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AN APPROACH TO THE CALCULATION OF NONPERTURBATIVE ENERGY-LEVEL SHIFTS IN HEAVY QUARKONIA

DESCRIPTION OF THE METHOD

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An approach to calculating shifts of heavy-quarkonium energy levels due to the interaction with the gluon condensate is developed. The problem is reduced to finding eigenvalues of an infinite matrix. The corresponding secular equation is then shown to lead after a series of approximations to results originally derived by Voloshin and Leutwyler using somewhat different methods.

ОБ ОДНОМ МЕТОДЕ ВЫЧИСЛЕНИЯ НЕПЕРТУРБАЦИОННЫХ СДВИГОВ ЭНЕРГЕТИЧЕСКИХ УРОВНЕЙ В ТЯЖЕЛЫХ КВАРКОНИЯХ

В работе разработан метод вычисления сдвига энергетических уровней в тяжелых кваркониях, обусловленного взаимодействием с глюонным конденсатом. Проблема сведена к нахождению собственных значений определенной бесконечной матрицы. Показано, что соответствующее вековое уравнение после ряда приближений приводит к результатам, впервые полученным Волошиным и Лейтвилером, которые однако использовали другие методы.

I. INTRODUCTION

Quantum chromodynamics [1] (for reviews see e.g. [2]) has become widely accepted as a promising candidate for the correct theory of strong interactions. Owing to its property of asymptotic freedom [3] it is possible to extract a whole variety of perturbative QCD predictions for the phenomena which are governed by a small effective coupling constant, i.e. for which the smallest distances are relevant [4]. On the other hand, the long-distance phenomena cannot be calculated reliably, since the effective coupling is large and perturbative methods are no longer applicable.

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After the discovery of the families of Ψ — [5] and Y-mesons [6] a new area for testing QCD predictions and QCD motivated ideas has been opened. These systems (called quarkonia) interpreted as bound states of heavy quark-antiquark pairs (the distinguishing quality is called charm in the former family and bottomness or beauty in the latter one) are singled because of three properties:

1) their constituents are heavy enough so that the systems are essentially nonrelativistic;

2) the effective coupling between the quark and the antiquark is sufficiently small so that they are similar to Coulombic systems (like positronium);

3) their characteristic radius is fairly small so that nonperturbative effects should show up rather simply.

Properties of the members of both families can be to a large extent accounted for by a simple potential model approach (most recent reviews are listed in Ref. [7]). This relies (in the first approximation) on the nonrelativistic Schrödinger equation with a static quark-antiquark potential. A QCD motivated potential of this sort has to reflect two features: first, the asymptotic freedom of QCD at small distances, and secondly, the apparent impossibility of the existence of free quarks. While the short-distance part of the potential is thus (at least in principle) calculable using the methods of perturbative QCD (see e.g. [8]) and in the lowest approximation is given by a Coulomb-like one-gluon-exchange term, the long-distance confining part of the potential is usually approximated by a linear term for which there is no reliable justification within QCD (except probably for lattice calculations [9]). Moreover, present-day data on charmonia and bottomonia are unable to fix the form of the potential unambigously, since their properties are sensitive mostly only to distances from ~ 0.1 to ~ 1 fermi and various types of potentials which are roughly identical in this region can be used [10].

The origin of the confining part of the static quark-antiquark potential is usually attributed to the complicated structure of the QCD vacuum fluctuations. An analysis of the hadron properties by the ITEP group using the QCD sum rules [11] has suggested a possibility to parametrize the characteristics of the QCD vacuum by a set of vacuum expectation values of combinations of quark and gauge fields (so-called condensates), e.g. the chiral symmetry breaking condensate $\langle 0|\bar{\Psi}\Psi|0\rangle$, or the gluon condensate $\langle 0|F^a_{\mu\nu}F^{a\mu\nu}|0\rangle$. ($\Psi$ and $F^a_{\mu\nu}$ denote the quark and gluon fields, respectively.)

