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Letters to the Editor

MEASUREMENT OF THERMAL DIFFUSIVITY BY THE FLASH METHOD WITH A COAXIAL SOURCE

ИЗМЕРЕНИЕ КОЭФФИЦИЕНТА ТЕМПЕРАТУРОПРОВОДНОСТИ С ПОМОЩЬЮ МЕТОДА ВСПЫШКИ С КОАКСИАЛЬНО РАСПОЛОЖЕННЫМ ИСТОЧНИКОМ

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In the pulse method for measuring thermal diffusivity, described by Parker et al. in 1961 [1], the front face of a small disc-shaped specimen is irradiated by an energy pulse of short duration and the resultant temperature rise of the rear face is recorded. The thermal diffusivity of the sample can be deduced from this time-temperature relation and the solution of the appropriate heat conduction problem.

If: a) the heat pulse is uniform over the sample surface, b) there are no heat losses from the surface, c) the heat pulse is instantaneous, d) the material properties are temperature and space independent, the thermal diffucivity can be evaluated by the simple relation

$$a = 0.139 l^2 / t_{1/2}, \tag{1}$$

where l is the sample thickness and $t_{1/2}$ is the experimentally obtained half time, i.e. the time corresponding to a rise in temperature to one half of the steady value.

If the absorbed energy flux on the sample front surface is not uniform (either due to a non-uniform flux density of the beam or a non-uniform absorbance), the heat flow is two-dimensional and one-dimensional mathematics no longer apply. Several investigators have studied the effects of two-dimensional conduction occurring simultaneously with surface radiation heat loss [2—5].

Donaldson [3] presented the mathematical solution of the transient two-dimensional conduction after an instantaneous energy pulse, whose diameter is less than that of the sample, has been applied to the front surface. In this theoretical model the sample is represented by a thin isotropic slab infinite in the radial direction. Based on this solution Donaldson and Taylor [4] measured thermal diffusivity in the radial and the axial direction on isotropic specimens and Chu, Taylor and Donaldson [5] on anisotropic ones.

The purpose of this paper was to find a solution of this problem with consideration to finite fimensions of a disc-shaped sample.

The system of radial and axial coordinates and dimensions is illustrated in Fig. 1.

The general equation of heat conduction in cylindrical coordinates with a heat source function $q(r, \varphi, z, t)$ and no other source or sinks is

$$\frac{\partial^2 \vartheta}{\partial r^2} + \frac{1}{r}\frac{\partial \vartheta}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \vartheta}{\partial \varphi^2} + \frac{\partial^2 \vartheta}{\partial z^2} + \frac{q(r, \varphi, z, t)}{k} = \frac{1}{a}\frac{\partial \vartheta}{\partial t}, \tag{2}$$

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where k is the thermal conductivity and a the diffusivity of the sample. $a = k/\varrho c$, where ϱ is the density and c the specific heat.

The terms in φ do not appear with axial symmetry and will be omitted.

For zero initial temperature, adiabatic boundary conditions and instantaneous source distribution $q(r, z) = q_1(r) \cdot q_2(z)$ at $t = 0$ the solution of Eq. (2) is the product of the component solutions:

$$\vartheta(r, z, t) = \Psi(r, t)\Phi(z, t). \tag{3}$$

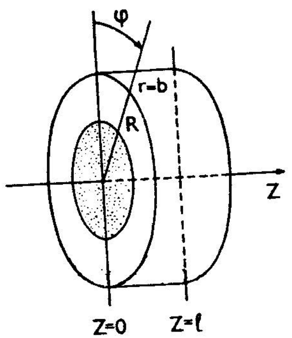

Fig. 1. The system of coordinates in the specimen.

