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MOBILITY OF ELECTRONS LIMITED BY SCATTERING ON AN IMPURITY-ION POTENTIAL WITH A SPATIALLY VARIABLE DIELECTRIC FUNCTION

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The model potential $V(r)=(e/4\pi\epsilon_0\epsilon_s r)\,[1+(\epsilon_s-1)\exp(-\beta r)]$ for charged centres examined in [1] is used for the evaluation of electron mobility. The temperature dependence of electron mobility for various values of the parameter β, related to the ionization energy of the centre, is found. At a given concentration of centres the mobility decreases with increasing ionization energy (decreasing β) and its temperature dependence becomes less expressive.

ПОДВИЖНОСТЬ ЭЛЕКТРОНОВ, ОГРАНИЧЕННАЯ РАССЕЯНИЕМ НА ИОННЫХ ПРИМЕСЯХ, ОПИСЫВАЕМЫХ ПОТЕНЦИАЛОМ С ПРОСТРАНСТВЕННОПЕРЕМЕННОЙ ДИЭЛЕКТРИЧЕСКОЙ ФУНКЦИЕЙ

В работе для вичисления подвижности электронов использован для заряженных центров модельный потенциал $V(r)=(e/4\pi\cdot\epsilon_0\cdot\epsilon_s\cdot r)\,[1+(\epsilon_s-1)\exp(-\beta r)]$, который был рассмотрен в работе [1]. Определена температурная зависимость подвижности электронов для различных значений параметра β, который характеризует энергию ионизации центра. При данной концентрации центров подвижность электронов с повышением энергии ионизации (падение β) падает и их температурная зависимость становится меньше ожидаемой.

I. INTRODUCTION

The problem of electron scattering by ionized impurities in semiconductors, recently reviewed by Chattopadhyay and Queisser [2], is still topical from various viewpoints. The most elaborated are the theories of scattering on shallow impurities with Coulombic or screened Coulombic (Yukawa) potentials. However,

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such potentials are not adequate for deeper electron trapping centres. In paper [1] the potential

$$V(r)=\frac{e}{4\pi\epsilon_0\epsilon_s r}[1+(\epsilon_s-1)\exp(-\beta r)] \tag{1}$$

was used for charged centres, which leads to larger ionization energies than those of hydrogen-like donor (when $\beta\to\infty$). In [1] the dependence of the ionization energy of centres with potential (1) on the parameter βa_1 was found ($a_1=4\pi\epsilon_0\epsilon_s\hbar^2/m^*e^2$ is the Bohr radius of a hydrogen-like centre in the crystal with static relative permittivity ϵ_s and electron effective mass m^*). In the numerical calculation the values of ϵ_s and m^* corresponding to GaAs were used ($\epsilon_s=13$, $m^*=0.067m_0$). The dependence of the ionization energy on βa_1 calculated in [1] is shown in Fig. 1.

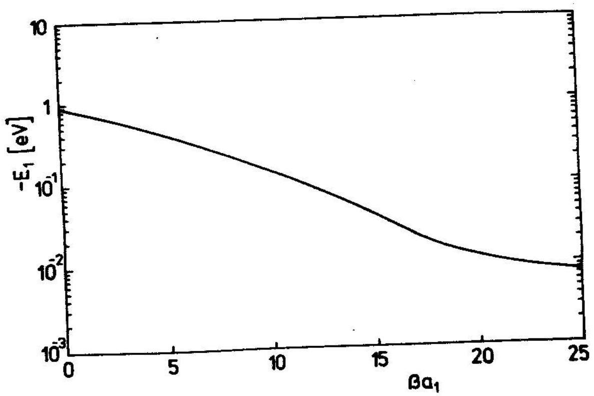

Fig. 1. The ionization energy — E_1 of a centre with potential (1) as a function of the parameter βa_1 calculated in [1] with $\epsilon_s=13$, $m^*=0.067m_0$.

Furthermore, in [1] also the photoionization cross-section and ionization energy lowering due to the applied electric field were calculated for various values of the parameter βa_1 for centres with potential (1). This potential is derived from the potential introduced by Csavinszky [3] providing that the screening by carriers and ionized impurities themselves can be neglected. It can be interpreted as a consequence of a spatially variable dielectric function.