Recently, Voloshin [12] and Leutwyler [13] (see also [14]) have attempted to calculate the effect of the gluon condensate on the characteristics of very heavy quarkonia. They argue that for sufficiently large quark masses (of the order of tens GeV/c^2) the quarkonium radius should be small enough compared to the characteristic length of vacuum fluctuations and thus one could use only the first few terms of the multipole expansion similar to that of quantum electrodynamics. Moreover, if the gauge field changes slowly not only in space but in time as well, one can in the first approximation face the problem of a quark-antiquark pair moving in a constant random gauge field. Using these assumptions and considering only the electric-dipole-like term of the multipole expansion Voloshin and Leutwyler found corrections to the Coulombic energy levels of very heavy quarkonia which were proportional to the gluon condensate $\langle 0|F^a_{\mu\nu}F^{a\mu\nu}|0\rangle$.

However, the authors' results are inapplicable to charmonium and bottomonium since their calculation is perturbative in the gluon condensate. In fact, the relevant parameter estimating the strength of the perturbation $\xi = 4\pi\alpha_s\langle 0|F^a_{\mu\nu}F^{a\mu\nu}|0\rangle/m^4\beta^6$ (here m is the quark mass, $\alpha_s = g^2/4\pi$ is the strong-interaction fine-structure constant, and $\beta = 4\alpha_s/3$) is far too large even for bottom quarks. Thus, the results of Voloshin an Leutwyler, though being of principal interest, should become applicable to real quarkonia only after the discovery of bound states of top quarks and antiquarks (if these do exist).

In this paper we are going to describe a method[1)] which could enable the nonperturbative (or at least systematic perturbative) calculation of heavy quarkonium energy-level shifts due to the gluon condensate. The problem will be shown to lead to solving a secular equation for finding eigenvalues of an infinite matrix[2)] (see Sect. II.). The simplifications leading to the Voloshin-Leutwyler formula for the shifts will be formulated at the end of Sect. II. Sect. III will comprise conclusions and an outline of a possible use of the described method.

II. DESCRIPTION OF THE METHOD

The system under study can be thought of as an isolated system consisting of the heavy quark-antiquark ($Q\bar{Q}$) pair together with the gluonic degrees of freedom. In the first approximation we can assume the pair to be immersed in a random vacuum gauge field (see Fig. 1) characterized by a four-vector potential $A^a_\mu(x)$ ($a = 1, 2, \ldots, 8$). Any physical state has to be singlet and hence consists of the $Q\bar{Q}$ pair in a singlet or octet state, and of the surrounding gluon field which must correspondingly be either in a singlet state (if the $Q\bar{Q}$ pair is singlet) or in an octet one (if the pair is octet). We shall denote the corresponding projectors on the singlet and octet states as P_S, P_A and Π_S, Π_A in the $Q\bar{Q}$ and gluon case, respectively.

If the quark mass m is large enough, the $Q\bar{Q}$ bound state becomes essentially Coulombic (see Eq. (6) below) and its characteristic radius is of the order of $r_q \sim (m\alpha_s)^{-1}$, while the characteristic period of the quark motion is of the order of $t_q \sim (m\alpha_s^2)^{-1}$. These values could be smaller than the (up to now not well known

[1)] A similar (though nonstationary) approach has been used by R. Meckbach (MPI München) for extracting some information on the static quark-antiquark potential [15].

[2)] Strictly speaking, the matrix elements themselves also depend on the eigenvalue that is looked for. The precise meaning of this statement will become clear from Eq. (28).

[12, 16]) mean scale R and period T of nonperturbative vacuum fluctuations, and one could restrict oneself to the fields constant in time, and make use of the QCD multipole expansion of the interaction of quarks with the constant gluon field.

Although the technique of calculation we shall use is in many respects similar to that describing the $Q\bar{Q}$ system in an external field[3], the physics behind it is different. What we consider is a coupled system consisting of a $Q\bar{Q}$ pair and gluons and the interaction is described in the long wavelength limit of gluonic excitations. The system is closed and thus it has sharp energy levels.

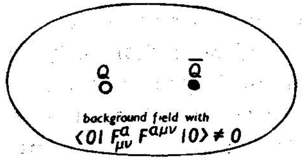

Fig. 1. $Q\bar{Q}$ pair in a constant vacuum gauge field — a sketch.