For the radial component $\Psi(r, t)$ we have

$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r}\frac{\partial \Psi}{\partial r} + \frac{q_1(r)}{k} = \frac{1}{a}\frac{\partial \Psi}{\partial t}, \qquad \begin{matrix} t > 0 \\ 0 \leqq r \leqq b \end{matrix} \tag{4}$$

with boundary conditions:

$$\Psi = 0 \text{ at } t = 0, \tag{5}$$

$$\frac{\partial \Psi}{\partial r} = 0 \text{ at } r = b, \tag{6}$$

$$q_1(r) \text{ occurs at } t = 0. \tag{7}$$

For the axial component $\Phi(z, t)$ we have

$$\frac{\partial^2 \Phi}{\partial z^2} + \frac{q_2(z)}{k} = \frac{1}{a}\frac{\partial \Phi}{\partial t}, \qquad \begin{matrix} t > 0 \\ 0 \leqq z \leqq 1 \end{matrix} \tag{8}$$

with boundary conditions:

$$\Phi = 0 \text{ at } t = 0, \tag{9}$$

$$\frac{\partial \Phi}{\partial z} = 0 \text{ at } z = 0, z = 1, \tag{10}$$

$$q_2(z) \text{ occurs at } t = 0. \tag{11}$$

Equations (4) and (8) may be solved by using standard results for the Green functions for axial and radial cases given by Carslaw and Jaeger [6].

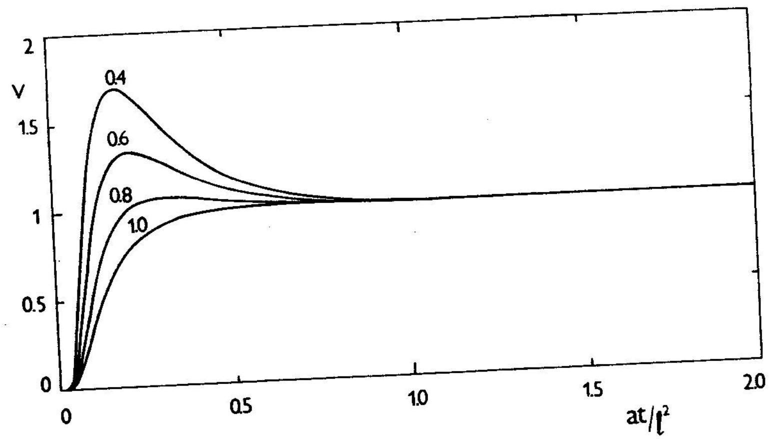

Fig. 2. Non-dimensional temperature in the centre of the rear face of the specimen. The numbers on the curves are values of f ($y = 0.65$).

If b is the radius of the sample and R the radius of a planar, circular-shape source at $t = 0$, $z = 0$ and $0 < R < b$, then the temperature at $t > 0$, $z = l$, $r = 0$ (centre of the rear face) is given by

$$\vartheta(0, l, t) = \frac{QR^2}{\varrho c l b^2}\left[1 + 2\sum_{n=1}^{\infty}(-1)^n \exp(-n^2\pi^2 at/l^2)\right] \times \tag{11}$$

$$\times\left[1 + 2\sum_{i=1}^{\infty}\frac{J_1(\alpha_i R/b)}{(\alpha_i R/b)J_0^2(\alpha_i)}\exp(-\alpha_i^2 at/b^2)\right],$$

where Q is the amount of heat supplied per unit areas, $J_n(x)$ are Bessel functions of the first kind, and α_i are the positive roots of $J_1(\alpha_i) = 0$.

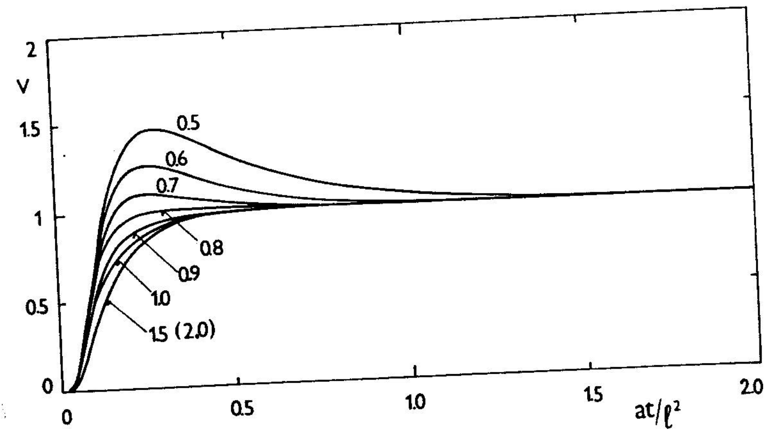

Fig. 3. Non-dimensional temperature V versus at/l^2. The numbers on the curves are values of y ($f = 0.7$).

