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NON-ISOTROPIC SCATTERING IN BINARY COLLISIONS IN THE HIGHER-ORDER SOLUTION OF THE ELECTRON BOLTZMANN EQUATION IN AN ELECTRIC FIELD[1]

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The recently developed technique to solve the Boltzmann equation with increasing order of the Legendre polynomial expansion of the electron velocity distribution function assuming isotropic scattering in binary collisions is generalized to situations with non-isotropic scattering. Also in this case the general solution of the hierarchy for the coefficients of a 2 l-term expansion contains l singular and l non-singular fundamental solutions of which the latter make the construction of the physically relevant solution possible. The first applications to a model plasma with non-isotropic elastic scattering demonstrate the impact of different scattering angle distributions on the converged electron velocity distribution function and the resulting macroscopic electron quantities.

НЕИЗОТРОПНОЕ РАССЕЯНИЕ В ПАРНЫХ СОУДАРЕНИЯХ В СЛУЧАЕ ВЫСШИХ ПОРЯДКОВ РЕШЕНИЯ УРАВНЕНИЯ БОЛЬЦМАННА ДЛЯ ЭЛЕКТРОНОВ В ЭЛЕКТРИЧЕСКОМ ПОЛЕ

В работе сделано обобщение недавно развитого метода решения уравнения Больцманна с возрастающим порядком разложения функции распределения скоростей электронов по полиномам Лежандра, которое предполагает изотропное рассеяние в парных соударениях, для случая неизотропного рассеяния. Общее решение иерархии для коэффициентов 2 l-ого члена разложения в этом случае также содержит l сингулярных и l несингулярных фундаментальных решений, последние из которых позволяют конструировать физически приемлемые решения. Уже первое применение к модели плазмы с неизотропным упругим рассеянием показывает влияние разных угловых распределений на функцию распределения скоростей электронов и на результирующие макроскопические величины, характеризующие электрон.

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I. INTRODUCTION

To determine the exact relationship between macroscopic electron quantities in a plasma such as transport and rate coefficients and the involved atomic data such as the collision cross sections characterizing the elementary processes, an accurate solution technique of the electron Boltzmann equation is necessary and was developed in [1] and outlined for a model plasma in our contributed paper [2]. This technique uses a higher even order approximation of the electron velocity distribution function $f(\mathbf{v})$ in Legendre polynomials in generalization of the conventional 2-term Lorentz approximation. But this technique is based on the assumption that scattering in binary elastic and exciting collisions between electrons and neutral gas particles is isotropic, which leads only to the impact of the total collision cross sections, i.e., the differential ones integrated over the scattering angle in the hierarchy for the coefficients of the expansion. As it is well known, the scattering in binary collisions is generally non-isotropic and often changes from slightly backward via transversal to predominantly forward scattering if the electron energy increases, which is necessarily reflected in the course of the velocity distribution and thus in the resulting macroscopic quantities. For elastic collisions this aspect is usually considered in the Lorentz approximation of the Boltzmann equation employing the transport cross section instead of the total cross section for this collision process. This paper outlines a solution of the electron Boltzmann equation in an arbitrary even order approximation for a plasma in an electric field $\mathbf{E}=E\mathbf{e}_z$ and with elastic and exciting collisions under the conditions of non-isotropic scattering in generalization of the technique employed in [2]. Using the expansion

$$f(\mathbf{v}) = \sum_{n=0}^{2l-1} F_n(v) P_n(v_z/v) \tag{1}$$

of the distribution function in Legendre polynomials P_n, the homogeneous and stationary Boltzmann equation yields the hierarchy

$$\frac{1}{3}U\frac{\mathrm{d}}{\mathrm{d}U}f_1+\frac{1}{3}f_1+\delta U^2 p_1\frac{\mathrm{d}}{\mathrm{d}U}f_0+\left[\delta\left(2Up_1+U^2\frac{\mathrm{d}}{\mathrm{d}U}p_1\right)-Uq_0\right]f_0+$$
$$+\sum_k (U+U_k^{ex})\,q_{k,0}(U+U_k^{ex})f_0(U+U_k^{ex})=0, \tag{2}$$

$$\frac{n}{2n-1}U\frac{\mathrm{d}}{\mathrm{d}U}f_{n-1}-\frac{n}{2n-1}\,\frac{n-1}{2}f_{n-1}+\frac{n+1}{2n+3}U\frac{\mathrm{d}}{\mathrm{d}U}f_{n+1}+\frac{n+1}{2n+3}\,\frac{n+2}{2}f_{n+1}-$$
$$-U(p_n+q_0)f_n+\sum_k (U+U_k^{ex})\,q_{k,n}(U+U_k^{ex})f_n(U+U_k^{ex})=0,$$
$$1\leqslant n\leqslant 2l-1,\quad l\geqslant 1,\quad f_{2l}\equiv 0$$