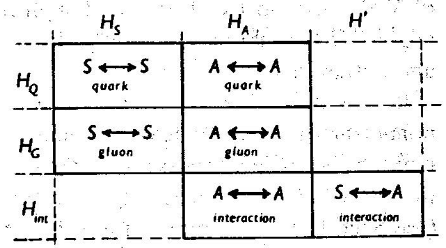

Fig. 2. Illustration of how the total singlet Hamiltonian can be separated in two different ways: 1) horizontally, according to which ($Q\bar{Q}$ or gluon) degrees of freedom are acted upon by parts of the Hamiltonian; 2) vertically, according to which colour states are connected by parts of the Hamiltonian.

The Hamiltonian of the singlet system consisting of the $Q\bar{Q}$ pair and the vacuum gauge field can now be separated into three parts

$$H = H_S + H_A + H', \tag{1}$$

where H_S and H_A do not mix singlet and octet states[4]

$$H_S = P_S \Pi_S H P_S \Pi_S, \tag{2}$$

$$H_A = P_A \Pi_A H P_A \Pi_A, \tag{3}$$

[3] Since $r_q \sim t_q \alpha_s^{-1} < t_q$ and one can expect $R \sim T$ for vacuum fluctuations, the condition $t_q \ll T \sim R$ is most essential and determines the range of applicability of the present method. If $t_q \sim T$, one should take into account explicitly the retardation effects and the interaction could not be formally like that of a $Q\bar{Q}$ system in a homogeneous external field. (I thank Prof. Voloshin for emphasizing this point to me).

[4] We should also include projector on the total singlet state (of the whole system). This will be for simplicity omitted, but tacitly implied.

while H' does

$$H' = P_S \Pi_S H P_A \Pi_A + P_A \Pi_A H P_S \Pi_S. \tag{4}$$

(Hereafter we shall adopt a shortened notation: $P_S \Pi_S \equiv P_1$ and $P_A \Pi_A \equiv P_8$; note that P_8 is not an octet projector, but projects on the singlet system consisting of a $Q\bar{Q}$-pair and gluon octet parts.) The same Hamiltonian can also be written in a different separated form, namely [12] (see also Fig. 2)

$$H = H_Q + H_G + H_{int}, \tag{5}$$

where H_Q is the Hamiltonian acting on dynamical variables of quarks, which for very quarks takes the form

$$H_Q = P_1 \left(-\frac{\Delta}{m} - \frac{4}{3}\frac{\alpha_s}{r} \right) P_1 + P_8 \left(-\frac{\Delta}{m} + \frac{1}{6}\frac{\alpha_s}{r} \right) P_8, \tag{6}$$

H_G is the Hamiltonian for gluonic degrees of freedom (whose energy spectrum will be assumed to start from zero), and H_{int} describes the interaction of quarkonium with vacuum fluctuations[5].

Using the multipole expansion [12, 13, 17] and neglecting higher terms H_{int} consists of two parts [12][6]

$$H_{int} = Q^a A_0^a(0) - \boldsymbol{d}^a \cdot \boldsymbol{E}^a(0) \tag{7}$$

(repeated indices are summed over), where

$$Q^a = \frac{1}{2} g t^a = \frac{1}{2} g (t_1^a + t_2^a), \tag{8}$$

$$\mathbf{d}^a = \frac{1}{2} g \xi^a \boldsymbol{r} = \frac{1}{2} g (t_1^a - t_2^a)\, \boldsymbol{r}, \tag{9}$$

and t_1^a, t_2^a are SU(3) colour generators for the quark and antiquark, respectively; $\boldsymbol{E}^a$ is the chromoelectric field of the vacuum fluctuation. Since t^a annihilates singlets, and ξ^a changes a singlet to an octet and an octet to a superposition of a singlet and an octet, H_{int} can be written as

$$H_{int} = [P_8 Q^a A_0^a(0)\, P_8 - P_8 \boldsymbol{d}^a \cdot \boldsymbol{E}^a(0)\, P_8] - P_1 \boldsymbol{d}^a \cdot \boldsymbol{E}^a(0)\, P_8 - P_8 \boldsymbol{d}^a \cdot \boldsymbol{E}^a(0)\, P_1. \tag{10}$$

[5] The separation of the Hamiltonian H into H_Q, H_G and H_{int} is in some sense arbitrary. Here the interaction of quarks with hard, short-wavelength gluons is included in H_Q and gives rise to the Coulombic interaction potential, while the long-wavelength gluonic fluctuations are contained in H_{int}. Any interaction between both these types of gluons is ignored.

