acta phys. slov. 35 (1985), No. 3

SOME COMMENTS ON THE DETERMINATION OF PLASMA PARAMETERS FROM THE SPECTRAL LINE PROFILE REGISTERED BY MEANS OF THE FABRY-PEROT INTERFEROMETER[1)]

A. BRABLEC[2)], F. ŠŤASTNÝ[2)], Brno

In the paper presented a method is suggested enabling to test the validity of programs for the evaluation of the initial (theoretical) spectral line profile from the registered one by means of the Fabry-Perot interferometer, with the apparatus function known. For the determination of the initial line parameters two different programs have been tested (the use of the Simpson three-point formula and the advantageous properties of the rational function). The method can be applied in the case of the investigated line having the Voigt profile.

НЕКОТОРЫЕ ЗАМЕЧАНИЯ ПО ПОВОДУ ОПРЕДЕЛЕНИЯ ПАРАМЕТРОВ ПЛАЗМЫ НА ОСНОВЕ ФОРМЫ СПЕКТРАЛЬНЫХ ЛИНИЙ, НАБЛЮДАЕМЫХ ПРИ ПОМОЩИ ИНТЕРФЕРОМЕТРА ФАБРИ-ПЕРО

В работе предложен метод, позволяющий проверять правильность программ для (теоретического) вычисления первоначальной формы спектральных линий, наблюдаемых при помощи интерферометра Фабри-Перо, когда функция прибора известна. Для определения параметров первоначальной линии были проверены две разных программы (использована трехточечная формула Симпсона и полезные свойства рациональной функции). Данный метод можно применять также в случае, когда исследуемая линия имеет форму Фойгта.

I. INTRODUCTION

Spectroscopical investigations into the spectral line profiles can bring valuable information about an emitting or an absorbing matter such as plasma. One of the most widely used equipment for determination of the line profile is the Fabry-Perot interferometer.

[1)] Contribution presented at the 5th Symposium on Elementary Processes and Chemical Reactions in Low Temperature Plasma. Štiavnické Bane, May 21—25, 1984.

[2)] Dept. of Physical Electronics, Faculty of Sciences, Purkyně University, Kotlářská 2, 611 37 BRNO, Czechoslovakia.

It can be shown that the relation between the initial (theoretical) spectral line profile $J_v(\lambda)$ and the so-called registered one $I(\lambda)$ is described as the convolution of $J_v(\lambda)$ with the apparatus function g(y) in the form

$$I(\lambda)=\int_{-\infty}^{\infty} J_v(\lambda-y)\,g(y)\,\mathrm{d}y \qquad (1)$$

where λ is the wavelength. Since both the apparatus function and the registered profile are always measured with a certain error the evaluation of the initial line parameters does not represent a proper problem from the mathematical point of view [1]. It means that the relatively small error of measurement can lead to a high error of the result. To eliminate this we must know the type of the initial spectral line profile. On the other hand, that problem can be solved only with the help of nonlinear minimization, which is always complicated.

In the optical diagnostics of plasma it is suitable to assume that the spectral line has the Voigt profile which results from the independent Doppler and Lorentz line broadening. The line shape is described by the convolution of a Gaussian with a Lorentzian

$$J_v(\alpha_L,\alpha_D,\sigma)=\frac{1}{\sqrt{\pi\ln 2}}\frac{1}{\alpha_D}V(x,y) \qquad (2)$$

where $V(x,y)=\frac{y}{\pi}\int_{-\infty}^{\infty}\frac{e^{-z^2}}{y^2+(x-y)^2}\mathrm{d}z$ is the Voigt function.

New introduced variables are defined as follows

$$\sigma=\frac{2d}{\lambda},\quad x=\frac{\sigma}{\alpha_D}\sqrt{\ln 2},\quad y=\frac{\alpha_L}{\alpha_D}\sqrt{\ln 2}.$$

The symbol d is the distance between the mirrors of the Fabry-Perot interferometer and the broadening parameters of the Lorentz and the Doppler mechanisms are denoted by α_L and α_D, respectively.

To determine the initial spectral line profile means to estimate several adjustable parameters (in our case we are interested mainly in α_L and α_D) of the assumed model. The actual values of the parameters are determined from the best fit of the model profiles to experimental data which are usually given by tabular values at a discrete set of points of $w_j=\sigma_j-INT(\sigma_j)$. Consequently the registered profile of the spectral line is given by a set of N points $[w_j, I(w_j)]$. Then with the starting value of the computed parameters the convolution of the initial line profile with the apparatus function (it is also given by a set of points) is carried out for all arguments of w_j by means of a suitable numerical procedure. The obtained values of $I_p(w_j)$ are compared with the registered profile at the corresponding point of w_j by means of a suitable minimization procedure to find such values of the adjustable parameters that the registered profile and the calculated one are fitted mutually as much as possible.

In every numerical solution of the above problem it is difficult to choose the suitable precision of individual steps in the course of the minimization procedure and to test the error of the calculated parameters.

