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NOTE ON THE EXISTENCE OF CONTINUOUS PHASE TRANSITIONS IN AN ANISOTROPIC FERROMAGNET WITH FIELD

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An anisotropic ferromagnet in an external magnetic field is considered. By renormalization group arguments necessary conditions for the existence of the continuous phase transitions are found. A general form of the interaction with the external field and a general form of anisotropy are taken into account.

О СУЩЕСТВОВАНИИ НЕПРЕРЫВНЫХ ФАЗОВЫХ ПЕРЕХОДОВ В АНИЗОТРОПНЫХ ФЕРРОМАГНЕТИКАХ, НАХОДЯЩИХСЯ ВО ВНЕШНЕМ МАГНИТНОМ ПОЛЕ

В работе рассмотрено поведение анизотропного ферромагнетика во внешнем магнитном поле. На основе использования группы ренормализации найдены необходимые условия существования непрерывных фазовых переходов в этом ферромагнетике. При этом учитываются общая форма взаимодействия ферромагнетика с внешним полем и общая форма анизотропии.

I. INTRODUCTION

A critical behaviour of anisotropic ferromagnets in the presence of a uniform, external magnetic field has been studied within the renormalization group theory by many authors (see e.g. [1—3]). The interaction with the field was given however, by the Zeeman term. There are systems in which the another form of interaction with the external magnetic field must be taken into account. It was shown in paper [4] that in some substances a more general type, the so-called Bleaney—Koster—Statz (BKS) term should be included in an effective spin Hamiltonian. The BKS term of interaction with the external field has the form

$$H_{BKS} = g\beta(\mathbf{B}\,.\,\mathbf{S}) + f\beta[B^x(S^x)^3 + B^y(S^y)^3 + B^z(S^z)^3] - \\ - 1/5\mathbf{B}\,.\,\mathbf{S}[3S(S+1)-1]; \tag{1.1}$$

for details see paper [5].

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Let us rewrite (1.1) in the form

$$H_{BKS} = -\sum_{i=1}^{3} h_i s_0^i - \sum_{i=1}^{3} h_i \int_q \int_{q_1} s_q^i s_{q_1}^i s^i_{-q-q_1} \tag{1.2}$$

where s_q^i — is a three-component classical spin of the wave vector q, h_i denotes the i-th component of the effective external field, the integrals in (1.2) are taken over $|q| \leq 1$.

In paper [6] there is considered an anisotropic ferromagnet described by the Landau—Ginzburg—Wilson (LGW) Hamiltonian of the form

$$H_{LGW} = -1/2 \sum_{i=1}^{3} \int_q (r_0^i + q^2) s_q^i s^i_{-q} - \tag{1.3}$$

$$- \sum_{i,j=1}^{3} u_{ij}^0 \int_q \int_{q_1} \int_{q_2} s_q^i s_{q_1}^i s_{q_2}^j s^j_{-q-q_1-q_2},$$

the thermal variables r_i^0 are given by

$$r_i^0 = a_i (T - \Theta_i). \tag{1.4}$$

For a spin system in which the interaction with the external field is given by the BKS Hamiltonian (1.2) an additional term to H_{LGW} (1.3), the so-called dipole-octupole term can appear. Such form of interaction between spin components has been introduced for description of ferromagnetic compounds [7], metamagnetic substances [8], paramagnetic ions [9]. The dipole-octupole term can be written as

$$H_{d-o} = \sum_{ij=1}^{3} v_{ij}^0 \int_q \int_{q_1} \int_{q_2} s_q^i s_{q_1}^j s_{q_2}^j s^j_{-q-q_1-q_2}. \tag{1.5}$$

In the following we assume the Hamiltonian of the system in the form

$$H = H_{LGW} + H_{BKS} + H_{d-o}. \tag{1.6}$$

By an appropriate choice of the parameters r_i^0, u_{ij}^0, v_{ij}^0 in (1.6) one can establish different forms of anisotropy of ferromagnetic systems.

In part II we will derive with the help of the renormalization group theory conditions for the existence of continuous phase transitions in systems described by Hamiltonians like (1.6).

We will generalize the method of Ritter and Sznajd [6] on our model, this will enable us to show analogies with models known in literature.

