THE LOW-FREQUENCY LIMIT OF COSMIC RAY FLUCTUATIONS IN THE DIFFUSION APPROXIMATION[1])

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Fluctuations of the cosmic ray intensity in the diffusion approximation are investigated. The power spectral density of the cosmic ray fluctuations is derived for the case when the large-scale inhomogeneities of the magnetic field over the whole modulation region share the formation of the cosmic ray fluctuations. Since the power spectrum of the cosmic ray fluctuations for frequencies above 10^{-5} Hz is determined only by the power spectrum of the stochastic magnetic field, the power spectrum of the fluctuations below 2×10^{-6} Hz is proportional to f^{-2} (f is the frequency).

НИЗКОЧАСТОТНЫЙ ПРЕДЕЛ ФЛУКТУАЦИЙ КОСМИЧЕСКИХ ЛУЧЕЙ В ДИФФУЗИОННОМ ПРИБЛИЖЕНИИ

В работе исследованы флуктуации интенсивности космических лучей в диффузионном приближении. Найдено спектральное распределение плотности энергии флуктуаций для случая, когда образование флуктуаций космических лучей обусловлено наличием крупномасштабного неоднородного магнитного поля во всей модуляционной области. Хотя энергетический спектр флуктуаций космических лучей для частот свыше 10^{-5} Гц был определен только на основе энергетического спектра стохастического магнитного поля, тем не менее энергетический спектр флуктуаций при частотах ниже $2 . 10^{-6}$ пропорционален f^{-2} (f — частота).

I. INTRODUCTION

Apart from a considerable development of a theory concerning problems of the distribution function fluctuations of cosmic ray (CR) particles [1—5], unsolved questions still exist. There is not a sufficiently common theory, which would comprise the whole frequency range of the observed fluctuations from 10^{-8} Hz to 10^{-2} Hz [6—9] and which would describe fluctuations of both the low-energy (tens

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of MeV) and the high-energy (hundreds of GeV) particles. It is necessary to explain and to describe the CR fluctuations in various approximations before constructing uniform theory and after verification by experiment it is possible to approach its generalization. There are developed various approximations [4, 5, 10] which more or less satisfy the observed CR fluctuations.

In the present paper, the diffusion approximation [4, 11] in the kinetic theory of CR fluctuations is investigated for the range of low frequencies, when the CR fluctuations are not immediately formed by small-scale pulsations of the turbulent magnetic field (for frequencies $f<10^{-6}$ Hz). The CR fluctuations with a great wave number are formed by the large-scale structure interplanetary medium as well as by diffusion fluxes ("waves") in the whole modulation region.

II. THE DIFFUSION APPROXIMATION AND THE POWER SPECTRAL DENSITY FOR LOW FREQUENCIES

The kinetic equation for the autocorrelation function of fluctuations in the CR distribution function, which was determined in many papers (see references in the Introduction) is

$$g(1,2)=F(1)F(2)+\int_0^{t_1} d1' \int_0^{t_2} d2' G(1,1')G(2,2')L_\alpha(1') B_{\alpha\lambda}(1',2')L_\lambda(2')g(1',2'), \quad (1)$$

where "1" denotes the set of variables $\{\boldsymbol{r}_1, \boldsymbol{p}_1, t_1\}$ in the phase space, $\boldsymbol{v}=\frac{1}{m}\boldsymbol{p}$ is the particle velocity, $\boldsymbol{L}=e/c[\boldsymbol{v}-\boldsymbol{u}, \partial/\partial\boldsymbol{p}]$ is the angle operator in the momentum space including the motion of magnetized cosmic plasma with velocity $\boldsymbol{u}$, $B_{\alpha\lambda}=\langle H_\alpha H_\lambda\rangle$ is the second-rank correlation tensor of the magnetic field, G is Green's function of the equation for the mean distribution function [1, 2, 4, 5].

Let us consider the first-order iteration, and change $g(1',2')\to F(1')F(2')$ on the left-hand side of equation (1). Let $d\Omega_p$ be the space angle element in the space of the momentum $\boldsymbol{p}$. Then after applying the method of the diffusion approximation [4] one obtains for the autocorrelation function in this approximation

$$N_{p_1p_2}(1,2)=\int d\Omega_{p_1}\, d\Omega_{p_2} g_{p_1p_2}(1,2) \quad (2)$$

the expression

$$N_{p_1p_2}(1,2)=\int_p^\infty dq_1 q_1^2 \int_p^\infty dq_2 q_2^2 \int_0^{t_1} d\tau_1\, d\varrho_1 \int_0^{t_2} d\tau_2\, d\varrho_2\; G_{p_1q_1}(\boldsymbol{r}_1, t_1; \varrho_1, \tau_1) G_{p_2q_2}(\boldsymbol{r}_2, t_2; \varrho_2, \tau_2) \Phi(\boldsymbol{q}_1, \varrho_1, \tau_1; \boldsymbol{q}_2, \varrho_2, \tau_2). \quad (3)$$