[6] The centre of mass of the $Q\bar{Q}$ pair is assumed to be placed at the origin of the coordinate system.

Comparing (1), (5) and (10) one sees immediately the relation between the two ways of separating the total singlet Hamiltonian into parts (Fig. 2)

$$H_S = P_1 H_Q P_1 + P_1 H_G P_1, \tag{11}$$

$$H_A = P_8 H_Q P_8 + P_8 H_G P_8 + P_8 H_{int} P_8, \tag{12}$$

and

$$H' = P_1 H_{int} P_8 + P_8 H_{int} P_1 = - P_1 d^a \cdot E^a(0) P_8 - P_8 d^a \cdot E^a(0) P_1. \tag{13}$$

All relevant information on the $Q\bar{Q}$ pair in the vacuum field could be extracted from the complete Green function for the Hamiltonian H,

$$G(E) \equiv (H-E)^{-1}. \tag{14}$$

However, our knowledge of the QCD vacuum is rather poor and does not enable to find a complete solution of the problem. A lot of useful information could nevertheless be obtained if we restricted our attention to the pure quarkonium[7)] Green function (i.e. the projection of the complete Green function on the colour singlet quarkonium states, averaged over the gluonic vacuum)

$$G_Q(E) \equiv P_S \langle 0_g | (H-E)^{-1} | 0_g \rangle P_S \tag{15}$$

($|0_g\rangle$ is the gluonic vacuum). Its poles, for example, correspond to quarkonium energy levels[8)].

The Green function $G_Q(E)$ remains to be an operator, acting on colour singlet quarkonium states. We shall now concentrate on deriving an operator equation which it must obey.[9)]

Using the well-known identity

$$(H-E)^{-1} = (H_S-E)^{-1} - (H_S-E)^{-1}(H-H_S)(H-E)^{-1} =$$
$$= (H_S-E)^{-1} - (H_S-E)^{-1}(H_A+H')(H-E)^{-1} \tag{16}$$

one can easily show that

$$G_Q(E) = P_A \langle 0_g | (H_S-E)^{-1} | 0_g \rangle P_S - \langle 0_g | P_1 (H_S-E)^{-1} P_1 H' P_8 (H-E)^{-1} P_1 | 0_g \rangle \tag{17}$$

[7)] Pure quarkonia consist of a singlet $Q\bar{Q}$ pair plus the gluonic vacuum.

[8)] Of course, poles of the pure quarkonium Green function are also poles of the complete Green function.

[9)] A similar approach was used to solve the coupled-channel problem in charmonium by Eichten et al. [18]. The quarkonium Green function $G_Q(E)$ was also introduced by Voloshin (Sect. III of Ref. [19]) and second-order perturbation expression (i.e. linear in $\langle 0|F^a_{\mu\nu}F^{a\mu\nu}|0\rangle$) was derived for it (treating H_G and H_{int}) as a perturbation).

where we utilized the fact that the gluonic vacuum is a color singlet state, and $P_1 H_A = 0$. This equation is illustrated in Figs. 3, 4a. A similar identity with H_S and H_A exchanged

$$(H-E)^{-1} = (H_A-E)^{-1} - (H_A-E)^{-1}(H_S+H')(H-E)^{-1} \tag{18}$$

can be used to express $P_8(H-E)^{-1}P_1$ (see Eq. (17)) through $P_1(H-E)^{-1}P_1$, viz.

