = eE/m \\ \Delta x &= v_{0x}T \qquad\qquad \Delta W = eE\Delta z \\ \Delta y &= v_{0y}T \end{aligned} \tag{5}$$

in which $E$ is the electric field, $e/m$ the charge to mass ratio of electron, $\Delta z$ the distance travelled along the $z$ direction ($\boldsymbol{E}$), $\Delta x$ and $\Delta y$ the position components with respect to the $x$ and $y$ axis, and $\Delta W$ the energy gain in the interval.

$v_{0x}$, $v_{0y}$ and $v_{0z}$ are the components of the initial velocity parallel to the respective axes and are given by

$$\begin{aligned} v_{0z} &= v_0 \cos \Theta \\ v_{0x} &= v_0 \sin \Theta \cos \Phi \\ v_{0y} &= v_0 \sin \Theta \sin \Phi \end{aligned} \tag{6}$$

in which $\Theta$ and $\Phi$ are the polar and azimuthal angles, respectively. If a collision occurs, $\Theta$ is calculated according to $\cos \Theta = 2R_2 - 1$ assuming that the scattering is isotropic in the laboratory coordinate system and $R_2$ is again a random number uniformly distributed between 0 and 1.

The total collision cross section is defined as

$$Q_T = Q_{el} + Q_{at} + Q_{ex} + Q_{ion} + Q_v, \tag{7}$$

where $Q_{el}$ is the elastic differential cross section which is replaced by the momentum transfer collision cross section $Q_m$ in the simulated collision, $Q_{at}$ is the attachment cross section, $Q_{ex}$ is the total electronic excitation cross section, $Q_{ion}$ is the total ionization cross section, and $Q_v$ is the vibrational collision cross section.

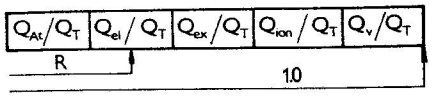


Fig. 2. Scheme for describing the nature of collision.

The type of collision is determined according to Fig. 2, where the position of the arrow indicates the nature of the collision. $Q_i/Q_T$ gives the probability of the coming process. Hence after the event of a collision, if the probabilities of inelastic collisions fail, the collision is deemed to be elastic and the loss of energy in the collision is $2m/M$ where $m$ and $M$ are the masses of electron and a $SF_6$ molecule, respectively.

If the electron is attached, it is lost in the swarm and its subsequent fate is ignored. For other inelastic processes the appropriate threshold energy of the process is subtracted from the electron energy. We have used the collision cross section from [3], [4]. Figure 3 summarizes the various cross sections.

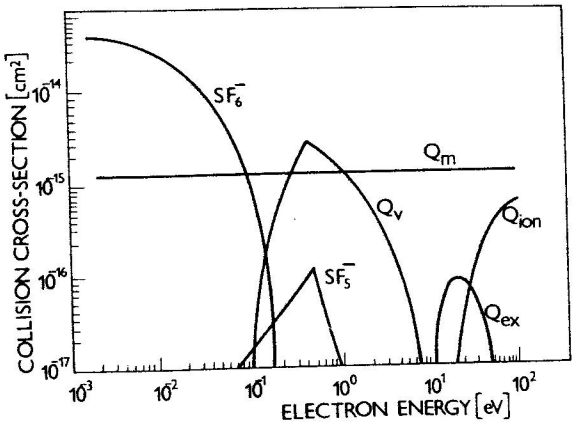


Fig. 3. Collision cross section in $SF_6$.

### A. Momentum transfer cross section

According to [4] we have adopted a cross section of $1.2 \times 10^{-15}$ cm² in the energy interval 0.5—75 eV.



### B. Electronic excitation cross section

The excitation cross section is given by

$$Q_{ex} = 0.95 \times 10^{-16} \frac{W - 9.8}{8.2} \exp\left(\frac{18 - W}{8.2}\right) \text{cm}^2 \tag{8}$$

Threshold energy is 9.8 eV.