In the lowest order relatively to the small parameter (u/v) there holds

$$\Phi(1',2')=\langle n'(1')n(2')\rangle+\langle n''(1')n''(2')\rangle, \quad (4)$$

where

$$n'=\frac{3e}{cvp}\varkappa_{\alpha\lambda}\varepsilon_{\lambda\beta\gamma}\frac{\partial H_\beta}{\partial r_\alpha}\varkappa_{\gamma\nu}\frac{\partial}{\partial r_\nu}n, \quad (5)$$

$$n''=\frac{e}{cvp}H_\alpha\varepsilon_{\alpha\beta\gamma}\mu_\beta\varkappa_{\gamma\nu}\frac{\partial}{\partial r_\nu}n, \quad (6)$$

$\varkappa_{\alpha\lambda}$ is the diffusion tensor and $n(\boldsymbol{r})$ is the CR particle density. One must realize relating to the source function Φ that the tensor $\langle H_\alpha H_\beta\rangle$ includes the contribution of the large-scale as well as the small-scale variations of the magnetic field, because the non-constant component of the magnetic field consists of two components:

$$\boldsymbol{H}=\boldsymbol{h}+\langle\boldsymbol{H}\rangle_{L_c},\ \langle\boldsymbol{h}\rangle_{L_c}=0, \quad (7)$$

where $\langle\boldsymbol{H}\rangle_{L_c}$ is the component, which has not the small-scale pulsations of the magnetic field with the characteristic size of inhomogeneities L_c and which is approximately constant at L_c. The mean value in the total ensemble of all realizations of the magnetic field in the whole modulation region during a long time is $\langle\boldsymbol{H}\rangle_{L_m}=\langle\langle\boldsymbol{H}\rangle_{L_c}\rangle_{L_m}=0$, where L_m is comparable with the size of the modulation region. By putting (7) into expression (4) one can find that the first term in (4) is negligible for the distances $r\lesssim L_c$, because $\langle n'^2\rangle/\langle n''^2\rangle\sim(u/v)^2(L_c/\Lambda)^2\lesssim1$. Λ denotes the mean transport path of particles in the irregular magnetic field. However, if $L_c\ll r\lesssim L_m$ holds, then $\langle n'^2\rangle/\langle n''^2\rangle\sim(u/v)^2(L_m/\Lambda)^2\gtrsim1$, and the second term is a negligible one.

From the observation of the interplanetary magnetic field it follows that the pulsations perpendicular to the regular (constant) magnetic field give the greatest contribution to the correlation tensor, since the parallel pulsations are ten-times smaller. The crosscorrelations between the different components are nearly zero [12]. Let us suppose in the following that the CR density gradient takes the radial direction, and the CR fluctuations are caused only by the diffusion flux $\boldsymbol{D}$, whose components are

$$\begin{aligned} D_x&=-\varkappa_A \sin\Psi\ \mathrm{grad}_r n,\\ D_y&=\varkappa_\perp \sin\Psi\ \mathrm{grad}_r n,\\ D_z&=-\varkappa_{//}\cos\Psi\ \mathrm{grad}_r n, \end{aligned} \quad (8)$$

where $\varkappa_{//}$ and $\varkappa_\perp$ are the diffusion coefficients parallel and perpendicular to the regular magnetic field, respectively; $\varkappa_A$ is the asymmetric part of the diffusion tensor. (The z-axis is along the regular magnetic field, the x-axis is perpendicular to the plane of the helioequator.)

After the Fourrier transformation of all coordinates and times in relations (3) and (4) one obtains in the case of the homogeneous and stationary distribution of CR fluctuations that

$$N_p(\boldsymbol{k},\omega)=\int_p^\infty \mathrm{d}q_1 q_1^2 \int_p^\infty \mathrm{d}q_2 q_2^2 G_{pq_1}(\boldsymbol{k},\omega)\, G_{pq_2}(-\boldsymbol{k},\omega)\,\Phi(\boldsymbol{k},\omega), \tag{9}$$

where

$$\Phi(\boldsymbol{k},\omega)=\left(\frac{3e}{cvp}\right)^2 D_\gamma D_\nu B_{\alpha\beta}(\boldsymbol{k},\omega)\,\varepsilon_{\lambda\beta\gamma}\varepsilon_{\mu\alpha\nu}\times \times\left\{k_\sigma k_\varepsilon \varkappa_{\sigma\lambda}\varkappa_{\varepsilon\mu}-\left(\frac{4}{3}\right)^2 u_\sigma u_\varepsilon \delta_{\sigma\lambda}\delta_{\varepsilon\mu}\right\}. \tag{10}$$