The scattering of electrons by an impurity-ion potential characterized by a spatially variable dielectric function was investigated by various authors [4, 5, 6]; however, the corresponding increase of the binding energy of the impurity has not been found.

The aim of this paper is to complete the results obtained in [1] by the formula for electron mobility corresponding to the scattering on centres with potential (1) and,

consequently, fo find how the deeper impurities or electron traps can influence the electronic transport in semiconductors.

II. THE ELECTRON MOBILITY

The formula for drift electron mobility will be derived analogically to the case of scattering on Coulombic (Conwell—Weisskopf) or Yukawa (Brooks—Herring) potentials, respectively, because the potential (1) is a superposition of these. The method of derivation is desribed, e. g., in [2].

Let us express the relaxation time for the scattering as

$$\frac{1}{\tau}=\frac{\Omega}{4\pi^2}\int_{k'}\int_{z=-1}^{1}(1-z)P(k,k')k'^2\,dk'\,dz, \tag{2}$$

where Ω is the crystal volume and

$$P(k,k')=\frac{2\pi}{\hbar}\frac{Ne^2}{\Omega}|V(q)|^2\delta(E'-E) \tag{3}$$

denotes the probability of transition of an electron from the state k to the state k' per unit time at an elastic scattering on the centre with potential $V(r)$ and

$$V(q)=\int V(r)\exp(-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}, \tag{4}$$

N is the concentration of the scattering centres, $\boldsymbol{q}=\boldsymbol{k}'-\boldsymbol{k}$, $z=\cos\vartheta$, $\boldsymbol{k}\cdot\boldsymbol{k}'=kk'\cos\vartheta$, $E=\hbar^2k^2/2m^*$, $\delta(E'-E)$ is the Dirac delta function which indicates the energy conservation law at scattering, thus yielding

$$q^2=2k'^2(1-z). \tag{5}$$

Substituting (1) into (4) we have

$$|V(q)|=\frac{e}{\epsilon_0\epsilon_s q^2}\left[1+(\epsilon_s-1)\frac{q^2}{q^2+\beta^2}\right] \tag{6}$$

and the relation for the relaxation time can be written as

$$\frac{1}{\tau}=\frac{Ne^4E^{-3/2}}{16\pi\sqrt{2}\,\epsilon_0^2\epsilon_s^2m^{*1/2}}\int_{-1}^{1}\frac{\mathrm{d}z}{1-z}\left[1+(\epsilon_s-1)\frac{1-z}{1-z+\frac{\hbar^2\beta^2}{4m^*E}}\right]. \tag{7}$$

Applying the same approximation for resolving the divergence of the integral $\int_{-1}^{1}\mathrm{d}z/(1-z)$ as Conwell and Weisskopf did (c. f. [2]) we obtain

$$\frac{1}{\tau}=\frac{Ne^4E^{-3/2}}{16\sqrt{2m^*}\pi\epsilon_0^2\epsilon_s^2}\left[\ln\left(1+\left(\frac{4\pi\epsilon_0\epsilon_sE}{e^2N^{1/3}}\right)^2\right)+\right. \tag{8}$$

$$\left.+(\epsilon_s^2-1)\ln(1+b)-(\epsilon_s-1)^2\frac{b}{1+b}\right],$$

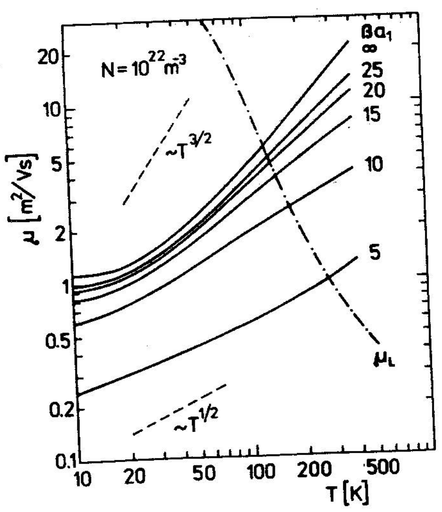

Fig. 2. Electron drift mobility as a function of temperature calculated according to the formula (9) with $\epsilon_s = 13$, $m^* = 0.067 m_0$, $N = 10^{22}\ \mathrm{m^{-3}}$, for various values of the parameter βa_1.