for the normalized expansion coefficients

$$f_n(U) = 2\pi(2/m)^{3/2} F_n[v(U)]/n_e \qquad (3)$$

where $U = mv^2/2$ and n_e is the electron concentration. Furthermore $p_n(U) = [Q_0^{el}(U) - Q_n^{el}(U)]/(e_0E/N)$, $q_{k,n}(U) = Q_{k,n}^{ex}(U)/(e_0E/N)$, $q_0 = \sum_k q_{k,0}$, $\delta = 2m/M$ with $Q_n^{el}(U) = \int \sigma^{el}(U, \cos\vartheta)\, P_n(\cos\vartheta)\, d\Omega$, $Q_{k,n}^{ex}(U) = \int \sigma_k^{ex}(U, \cos\vartheta)\, P_n(\cos\vartheta) d\Omega$ and $d\Omega = \sin\vartheta\, d\vartheta\, d\varepsilon$ as the solid angle of scattering, σ^{el} and σ_k^{ex} as the differential cross section for elastic collisions and for kth excitation process with the threshold U_k^{ex} respectively. The Q_n^{el} and $Q_{k,n}^{ex}$ are generalized total cross sections weighted by means of the Legendre polynomial P_n which describe the selective dissipation of the different coefficients f_n due to the non-isotropic character of the scattering in collisions, with $Q_n^{el} = Q_{k,n}^{ex} = 0$ for $n \geq 1$ following for isotropic scattering. Finally, using (3) f_0 is to be normalized according to

$$\int_0^\infty U^{1/2} f_0(U)\, dU = 1. \qquad (4)$$

II. ASPECTS OF THE GENERAL SOLUTION AND SOLUTION TECHNIQUE

The linear system of ordinary differential equations (2) constitutes, as in the case of isotropic scattering, a weakly singular system at small U and a strongly singular system at large U, with $U = 0$ and $U = \infty$ as the singular points if appropriate power series and asymptotic series representations respectively for $Q_n^{el}(U)$ and $Q_{k,n}^{ex}(U)$ are assumed. Separate considerations of the structure of the general solution of the hierarchy (2) in the region of small as well as large energies yield that this contains l non-singular (i.e. normalizable) and l singular fundamental solutions in both energy regions. Though the fundamental solutions and the contributions to the particular solution due to the consideration of the inscattering terms of (2) as formal inhomogeneities of the system are modified by the non-isotropic scattering, the physically relevant solution can again be uniquely determined by the construction of the nonsingular part of the general solution (NSPGS) at low as well as high energies, by continuous connection of both these NSPGS's at an appropriate connection point U_c and by additional normalization according to (4). Finally, in analogy to [2], the distribution function f in the even order approximation ($2l$) is numerically found by l-fold backward integration of (2) down to U_c starting from a sufficiently large energy U_∞ and by $2l$-fold forward integration of (2) up to U_c starting from $U = 0$. Special measures are to be taken to preserve the linear independence of the contributions to the NSPGS, particularly during the l-fold backward integration, and to start the $2l$ forward integrations directly at the singular point $U = 0$.

III. MODEL, RESULTS AND DISCUSSION

A model plasma with elastic collisions and one excitation process ($k = 1$) is dealt with considering non-isotropic scattering in elastic collisions but still isotropic scattering in exciting collisions (i.e. $Q_{1,n}^{ex} = 0$ for $n \geq 1$). To illustrate the impact of non-isotropic scattering we take the simple differential elastic cross section to be

$$\sigma^{el}(U, x) = Q_0^{el}(U) \frac{1}{2\pi} R(x), \quad x = \cos\vartheta \qquad (5)$$

with the normalization $\int_{-1}^{+1} R(x)\, dx = 1$ where the scattering angle distribution R is supposed to be independent and modelled by the profile

$$R(x) = \frac{1}{a} \exp[-(x - x_M)^2/s^2], \qquad a = \int_{-1}^{+1} \exp[-(x - x_M)^2/s^2]\, dx \qquad (6)$$