In the following we can neglect Green's function dependence on the energy and we use Green's function of the transport equation for CR particles with the constant solar wind velocity [4]:

$$G_{p_1p_2}(\boldsymbol{k},\omega)=p_1^{-2}\delta(p_1-p_2)[\varkappa_{\alpha\beta}k_\alpha k_\beta-\mathrm{i}(\omega-\boldsymbol{uk})]^{-1}. \tag{11}$$

Since the solar wind velocity is supersonic, we neglect the $B_{\alpha\beta}$ dependence on the frequency and we shall investigate the steady fluctuations in the modulation region. From relations (9)—(11) we obtain a cumbersome expression in which we keep only the term $B_{xx}\approx B_{yy}$. We take the result in two extreme cases (concerning the direction of the wave vector $\boldsymbol{k}$). If $k_{\cdot}\to 0$, then

$$N(\boldsymbol{k})=\left(\frac{3e}{cvp}\right)^2 B_{xx}(\nabla_r n)^2\cos^2\Psi\,\frac{1}{k^2}\,\varkappa_{\cdot}^2\left(1+\frac{\varkappa_A^2}{\varkappa_\perp^2}\right)\left(1+\frac{k_1^2}{k^2}\right), \tag{12}$$

where

$$k_1^2=\left(\frac{4}{3}\right)^2 u^2\sin^2\Psi\,\frac{(\varkappa_{//}-\varkappa_\perp)^2+\varkappa_A^2}{\varkappa_{//}^2(\varkappa_A^2+\varkappa_\perp^2)},$$

and Ψ is the angle between the solar wind velocity and the regular magnetic field direction.

In the case of $k_\perp\to 0$

$$N(\boldsymbol{k})=\left(\frac{3e}{cvp}\right)^2 B_{xx}(\nabla_r n)^2\sin^2\Psi\,\frac{1}{k^2}(\varkappa_\perp^2+\varkappa_A^2)\left(1+\frac{k_2^2}{k^2}\right), \tag{13}$$

where

$$k_2^2=k_1^2\operatorname{ctg}^2\Psi.$$

The power spectrum of the fluctuations is defined by the relation $N(f)=\int \mathrm{d}\boldsymbol{k}\delta(2\pi f-\boldsymbol{uk})N(\boldsymbol{k})$. Then the power spectrum of the CR fluctuations perpendicular to the regular magnetic field is

$$N(f)=\frac{4\pi\cos^2\Psi}{u|\sin\Psi|}\left(\frac{3e}{cvp}\right)^2 B_{xx}(\nabla_r n)^2\varkappa_{\cdot}^2\left(1+\frac{\varkappa_A^2}{\varkappa_\perp^2}\right)\left(1+\frac{f_1^2}{f^2}\right), \tag{14}$$

where

$$f_1=\frac{2|\sin\Psi|}{3\pi}u^2\frac{(\varkappa_{\cdot}-\varkappa_\perp)^2+\varkappa_A^2}{\varkappa_{\cdot}^2(\varkappa_\perp^2+\varkappa_A^2)},$$

and the power spectrum of the parallel CR fluctuations is

$$N(f)=\frac{4\pi|\sin\Psi|}{u}\left(\frac{3e}{cvp}\right)^2 B_{xx}(\nabla_r n)^2(\varkappa_\perp^2+\varkappa_A^2)\left(1+\frac{f_2^2}{f^2}\right), \tag{15}$$

where $f_2^2=f_1^2\operatorname{ctg}^2\Psi$.

III. CONCLUSIONS

It follows from the relations (12)—(15) that the decrease $\sim f^{-2}$ of the power spectrum of the CR fluctuations for $f\lesssim f_1$ can be explained by the enough simple diffusion approximation in the kinetic theory of the CR fluctuations. Let the inequalities $\varkappa_\perp\ll\varkappa_A\ll\varkappa_{//}$ hold. Then from (14) it follows that

$$N(f)\sim B_{xx}(f)\left(1+\frac{f_1^2}{f^2}\right), \tag{16}$$

where $f_1\doteq\frac{2}{3\pi}|\sin\Psi|\frac{u^2}{\varkappa_A}$.

If $u=4\times10^7\,\mathrm{cm\,s^{-1}}$, $\varkappa_{\cdot}=10^{21}\,\mathrm{cm^2s^{-1}}$ and $\varkappa_A/\varkappa_{\cdot}=0.15$, then $f_1\doteq2\times10^{-6}$ Hz. Since $B(f)$ weakly depends on the frequency for $f<f_0=10^{-5}$ Hz, $N(f)\sim f^{-2}$ for $f<f_1$, as $N(f)\sim B(f)$ for $f>f_0$. We can remark that these relations satisfy very well the observed power spectra (see the following Paper in this Symposium).

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