Testing the validity of the program for the calculation of the parameters determining the initial spectral line profile it is mostly assumed that the apparatus function is also given as a Voigt function, which is then convoluted with the second Voigt function. In this way we generate the registered profile (it is also the Voigt function) whose parameters are in simple relation with the parameters of both given Voigt functions. However, this test cannot eliminate all the discrepancies in the program. For example, one of the reasons is the fact that the Voigt function must be calculated numerically and that does not describe the sight behaviour in boundary cases.

II. TESTING OF PROGRAMS

To test all possibilities of our programs there were used conclusions presented in [1, 2].

The apparatus function of the ideal Fabry-Perot interferometer (the interferometer with absolutely plane mirrors and absolutely parallel ones) can be expressed by means of Airy's function as follows:

$$I_i(w) = \frac{(1-R)^2}{1+R^2-2R\cos 2\pi w} \tag{3}$$

where R is the reflexivity of mirrors. If the initial spectral line profile is described by the Voigt function (2), then the registered profile of the ideal Fabry-Perot interferometer can be expressed as

$$I(\alpha_L, \alpha_D, w) = \frac{1}{I_{max}(\alpha_L, \alpha_D)} \frac{1-R}{1+R} \times \left[1 + 2\sum_{n=1} R_e^n \left[\exp - \right.\right.$$
$$\left.\left. - \left(\frac{n^2 Q^2}{4}\right)\right] \cos 2\pi w\right] \tag{4}$$

where $Q = \frac{2\pi}{\sqrt{\ln 2}} \alpha_D$, $R_e = R\exp(-2\pi\alpha_L)$.

Using the relation (4) the registered profile can be generated for a chosen reflexivity of R and the parameters (α_L, α_D) of the initial line profile. Choosing the suitable values of R, α_L, α_D the applicability of the program can be tested for a different ratio of the width of the registered profile and the apparatus because the accuracy of calculation depends on this ratio.

Convoluting the apparatus function with the initial spectral line profile we used two different procedures. In the first case the apparatus function was interpolated by a cubic spline and then it was convoluted with the initial line profile (Voigt function) using the Simpson three-point formula. It was shown to be able to represent the line profile by points in that $I(w) \geq 0.03$. The numerical integration was carried out in 99 points on the wavelength scale. In the second method the very advantageous properties of rational functions are used because they are suitable for approximations of line profiles with only little computing effort. Another definite advantage is the possibility of an easy convolution of several rational functions, which is used to predict line profiles [3]. In our case the apparatus function was approximated by two-term approximations which give good results. Now, the convolution with the Voigt function can be made without direct numerical integration.

Next, the Marquardt-Levenberg algorithm was used in both cases for finding values of the mentioned parameters according to the requirement of minimization of the mean deviation from the approximated profiles according to the method of the least squares.

III. DISCUSSION AND CONCLUSION

In Tab. 1 there are presented results of our computations for several typical values of α_L and α_D. Symbols introduced in this Table mean E_L and E_D are the relative errors of α_L, α_D respectively. SSD is the squares sum of deviations. Numbers 1, 2 denote the first and the second method mentioned above.

We see that the direct integration (the first method) gives more precise results. The highest discrepancy appeared in determining α_L. However, calculations made in this way require at least three times as much time than those by the second method, with the processing of one profile taking over 15 minutes using the HP 9825 A topdesk computer.

As it follows from the results presented it is necessary to have at the disposal of a sufficiently effective testing method of programs to write a suitable program for the computation of spectral line parameters. Even in the case of the problem not being solved in its whole generality (e.g. presence of systematic deviations of measurement error and influence of back-ground, ...) it is not easy to achieve the required coincidence of given and calculated parameters (twelve-digit mantisa) from which it follows to be careful to make conclusions from the calculated parameters. It means to increase the efficiency of our procedure not only in the accuracy of evaluations but also in speed so as to be able to process the measured spectral line profile on line by means of a similar computer such as the HP 9825 A one. We suppose that the suggested method makes it possible to determine the influence of measurement errors on the accuracy of calculations of the spectral line

Table 1

	a_L	E_L	a_D	E_D	SSD
given value	0.01700		0.01700		
1.	0.01742	2.5%	0.01662	2.3%	4.1×10^{-5}
2.	0.01772	4.2%	0.01629	4.4%	5.9×10^{-5}
given value	0.01700		0.03400		
1.	0.01755	3.2%	0.03372	0.8%	7.7×10^{-4}
2.	0.01822	7.2%	0.03316	2.5%	4.0×10^{-5}
given value	0.01700		0.05100		
1.	0.01735	2.1%	0.05108	0.2%	2.3×10^{-4}
2.	0.01876	10.0%	0.04997	2.1%	1.9×10^{-5}

parameters, which appears to be fundamental problem of the interpretation of results when the spectral line profile is measured.

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Received June 8th, 1984