$$P_8(H-E)^{-1}P_1 = - P_8(H_A-E)^{-1}P_8 H' P_1 (H-E)^{-1} P_1. \tag{19}$$

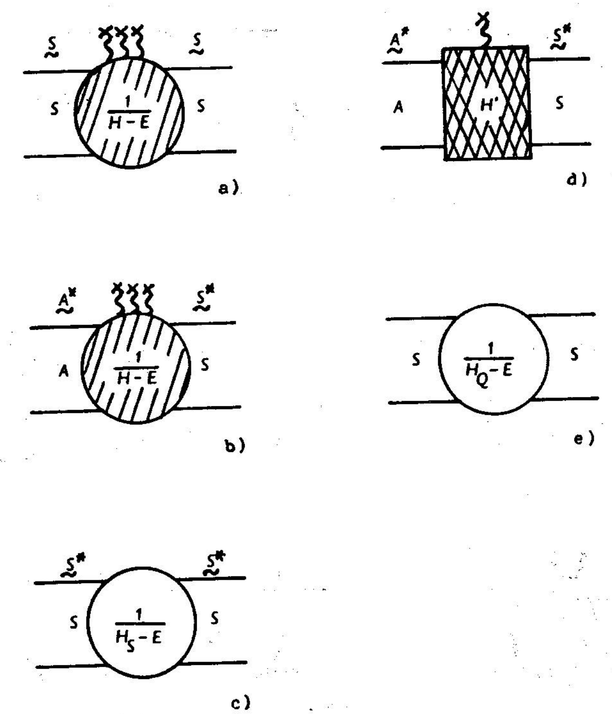

Fig. 3. Vocabulary of elements entering Eqs. (17), (19), (20) and (22), and Figs. 4, 5; a) complete pure quarkonium Green function; b) propagation of quarkonium and condensate with interaction in which gluonic quantum numbers are transferred from the quarkonium to the condensate (A ↔ S, A* ↔ S*) transitions occur); c) propagation of quarkonium and condensate in singlet states (without interacting); d) the interaction of quarkonium with the condensate in which gluonic quantum numbers are transferred; e) propagation of singlet quarkonium without any interaction with the condensate. (S, S*, A* denote the states of the gluonic background (gluonic vacuum, singlet gluonic excitation, octet gluonic excitation, respectively), while S and A denote the $Q\bar{Q}$ pair colour state (singlet and octet)).

Inserting this into Eq. (17) we get (Fig. 4c)

$$G_Q(E) = P_S(H_Q - E)^{-1}P_S + \\ + \langle 0_g|P_1(H_S - E)^{-1}P_1H'P_8(H_A - E)^{-1}P_8H'P_1(H - E)^{-1}P_1|0_g\rangle, \quad (20)$$

where use was also made of $\langle 0_g|(H_S - E)^{-1}|0_g\rangle = (H_Q - E)^{-1}$.

