A DELOCALIZATION EFFECT OF AN EXTERNAL ELECTRICAL FIELD IN A ONE-DIMENSIONAL ANDERSON MODEL

ЭФФЕКТ ДЕЛОКАЛИЗАЦИИ ВНЕШНЕГО ЭЛЕКТРИЧЕСКОГО ПОЛЯ В ОДНОМЕРНОЙ МОДЕЛИ АНДЕРСОНА

P. MARKOŠ[1], Bratislava

A one-dimensional random walk is studied to find a delocalization effect of an external electrical field. Threshold behaviour of a localization length as a function of the disorder parameter is found.

Allen [1] has given an interesting analogy relating to the Anderson localization. His quantum-mechanical model exhibits similar localization — delocalization properties as the theory of the Anderson localization does. Namely, an electron in the one-dimensional disordered lattice is, according to Allen, always localized.

Because of the simplicity and objectivity of this model, we have used it to study how the external electrical field influences the electron localization in the one-dimensional case. We do not discuss principles upon which the model is based nor do we give any theoretical arguments to support it. Nevertheless, by comparing our results with experimental data we can give some arguments for accepting Allen's model.

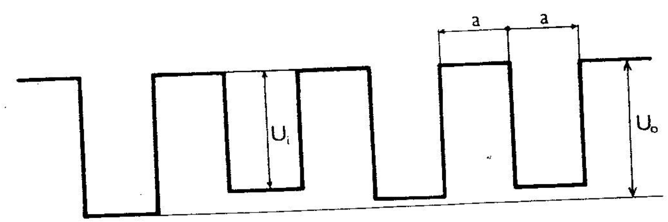

Fig. 1.

Let us briefly summarize the basic ideas of Allen's work. In Fig. 1 a one-dimensional chain of potential wells of the random depth U_i and of the width a, separated from each other by barriers of the same width, is depicted. We suppose the energy of the electron to be spread in an interval of the width V; $V \neq 0$ because of the different depths of the wells.

[1] Institute of Physics, EPRC, Slov. Acad. Sci. Dúbravská cesta, 842 28 BRATISLAVA, Czechoslovakia.

The electron makes random hops from each well to the first neighbouring ones at a rate of $W/2h$ hops per second. During the time interval $(0, t)$ it makes

$$n(t) = \frac{Wt}{2\hbar} \tag{1}$$

hops, and visits $S(n(t))$ distinct wells. This process is characterized by an uncertainty in the energy of the electron $\Delta E = \min|E_i - E_{S(n)}|$ where E_i is the energy of the electron in the i-th, well, and the minimum is taken with respect to the energy spectrum of the electron in the $S(n)$-th well. We postulate, according to Allen, the statistical average $\langle E(t)\rangle$ by the expression

$$\langle E(t)\rangle = \frac{V}{S(n(t))}. \tag{2}$$

We define the region of localization as the one in which the electron can move in a way of random hopping, without cancelling the inequality $\langle E(t)\rangle t < \hbar$. If $\langle E(t)\rangle t > \hbar$, the motion of the electron from the first well to the last is "observable", e.i. electron leaves its localization region during the time t. Thus, we can determine the localization length ξ as a mean distance, over which the electron can travel without being observed. In other words, the electron reaches the distance ξ in a time T for which the equality

$$\langle E(T)\rangle T = \hbar \tag{3}$$

is fulfilled.

Define a disorder parameter

$$u = \frac{S(N)}{N} = \frac{2V}{W} \tag{4}$$

where $N = n(T)$.

According to the theory of the random walk, we have the relation

$$\xi = 2aN^{1/2}. \tag{5}$$

Asymptotic values of $S(N)$ for large N can be calculated by the method of Montroll and Weis [2], which gives, in the one-dimensional case, the formula

$$S(N) = \sqrt{\frac{8N}{\pi}}. \tag{6}$$

Using formulae (1), (2), for $t = T$, we obtain a u-dependence of the localization length

$$\frac{\xi}{2a} = \frac{1}{u}\sqrt{\frac{8}{\pi}} \tag{7}$$

which gives the localization of the electron for all the values of u, $u \neq 0$.

which gives the localization of the electron for all the values of u, $u \neq 0$.
Expression (6) is valid only if the probabilities p, q of an electron hopping to the right and to the left are equal. There is no argument for considering any asymmetry of the problem. If the external electrical field Q is applied, however, the symmetry is broken (Fig. 2) $p - q = \Delta > 0$, $(p + q = 1)$. We are interested in how this asymmetry influences the u-dependence of the localization length. Using once again the method of Montroll an Weis we obtain a new asymptotic formula for $S_Q(N)$:

$$S_Q(N) = \sqrt{\frac{8N}{\pi}}\left(1 + \frac{1}{2}\Delta^2 N\right)^{1/2}. \tag{8}$$

Then the u-dependence of ξ shows a non-analytic behaviour

$$\frac{\xi}{2a} = \sqrt{\frac{8}{\pi}}(u^2 - u_c^2)^{-1/2} \tag{9}$$

where

$$u_c = \frac{2}{\sqrt{\pi}}\Delta(Q) \tag{10}$$

is a critical value of the disorder parameter.