Dividing both sides of equation (11) by $QR^2/(\varrho clb^2)$, we have for the non-dimensional temperature

$$V=\left|1+2\sum_{n=1}^{\infty}(-1)^n \exp(-n^2\pi^2 at/l^2)\right|\times \qquad (12)$$

$$\times\left[1+2\sum_{i=1}^{\infty}\frac{J_1(\alpha_i R/b)}{(\alpha_i R/b)J_0^2(\alpha_i)}\exp(-\alpha_i^2 at/b^2)\right|.$$

The numerical work is simplified by using the dimensionless parameters $\tau=at/l^2$, $y=l/b$, $f=R/b$. For the non-dimensional temperature we now have

$$V=\left[1+2\sum_{n=1}^{\infty}(-1)^n \exp(-n^2\pi^2\tau)\right]\left[1+2\sum_{i=1}^{\infty}\frac{J_1(\alpha_i f)}{\alpha_i f J_0^2(\alpha_i)}\exp(-\alpha_i^2 y^2\tau)\right] \qquad (13)$$

In Fig. 2 V is plotted against at/l^2 for values 0.4; 0.6; 0.8; 1.0; of f and $y=0.65$. Deviations of the curves for $f<1$ from ideal ($f=1$) are evident. The calculated value of a increases with decreasing f and using Eq. (1) can lead to substantial errors.

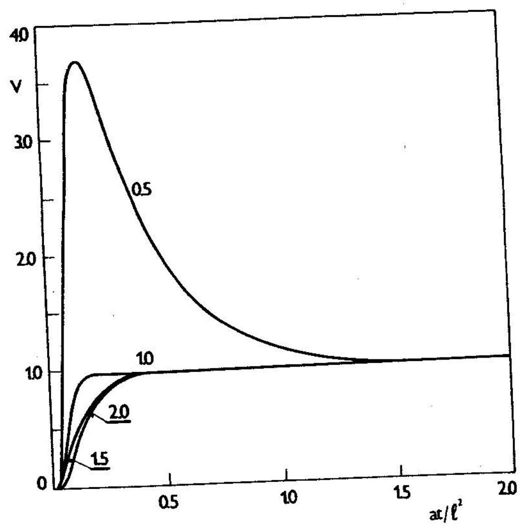

Fig. 4. Non-dimensional temperature of the specimen in the case of $R=0$. The numbers on the curves are the values of y.

In Fig. 3 values of V calculated from (13) for various values of y and $f=0.7$ are plotted against τ. Curve of V for $y=2$, $f=0.7$ is practically identical with the ideal curve for $f=1$ and the effect of non-uniform heating of the sample is negligible. From Eq. (1) the calculated value of a increases with increasing y. If $y<2 (f<1)$, the numerical factor in (1) must be corrected solving (13) for unknown $\tau_{1/2}$ for appropriate f and y in the usual way. (Putting $V=0.5$ and solving Eq. (13) for unknown $\tau_{1/2}$ we have for thermal diffusivity $a=\tau_{1/2}l^2/t_{1/2}$).

In the case of $R=0$ (the point instantaneous source at the centre of the front face of the sample) using a similar procedure for solving (4) and (8), we have

$$V=\left|1+2\sum_{n=1}^{\infty}(-1)^n \exp(-n^2\pi^2\tau)\right|\left|1+\sum_{i=1}^{\infty}\frac{1}{J_0^2(\alpha_i)}\exp(-\alpha_i^2 y^2\tau)\right|. \qquad (14)$$

Equation (14) is plotted in Fig. 4 for various y. The value of error from using (1) depends on the geometry on the sample.

In the limiting case of $R=b$ the radial component is equal to 1 and we have

$$V=\left|1+2\sum_{n=1}^{\infty}(-1)^n \exp(-n^2\pi^2\tau)\right|. \qquad (15)$$

Equation (15) corresponds to a result derived by Parker et al. [1].

Based on numerical analyses of Eqs. (13) and (14) it can be shown that the effect of non-uniform heating of the sample is negligible if $y\geq 2$. This result corresponds to one given by Platunov and Rykov [7]. If $y<2$, one can compute the value of $\tau_{1/2}$ using equations (13) and (14) for appropriate y and f.

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