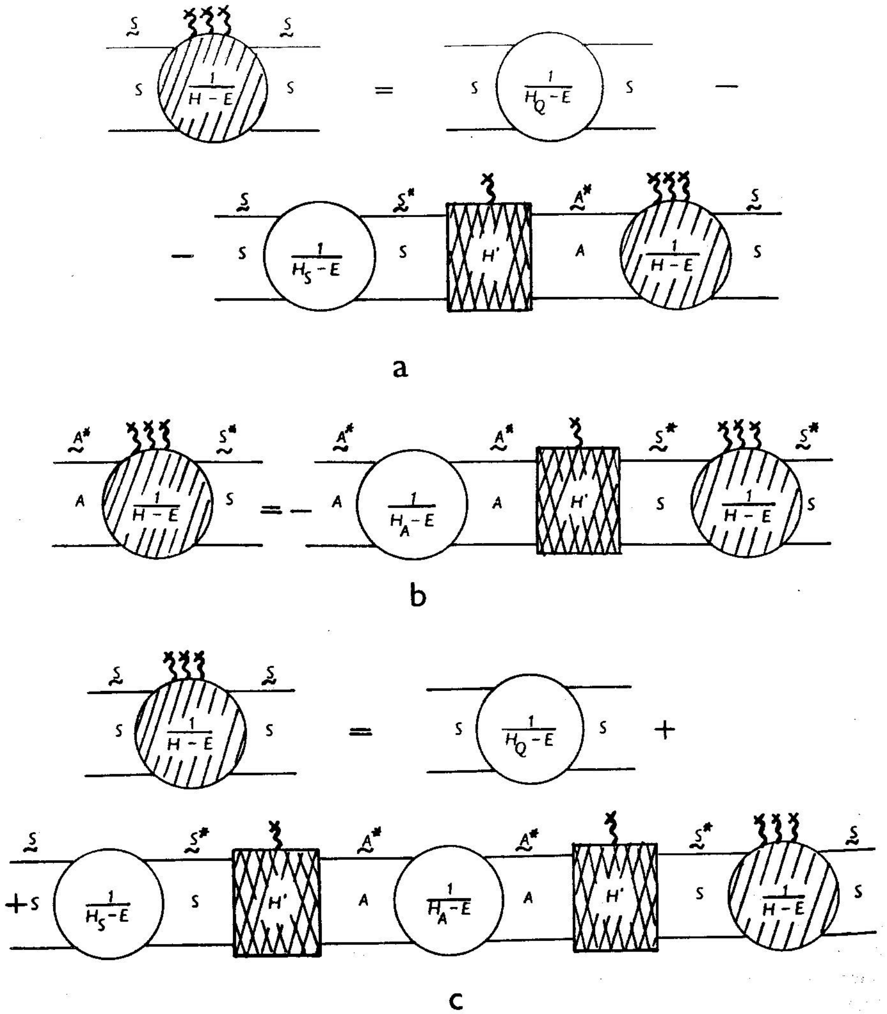

Fig. 4. Diagrammatical illustration of steps in deriving the operator equation for the quarkonium Green function: a) Eq. (17); b) Eq. (18); c) Eq. (20).

To further simplify this equation, one has to use some physical assumption on the nature of the problem. We shall assume that in $P_1 = P_S\Pi_S$

$$\Pi_S \simeq |0_g\rangle\langle 0_g|. \quad (21)$$

Doing this we omit excited colour singlet states of the gauge field, i.e. we do not care about the so-called gluonia. In fact we thus neglect the possible admixture of gluonia to the $Q\bar{Q}$ wave function. The reasons are twofold: first, this assumption enables us to find a closed equation for $G_Q(E)$ and it is interesting to explore its consequences, and, second, we do not exclude gluons completely, we allow for the emission of gluons with the corresponding transition of the $Q\bar{Q}$ pair from a colour singlet to an octet state. Thus, the assumption (21) does *not* destroy the possibility of singlet ↔ octet transitions that significantly change the quark-antiquark interaction in quarkonia [12].

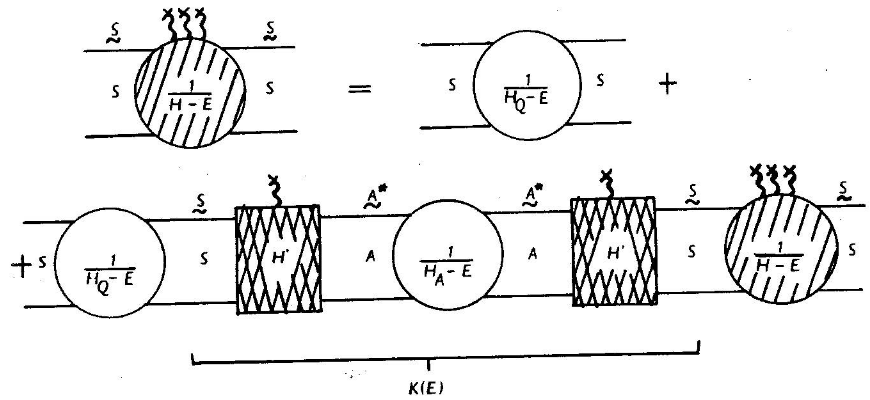

Fig. 5. The closed operator equation for the pure quarkonium Green function (Eq. (22)).