of the Gaussian type with the two parameters x_M and s which allow to vary the position x_M of the maximum scattering probability as well as the width s of the scattering angle distribution R. Fig. 1 shows for selected values of x_M and s this distribution which illustrates besides the isotropic scattering predominantly forward ($x_M = 1$), transversal ($x_M = 0$) and backward ($x_M = -1$) scattering. The elastic collisions are described in (2) only by the cross section differences $Q_0^{el}(U) - Q_n^{el}(U)$ which, using (5), can be expressed by

$$Q_0^{el}(U) - Q_n^{el}(U) = \alpha_n Q_0^{el}(U), \qquad \alpha_n = 1 - \int_{-1}^{+1} R(x)\, P_n(x) dx, \qquad (7)$$

i.e. by a factorial alteration of the total elastic cross section Q_0^{el}. Table 1 shows the change of this factor α_n with n for different non-isotropic scattering conditions whereas for isotropic scattering ($s \to \infty$) $\alpha_n = 1$, $n \geq 1$ is obtained. It is evident that the non-isotropic scattering has a marked impact in particular on the first differences $Q_0^{el} - Q_n^{el}$ and thus via the change of the dissipation by elastic collisions on the first equations for the coefficients f_n. This impact differs widely for different x_M and becomes large if the scattering angle distribution narrows.

Furthermore the cross sections Q_0^{el} and $Q_{1,0}^{ex}$ are assumed to be energy independent but with a linear increase of the latter from 0 to its final constant value Q^* over a small energy region $U_1^{ex} \leq U \leq U_1^{ex} + 0.2$ eV with the special values $Q_0^{el} = Q^* = 6 \times 10^{-16}$ cm^2 and $U_1^{ex} = 1$ eV for which a large anisotropy of the velocity distribution $f(\mathbf{v})$ is to be expected.

Further a plasma of atoms with a mass M of four atomic units and with a normalized field strength $E/N = 50$ Td is considered. In Fig. 2 the converged isotropic distribution f_0, i.e. that of the 8-term approximation (ETA), is presented for the case of forward scattering ($x_M = 1$) with s as a parameter of the curves. With decreasing width s of forward scattering the isotropic distribution f_0 increases at larger energies U up to orders of magnitude, which results predominantly from the reaction of the reduced dissipation of the first f_n, $n \geqslant 1$ and in particular of f_1 in elastic collisions to the isotropic distribution. To illustrate the insufficient accuracy of solving the Boltzmann equation under the same conditions by a 2-term approximation (TTA), Fig. 3 shows for $x_M = 1$ the ratio of the converged f_0 to that resulting from the TTA. It can be seen that with increasing energy a large change of the isotropic distribution f_0 from the TTA to the converged approximation occurs, which holds for all widths s of the scattering angle distribution but shows a non-monotone behaviour with this parameter. Finally, Fig. 4 shows the behaviour of the drift velocity $W = -(1/3)\,(2/m)^{3/2} \int_0^\infty U f_1(U)\,\mathrm{d}U$ for $x_M = 1$, 0 and -1 as

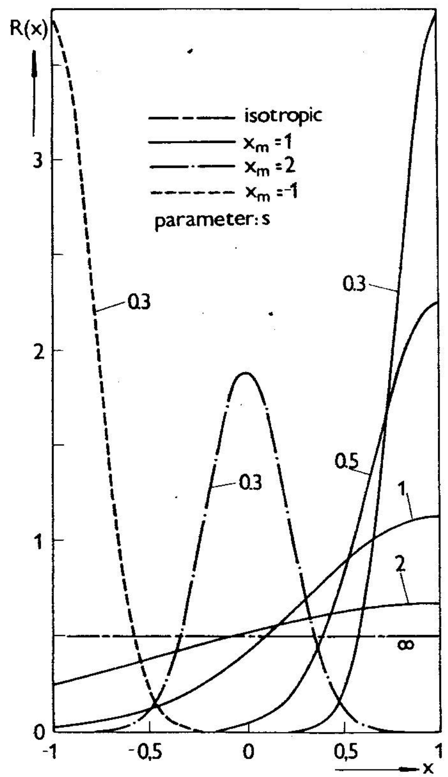

Fig. 1. The normalized scattering angle distribution $R(x)$ for several cases of the parameters x_M and s.