### C. Attachment collision cross section

In the present simulation there were used the relationships

$$\begin{aligned} Q_{at}(SF_6^-) &= 5.2 \times 10^{-14} \exp\left(-\frac{W}{0.0177}\right) \text{cm}^2 \\ &\qquad 0 \leq W \leq 0.045 \text{ eV} \\ &= 4.5 \times 10^{-14} \exp\left(-\frac{W}{0.02}\right) \text{cm}^2 \\ &\qquad 0.045 \leq W \leq 2 \text{ eV} \end{aligned} \tag{9}$$

$$\begin{aligned} Q_{at}(SF_5^-) &= 2.94 \times 10^{-16} W^{0.899} \text{ cm}^2 \\ &\qquad 0.068 < W \leq 0.4 \text{ eV} \\ &= 7.047 \times 10^{-18} W^{-3.32} \text{ cm}^2 \\ &\qquad 0.4 < W < 0.93 \text{ eV} \end{aligned} \tag{10}$$

The total $Q_{At}$ is then

$$Q_{At} = Q_{at}(SF_6^-) + Q_{at}(SF_5^-). \tag{11}$$

### D. Ionization collision cross section

The ionization cross section of Rapp et al. [5] is used with an ionization threshold energy of 15. 8. eV.

## III. RESULTS

Our computations have been carried out with $N_0 = 40$—60 electrons. Each electron has collided 2000 times. According to Nanbu [9] the relative error of the computed parameters is proportional to $1/(N_0 k)$, $k$ is number of collisions.

The energy distribution normalized according to $\int_0^\infty F(W)\mathrm{d}W = 1$ is shown in Fig. 4. The full lines show the Maxwellian distribution at the same mean energy. The drift velocity was computed by

$$v_d = \left(\sum_{i=1}^{N_0} \frac{\bar{d}_i}{t_i}\right) / N_0 \qquad (12)$$

where $\bar{d}_i$ is the average distance traversed in the field direction; $t_i$ is the time of observation of the $i$-electron. The calculated drift velocities are shown in Fig. 5 and

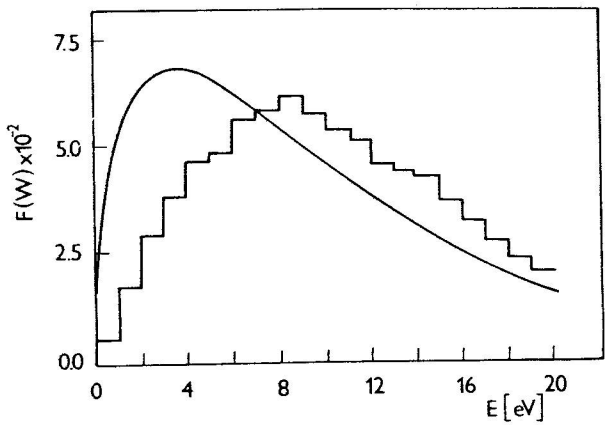


Fig. 4. Energy distributions for $E/p = 90\ \mathrm{Wm^{-1}\,Pa^{-1}}$. Full curve — Maxwellian at the same mean energy.

compared with the measured values of Teich and Sangi [6], Naidu and Prasad [7], and the calculated values of Novak and Frechette [8] on the basis of the Boltzmann equation, and the values of Dincer and Govinda Raju on the basis of the Monte-Carlo technique [4]. The agreement between the present result and those referred to above is very good, particularly at higher $E/p$ values.

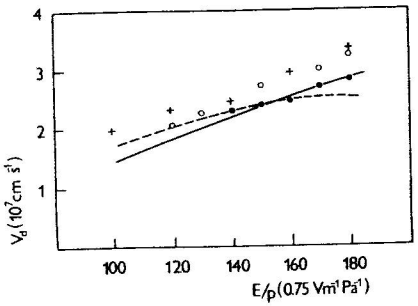


Fig. 5. Drift velocities in $SF_6$. Open circle — Naidu and Prasad (experimental) [7], full circle — Teich and Sangi (experimental) [6], broken line — Novak and Frechette [8], full line— Dincer and Govinda Raju [4] and + is the present work.



## IV. CONCLUSIONS

Swarm parameters evaluated by the Monte-Carlo technique are in good agreement with the experimental values. Energy distributions obtained by the simulation indicate Maxwellian tail behaviour at corresponding mean energies.

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