II. DISCUSSION

In the following we show that the Hamiltonian (1.6) can be transformed into a form which has been considered in literature. Having that, we establish by

analogy the conditions for the existence of continuous phase transitions. Primarily we eliminate from (1.6) terms which are linear in spin variables. This can be done by the insertion of new spin variables into (1.6) defined by the relation

$$s_q^i \rightarrow s_q^i + M_i \delta_{q,0}. \tag{2.1}$$

The linear term in (1.6) vanish if M_i are taken as a root of the equation

$$h_i(1 + 3M_i^2) = M_i \left| r_i^0 + 4 \sum_j u_{ij}^0 M_j^2 + 3 \sum_j v_{ij}^0 M_j^2 \right| + \sum_j v_{ij}^0 M_j^3. \tag{2.2}$$

After this transformation the Hamiltonian (1.6) takes the form

$$H = -1/2 \sum_{i,j=1}^{3} \int_q (r_{ij} + q^2 \delta_{ij}) s_q^i s^j_{-q} - \sum_{i,j=1}^{3} w_{ij} \int_q \int_{q_1} s_q^i s_{q_1}^j s^j_{-q-q_1}$$

$$- \sum_{i,j=1}^{3} \int_q \int_{q_1} \int_{q_2} u_{ij} s_q^i s_{q_1}^i s_{q_2}^j s^j_{-q-q_1-q_2} \tag{2.3}$$

$$- \sum_{i,j=1}^{3} v_{ij} \int_q \int_{q_1} \int_{q_2} s_q^i s_{q_1}^j s_{q_2}^j s^j_{-q-q_1-q_2}$$

where the coefficients r_{ij} and w_{ij} are given by

$$r_{ij} = r_{ij}^0 \delta_{ij} + 8 M_i M_j u_{ij} + 4 \delta_{ij} \sum_k M_k^2 u_{ik} \tag{2.4}$$

$$- 3 v_{ij}^0 (M_i^2 + M_j^2) + 3 \gamma h_i (M_i + M_j) \delta_{ij} + 6 \delta_{ij} \sum_k M_k M_j v_{kj}^0$$

and

$$w_{ij} = (\gamma h_i + v_{ij}^0 M_i) \delta_{ij} + 4 M_j u_{ij} + 3 M_j v_{ij}^0. \tag{2.4}$$

The M_i have a meaning of magnetization of the paramagnetic phase when it arises from the interaction with the external field.

In the presence of the external magnetic field phase transition occurs for systems for which an idependent on the field ordering mechanism exists. The direction along the magnetization $\boldsymbol{M}$ cannot be critical since magnetization in the presence of the external magnetic field is non zero at any temperature. Hence only two directions perpendicular to $\boldsymbol{M}$ can be critical.

Let us make a transformation into a new set of spin variables, in which one component is directed along the magnetization. After transformation

$$s_q^i = \sum_j T_j^i S_q^j, \tag{2.5}$$

the Hamiltonian (2.3) takes the form

$$H = -1/2 \sum_{i,j=1}^{3} \int_q (\tilde{r}_{ij} + q^2\delta_{ij}) S^i_q S^j_{-q} - \sum_{i,j=1}^{3} \tilde{w}_{ij} \int_q \int_{q_1} S^i_q S^j_{q_1} S^j_{-q-q_1} - \quad (2.6)$$

$$-2w \int_q \int_{q_1} S^1_q S^2_{q_1} S^3_{-q-q_1} - \sum_{i,j=1}^{3} \int_q \int_{q_1} \int_{q_2} \tilde{u}_{ij} S^i_q S^i_{q_1} S^j_{q_2} S^j_{-q-q_1-q_2} -$$

$$-4 \int_q \int_{q_1} \int_{q_2} \sum_{i,j=1}^{3} \tilde{v}_{ij} S^i_q S^i_{q_1} S^i_{q_2} S^j_{-q-q_1-q_2} - \sum_{m\neq j\neq k} m_{ijk} \int_q \int_{q_1} \int_{q_2} S_q S_{q_1} S_{q_2} S_{-q-q_1-q_2}.$$

The expressions for $\tilde{r}_{ij}$, w, $\tilde{w}_{ij}$, $\tilde{u}_{ij}$, $\tilde{v}_{ij}$, m_{ijk} are given in the appendix.