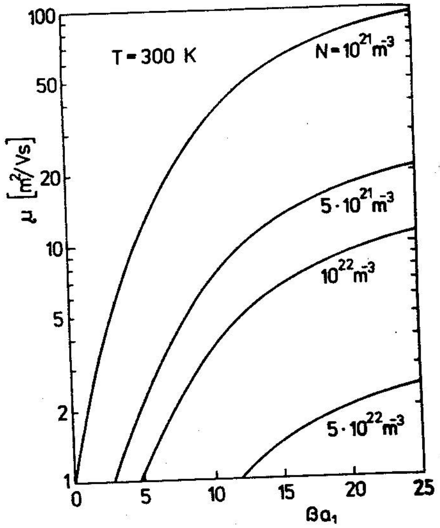

Fig. 3. Electron drift mobility at 300 K as a function of the parameter βa_1, calculated according to the formula (9) with $\epsilon_s = 13$, $m^* = 0.067 m_0$, for various values of concentration N of the centres.

where $b = \dfrac{8m^*E}{\hbar^2\beta^2} = 4\dfrac{E}{E_0}\dfrac{1}{(\beta a_1)^2}$, $E_0 = \hbar^2/(2m^* a_1^2)$ being the ionization energy of a shallow hydrogen-like centre.

Using Eq. (8) and assuming a parabolic dispersion law and the Maxwell—Boltzmann distribution function, the electron drift mobility can be expressed as

$$\mu = AT^{3/2}\int_0^\infty \frac{x^3\,e^{-x}\,dx}{\ln\left(1+\dfrac{T^2}{\Theta_1^2}x^2\right)+(\epsilon_s^2-1)\ln\left(1+\dfrac{T}{\Theta_2}x\right)-(\epsilon_s-1)^2\,\dfrac{\dfrac{T}{\Theta_2}x}{1+\dfrac{T_2}{\Theta}x}}\,, \qquad (9)$$

where $A = \dfrac{64\sqrt{2\pi}\epsilon_0^2\epsilon_s^2 k_B^{3/2}}{3m^{*1/2}e^3N}$, $x = E/k_BT$, $\Theta_1 = e^2N^{1/3}/4\pi$, $\Theta_2 = (\beta a_1)^2 E_0/4k_B$. For $\Theta_2 \to \infty$ the formula (9) is identical with the formula of Conwell and Weisskopf.

III. NUMERICAL RESULTS AND DISCUSSION

The numerical calculation of mobility from Eq. (9) was performed using $\epsilon_s = 13$, $m^* = 0.067 m_0$, and various values of βa_1, concentrations of centres N, and temperatures T.

Fig. 2 shows that at a given concentration N of the scattering centres with potential (1) the mobility decreases with the decreasing value of the parameter βa_1 (increasing ionization energy of centres). The dependence of mobility on temperature also varies with different βa_1 values. Within the temperature range 200 K to 400 K it is $\mu \sim T^n$, where n has values between 0.5 ($\beta a_1 \simeq 10$) and 1.2 ($\beta a_1 \to \infty$).

Further, Fig. 2 exhibits also the calculated electron drift mobility of pure (intrinsic) GaAs (lattice limited mobility) μ_L according to Rode and Knight [7]. It is evident that the deeper centres (the lower βa_1 value) will shift the maximum of the total electron drift mobility to higher temperatures as compared with the shallow impurities with the same concentration.

In Fig. 3 we illustrate the electron mobility limited by scattering on centres with potential (1) as a function of βa_1 at 300 K and various values of the concentration of centres.

Although we have not yet experimentally verified the conclusions which follow from the use of the model potential (1) for charged centres in semiconductors, we consider the search for adequate potentials for non-hydrogen-like centres to be an actual problem especially in semiinsulating materials where such centres are very important. Without the knowledge of the potentials of non-shallow centres the characterization of semiinsulating materials remains questionable. Therefore, the analysis of consequences of various reasonable potentials would be very useful.

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