It is now clear that the external electrical field may, at least in principle, cause a delocalization of electrons even in the one-dimensional case.

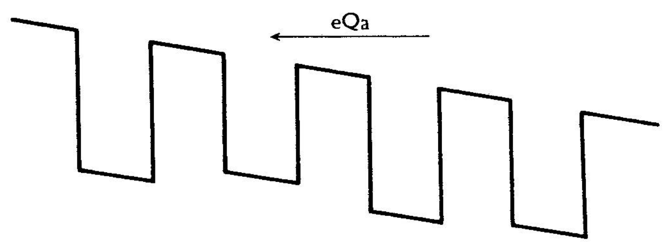

Fig. 2.

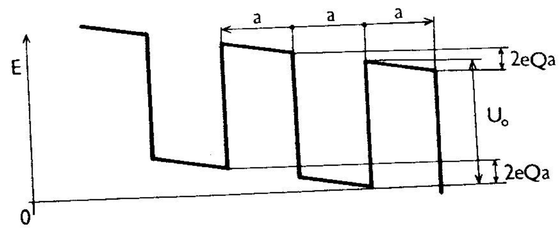

Fig. 3.

In the following we will find the critical electrical field which causes the delocalization of the electrons for a given disorder parameter u. We consider the most simple model: an electron can hop from one well to the neighbouring one only in a way of the quantum-mechanical tunnelling through the potential barrier. We approximate the depths of both wells by the mean value U_0. Then Δ depends on Q and on the energy E of the electron in the well. We take

$$\Delta(Q, E) = \frac{p_+ - p_-}{p_+ + p_-} \tag{11}$$

where $p_+(p_-)$ is proportional to the probability of quantum-mechanical tunnelling to the right (to the left):

$$p_- \sim \exp\left[-\int_0^a dx\,\sqrt{\frac{2m}{\hbar^2}(U_0 + eQx - E)}\,\right]$$

$$p_+ \sim \exp\left[-\int_0^a dx\,\sqrt{\frac{2m}{\hbar^2}(U_0 - eQx - E)}\,\right] \tag{12}$$

for $E \in (2eQa, U_0)$, and $\Delta(Q, E) = 1$ for $E < 2eQa$ (Fig. 3).

To obtain $\Delta(Q)$, we average $\Delta(Q, E)$ over the electron energy E:

$$\Delta(Q) = \frac{1}{U_0}\int_0^{U_0} dE\,\Delta(Q, E) \simeq \frac{1}{U_0}[2eQa + (U_0 - 2eQa)\Delta(\bar{E})] \tag{13}$$

where we have taken $\bar{E} = \frac{1}{2}[U_0 + 2eQa]$.

Denoting $z = 2eQa/U_0$, $C = [(2ma^2/\hbar^2)(U_0/2)]^{1/2}$ we obtain

$$p_+(E) \sim \exp\left[-\frac{C}{2}((1-z)^{3/2} - (1-2z)^{3/2})\right] \quad z < \frac{1}{2}$$

$$p_+(E) \sim \exp\left[-\frac{C}{2}(1-z)^{3/2}\right] \quad \frac{1}{2} < z < 1$$

$$p_-(E) \sim \exp\left[-\frac{C}{2}(1-(1-z)^{3/2})\right]$$

$$\Delta(Q) = z + (1-z)\Delta(E).$$

The Q-dependence of u_c is depicted in Fig. 4. As typical values of the parameters U_0, a, we take $U_0 = 0.15$ eV, $a = 2.4 \times 10^{-9}$ m. From formula (7) we suppose the values of the disorder parameter u to

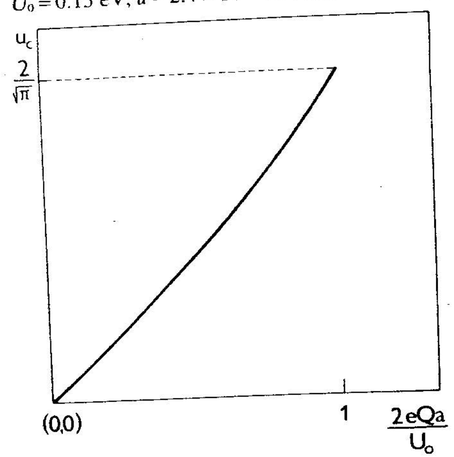

Fig. 4.

lie in the interval (0.1, 0.5). The critical electrical field, which corresponds to this disorder parameter, is estimated by the values

$$Q \sim 1 - 2 \times 10^7 \text{ V/m}.$$

This is in good agreement with the critical electrical field, observed experimentally in the bulk samples (10^7—10^8) V/M. This result is encouraging. Therefore, in near future we will calculate the critical electrical field for the cases of a dimension greater than one. A more realistic model for calculating Δ will also be used.

I thank Dr. E. Majerníková for introducing me to the problem.

REFERENCES

[1] Allen, P. B.: J. Phys. C. Solid St. Phys. *13* (1980), L667.
[2] Montroll, E. W., Weis, G. H.: J. Math. Phys. *6* (1965), 167.

Received December 2nd, 1982
Revised version received January 17th, 1984