The Hamiltonian (2.6) has the form as that given by eqs. (10) of [6], with the coefficients $\tilde{r}_{ij}$, w, $\tilde{w}_{ij}$, $\tilde{v}_{ij}$, $\tilde{u}_{ij}$ and M_i of Ritter and Sznajd [6] replaced by eqs. (2.2), (2.4) and the formulae in the appendix. Following further the analysis of Ritter and Sznajd we can immediately write down the conditions for the existence of continuous phase transitions in the considered system. There are two types of them: for the existence of the $X-Y$ like transition (two critical directions) and for the Ising-like one (one critical direction). The critical exponents are equal to the classical $X-Y$ and the Ising critical exponents, respectively.

The conditions for the $X-Y$ like critical behaviour are

$$\tilde{r}_{33} = \tilde{r}_{22} < \tilde{r}_{11}, \quad \tilde{r}_{ij} = 0 \text{ for } i \neq j \quad (2.7)$$

$$\tilde{w}_{ij} = 0 \text{ for } i, j = 2, 3$$

$$\tilde{v}_{ij} - w\tilde{w}_{ij}/(2\tilde{r}_{11}) = 0 \text{ for } i \neq j;\ i, j = 2, 3$$

$$0 < [\tilde{u}_{23} - (2w^2 - \tilde{w}_{12}\tilde{w}_{13})/(2r_{11})] < \Lambda$$

with

$$\Lambda < (3\bar{\tilde{u}}_{22}\bar{\tilde{u}}_{33}), \quad \bar{\tilde{u}}_{ii} = \tilde{u}_{ii} - \tilde{w}^3_{1i}/(2\tilde{r}_{11}) \quad (2.8)$$

the $\tilde{r}_{11}$ is associated with the noncritical direction along M.

$$\tilde{r}_{33} < r^{(1)}_{22},\ r^{(1)}_{11},\ r^{(1)}_{ij} = 0 \text{ for } i \neq j \quad (2.9)$$

$$\tilde{w}_{33} = 0, \quad \tilde{u}_{33} = \sum_{i=1}^{2} [w^{(1)}_{i3}]^2/(2r^{(1)}_{ii}) < 0;$$

the superscript "1" denotes the coefficients $\tilde{r}_{ij}$, $\tilde{w}_{ij}$ after rotation around the critical direction "3".

Conditions (2.7) and (2.9) form restrictions on the Hamiltonian coefficients and fields, which must be satisfied if continuous phase transitions are to occur. When none of these conditions is fulfilled the system undergoes the first order phase transition. The Hamiltonian (1.6) is represented in its most general form of interaction between classical spins in the presence of an external uniform field of an arbitrary direction. The qualitative picture of a critical behaviour in such a system remains in comparison with the simpler model of Ritter and Sznajd [6], but the parameters of BKS and the dipole-octupole interaction modify the necessary conditions (2.7) and (2.9) for the existence of continuous phase transitions.

APPENDIX

$$r_{ij} = \sum_{kl} r_{kl} T^k_i T^l_j;$$

$$w = \sum_{ij} w_{ij} T^1_i T^2_j T^3_j;$$

$$w_{ij} = \sum_{kl} w_{kl} [T^i_k T^i_l T^j_l + 2T^i_k T^i_l T^j_l],$$

$$u_{ij} = \sum_{kl} [u_{kl}(T^i_k T^i_k T^j_l T^j_l + 2T^i_k T^i_k T^j_l T^j_l) + 3v_{kl} T^i_k T^i_k T^j_l T^j_l];$$

$$v_{ij} = \sum_{kl} [4u_{kl} T^i_k T^i_k T^i_l T^j_l + (v_{kl}(T^i_k T^i_l T^i_l T^j_l + 3T^i_l T^i_k T^i_l T^j_l))];$$

$$m_{hij} = -u_{hi} - u_{hj} + 2\sum_{kl} u_{kl} [T^h_k T^h_k T^i_l T^j_l + 2T^h_k T^h_l T^i_k T^j_l] +$$

$$+ 3\sum_{kl} v_{kl} [T^h_k T^h_l T^i_l T^j_l + T^h_k T^h_k T^i_l T^j_k].$$

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