Table 1

The factor α_n for the alteration of the cross section difference $Q_0^{el} - Q_n^{el}$ due to non-isotropic scattering in elastic collisions

	$x_M = 1$		$x_M = 0$		$x_M = -1$	
n	$s = 1$	0.3	1	0.3	1	0.3
1	0.556	0.169	1.00	1.00	1.44	1.83
2	0.951	0.440	1.12	1.43	0.951	0.400
3	1.03	0.716	1.00	1.00	0.968	1.28
4	1.01	0.920	0.989	0.767	1.01	0.920
5	1.00	1.03	1.00	1.00	1.00	0.975
6	0.999	1.05	1.00	1.12	0.999	1.05
7	1.00	1.03	1.00	1.00	1.00	0.965

dependent on the width. The dashed curves give the results from the TTA and the full curves those from the converged approximation i.e. from the ETA, whereas the two dashes on the right margin denote the values for isotropic scattering in TTA and ETA, respectively. The behaviour of W is quite different for predominantly forward, symmetrically transversal and backward scattering whereas for a large width s the values of the drift velocity for isotropic scattering are approached in each case. Since, passing from the TTA to the ETA, the change of the function f_1 is

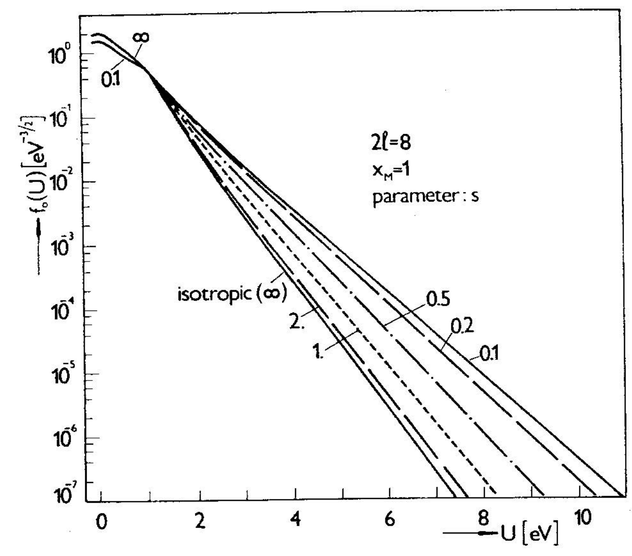

Fig. 2. The converged isotropic distribution for isotropic scattering in comparison to those for forward scattering with different widths in elastic collisions.

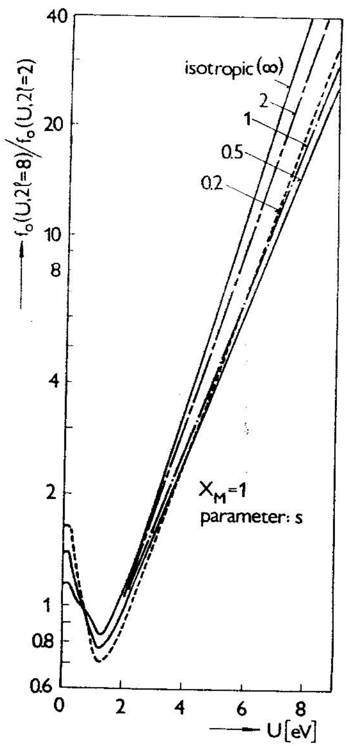

Fig. 3. The ratio of the converged f_0 to f_0 of the TTA under the same conditions as in Fig. 2.

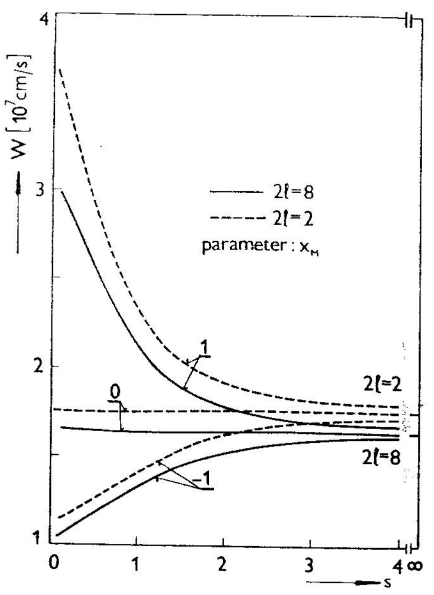

Fig. 4. The drift velocity for different widths of forward, symetrically transversal and backward scattering in elastic collisions (full lines — converged values, dashed lines — TTA).

very similar to that of f_0 shown in Fig. 3, remarkable changes of the drift velocity W occur in the transition from TTA to the converged approximation.

Summarizing it can be concluded that a remarkable anisotropy in the elastic scattering leads to a pronounced impact on the velocity distribution. Thus, if such conditions are given in real plasmas, this impact must be taken into account for a precise determination of the velocity distribution.

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