Using (21) Eq. (20) changes to (see Fig. 5)

$$G_Q(E) = G_Q^{(S)}(E) + G_Q^{(S)}(E)K(E)G_Q(E), \quad (22)$$

where $G_Q^{(S)}(E)$ is the Green function for a singlet Coulombic quarkonium

$$G_Q^{(S)}(E) = P_S(H_Q - E)^{-1}P_S\,; \qquad P_SH_QP_S = -\frac{\Delta}{m} - \frac{4}{3}\frac{\alpha_s}{r} \quad (23)$$

and the operator $K(E)$ is

$$K(E) = P_S\langle 0_g|H'P_8(H_A - E)^{-1}P_8H'|0_g\rangle P_S. \quad (24)$$

The way how the assumption (21) simplifies Eq. (20) can simply be visualized by comparing Figs. 4c and 5. Neglecting possible singlet gluon excitations ($S^*_\sim$ in Fig. 4c) at intermediate stages immediately leads to Eq. (24) (Fig. 5, each $S^*_\sim$ is simple replaced by $\underset{\sim}{S}$, where $\underset{\sim}{S}$ denotes the ground state of the system).

Having derived an (approximate) operator equation for the quarkonium Green function $G_Q(E)$ we can use it in a straightforward way for finding quarkonium energy levels. Let ϵ_n and $|n\rangle$ be eigenvalues and eigenstates of the singlet (Coulombic) quarkonium Hamiltonian $P_S H_Q P_S$, and let $\Psi_n(r)$ be the corresponding wave functions. Then

$$\langle r'|G_Q(E)|r\rangle = \sum_n \Psi_n(r')\langle n|G_Q(E)|r\rangle, \tag{25}$$

where, using Eq. (22),

$$\langle n|G_Q(E)|r\rangle = \frac{-1}{E-\epsilon_n}[\Psi_n^*(r) + \sum_m \langle n|K(E)|m\rangle\langle m|G_Q(E)|r\rangle], \tag{26}$$

or

$$\sum_m [(E-\epsilon_n)\delta_{nm} + K_{nm}(E)]\langle m|G_Q(E)|r\rangle = -\Psi_n^*(r), \tag{27}$$

with $K_{nm}(E) \equiv \langle n|K(E)|m\rangle$. At a quarkonium energy $\langle r'|G_Q(E)|r\rangle$ must have a pole; this is only possible if the infinite system of equations (27) has no solution for E equal to quarkonium energies. This condition can formally be expressed in the following form:

$$\det[(E-\epsilon_n)\delta_{nm} + K_{nm}(E)] = 0. \tag{28}$$

This equation could provide a basis for a systematic calculation of quarkonium energy level shifts due to the interaction with the vacuum gluon fields. Though the task seems hopeless since one has to cope formally with an infinite determinant, we do believe that a series of reasonable approximations ought to brinng Eq. (28) into a manageable form.

To illustrate this point we shall show the approximations that lead to the energy-level shifts calculated by Voloshin [12] and Leutwyler [13]. Let us, however, first recall assumptions leading to their result. Aside from a natural assumption of the invariance of the gluonic vacuum with respect to global colour and space rotations they assume the following:

1) the interaction of the $Q\bar{Q}$ pair with vacuum fluctuations is small so that one can use the quantum-mechanical second-order perturbation expression for energy shifts;

2) energies of colour octet gluonic excitations are small compared to the difference between colour octet and singlet quarkonium energies (this corresponds to neglecting higher terms in the operator product expansion, see [19]).

It is easy to see what must be done with Eq. (28) to recover the Voloshin—Leutwyler formula. First, one has to neglect all non-diagonal terms of $K_{nm}(E)$ and replace E by its zeroth approximation, ϵ_n

$$K_{nm}(E) \simeq K_{nm}(\epsilon_n)\delta_{nm}. \tag{29}$$

This clearly corresponds to the first above-mentioned assumption. The shifts are then simply

$$\Delta E_n = -K_{nn}(\epsilon_n) = -\langle n|\langle 0_g|H' P_8 (H_A - \epsilon_n)^{-1} P_8 H'|0_g\rangle|n\rangle. \tag{30}$$

Secondly, working to the first order in g^2, one retains only the quarkonium part of the Hamiltonian H_A (see Eq. (12)) and gets using Eqs. (9) and (13)

$$\Delta E_n = -\langle 0_g|\pi\alpha_s F_i^a E_j^b|0_g\rangle\langle n|r_i\xi^a G_Q^{(A)}(\epsilon_n) r_j \xi^b|n\rangle \tag{31}$$

with $G_Q^{(A)}$ being analogous to $G_Q^{(S)}$

$$G_Q^{(A)} = P_A(H_Q - E)^{-1}P_A; \qquad P_A H_Q P_A = -\frac{\Delta}{m} + \frac{1}{6}\frac{\alpha_s}{r}. \tag{32}$$

To show the relation of this assumption to that of Voloshin and Leutwyler we can rewrite $P_8(H_A - \epsilon_n)^{-1}P_8$ in the following form

$$P_8(H_A - \epsilon_n)^{-1}P_8 = P_A \sum_{m_g^A, n_g^A} |m_g^A\rangle\langle m_g^A|(P_A H_Q P_A + P_A H_G P_A +$$

$$+ P_A H_{int} P_A - \epsilon_n)^{-1}|n_g^A\rangle\langle n_g^A|P_A = P_A \sum_{m_g^A, n_g^A} |m_g^A\rangle\langle m_g^A|(P_A H_Q P_A +$$

$$+ P_A H_{int} P_A + \epsilon_{m_g} - \epsilon_n)^{-1}|n_g^A\rangle\langle n_g^A|P_A. \tag{33}$$

If we neglect $P_A H_{int} P_A$ (otherwise higher than second-order terms in the gluon field would be retained) and ϵ_{m_g} (that corresponds to the Voloshin—Leutwyler second assumption), we get from Eq. (33)

$$P_8(H_A - \epsilon_n)^{-1}P_8 = P_A(H_Q - \epsilon_n)^{-1}P_A \tag{34}$$

and arrive at Eq. (31). Using colour and rotation invariance of the gluonic vacuum

$$\langle 0_g|E_i^a E_j^b|0_g\rangle = -\frac{1}{96}\delta^{ab}\delta_{ij}\langle 0_g|F_{\mu\nu}^a F^{a\mu\nu}|0_g\rangle \tag{35}$$

and

$$\langle \text{singlet}|\xi^a \text{ (singlet operator) } \xi^b|\text{singlet}\rangle = \frac{2}{3}\delta^{ab}, \tag{36}$$

we see that (31) is identical to Voloshin's form [12]

$$\Delta E_n = \eta \langle n | r G_Q^{(A)}(\epsilon_n) r | n \rangle, \tag{37}$$

where η is proportional to the gluon condensate: $\eta = (\pi^2/18) \langle 0 | \frac{\alpha_s}{\pi} F^a_{\mu\nu} F^{a\mu\nu} | 0 \rangle$. Thus, using approximations essentially identical to that of Voloshin and Leutwyler, our expression (28) reproduces their result.

III. CONCLUSIONS

The method described in the present paper could provide a basis for a systematic calculation of the shifts of heavy-quarkonium energy levels due to the gluon condensate. An approximate operator equation for the pure-quarkonium Green function has been derived using a single assumption that neglects the admixture of gluonia in quarkonium states. We do not consider this simplification to be vitally important since the essential feature of the model — the possibility for the quark-antiquark pair to jump from singlet to octet states (and vice versa) with simultaneous jumps of the gluonic background — has been retained. The operator equation has then been used to derive an equation for finding quarkonium energy levels. However, the latter cannot be solved without any further assumptions or simplifications.

The most important task now remains to find and justify some approximation which would: 1) reduce Eq. (28) to a manageable form; 2) enable to include some of the effects omitted in the original work of Voloshin and Leutwyler. In fact this should consist of two steps: 1) using reasonable information or assumptions on the dynamics of gluons in the QCD vacuum (since the complete kernel $\tilde{K}(E)$, Eq. (24), contains also a generally non-negligible gluonic piece, see Eq. (12)); 2) truncating the infinite determinant, Eq. (28). If this proves to be possible, the proposed method should enable a more reliable calculation of the quarkonium level shifts due to the gluon condensate.

Some clues to this problem will form the subject of a later publication.

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