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# PROPAGATION OF A ONEDIMENSIONAL WAVE IN A NONLINEAR SELECTIVELY AMPLIFYING MEDIUM[1]

## РАСПРОСТРАНЕНИЕ ОДНОРАЗМЕРНОЙ ВОЛНЫ В СРЕДЕ С НЕЛИНЕЙНО-ИЗБИРАТЕЛЬНЫМ УСИЛЕНИЕМ

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Transition from regular to chaotic motion of waves in an amplifying medium can be explained by a spontaneous subharmonic modulation of the waves. Instability leading to this modulation in a discharge plasma is discussed and its use for plasma diagnostics is mentioned.

Processes in plasma can often be investigated by observing the nonlinear changes in waves propagating through the plasma. Higher harmonics generation is the best known influence of nonlinearities, while generation of subharmonic frequencies, observed in some cases (e.g. in laser beams entering a plasma target [1]), has been less investigated and is considered usually to be a consequence of forced excitation of some lower resonant plasma frequency.

In the presented contribution, an example is shown where the subharmonic modulation is not due to a resonance, but is connected with nonlinear properties of a strongly nonequilibrium plasma. The modulation should offer, in principle, new information about the dynamics of processes maintaining the nonequilibrium plasma state.

An optical wave (laser beam) used most often for plasma diagnostics, has too high frequency to allow -at least for the present time- experimental tracing of the nonlinear wave shapes and of the motion of

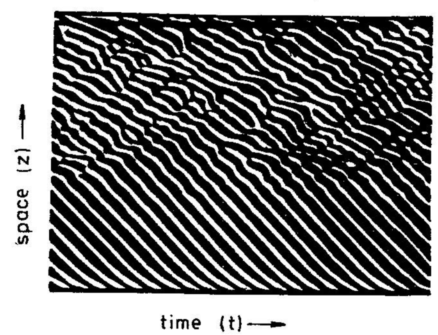


Fig. 1. Space-time diagram of waves moving in a discharge plasma. Copy of a rotating drum photography.

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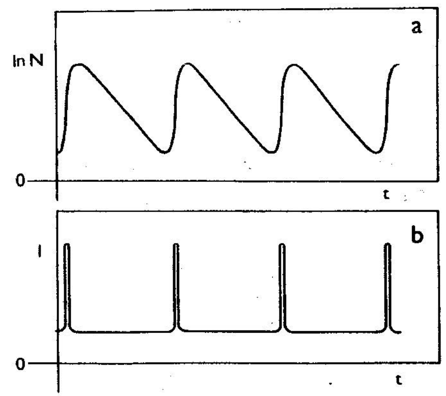


Fig. 2. Numerically calculated time-dependence of a) the logarithm of ion (and electron) density changes, and b) ionization rate.

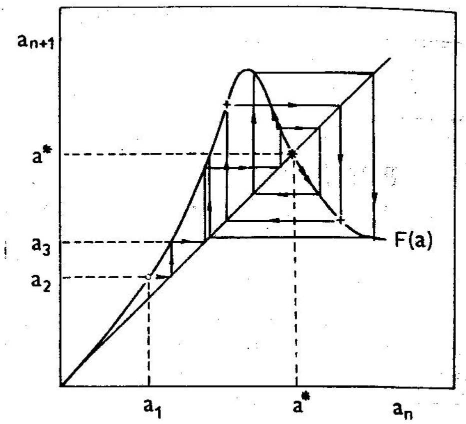


Fig. 3. Calculated transfer function determining the $a_{n+1}$ amplitude, if the $a_n$ amplitude is known. Point of unstable equilibrium is marked by an asterisk, the two points representing the new subharmonic state are marked by crests. An example of determining for an arbitrary initial amplitude $a_1$ the subsequent amplitudes $a_2$, $a_3$, etc. is given.

individual wave crests. With slow waves, e.g. with ionization waves, it is easily possible to observe the wave motion in detail: in Fig. 1 a space-time display of such a wave, propagating and amplified in a discharge plasma, is shown. The subharmonic spontaneous modulation and subsequent onset of turbulent wave is distinctly seen in the picture directly in the motion of individual bright regions representing the crests of the wave.

For a theoretical description of the plasma we have chosen the system of differential equations published by Grabec (see [2]) which, in spite of some simplifications, gives correct dispersion and amplification curves for small amplitude waves:

$$\frac{\partial N}{\partial t}=\frac{\partial^2 N}{\partial x^2}-N+NI$$

$$I=\exp\,[B(1-1/W)]$$

$$1=NF+\frac{2}{3}W\frac{\partial N}{\partial x}$$

$$\frac{\partial W}{\partial x}=-F+C_1NW+C_2NI.$$

All four variables, $N$ — ion density, $I$ — ionization frequency, $F$ — electric field intensity and $W$ — mean electron energy are dependent on $t$ — time and $x=az$ axial coordinate ($a=qE_0/w_0$). They are normalized to unity for the equilibrium state conditions. Constants are $D=a^2D_a/Z_0$ — coefficient of ambipolar diffusion, $B=qV_i/KT_\infty$ — ionization potential and $C_1$, $C_2$ — relaxation constants times of electron energy due to elastic collisions and ionization losses, respectively, with $C_1+C_2=1$.



To simplify the numerical solution of this system of nonlinear differential equations we have postulated constant phase velocity of the resulting waves (so-called "constant profile waves"). Curves shown in Fig. 2 are obtained for the shapes of waves periodic in time and space. They correspond to limit cycles of the obtained nonlinear oscillator, which may be called the ionizator. The ionization and the light intensity occur in short narrow bursts, while the charged particles density decays exponentially between the bursts.

The solution giving the strictly periodic state of the strongly nonlinear amplitude-saturated wave appears, however, unstable in some cases. The instability is best demonstrated by constructing the transfer function from the numerical solutions for waves (oscillations) with disturbed amplitudes: Fig. 3. This function, $F(a)$, allowing to determine the value $a_n$ of the $n$-th wave crest by means of the value $a_{n-1}$ of the $(n-1)$ st crest, crosses the bisectrix with negative slope in our case. This slope may — depending on the values of parameters in the used equations — be even smaller than $-1$, i.e. $F'(a)<-1$. The equilibrium state $a^*$ is unstable in that case: the wave inevitably leaves the point $a^*$ and tends asymptotically to a new, also periodic, motion, but with a doubled period, i.e. with second subharmonic frequency. In case of a very steep descent ($F'(a)\ll-1$), *only a still higher subharmonic motion* ($\omega_0/4$, $\omega_0/8$, etc.) may appear stable (see e.g. [3]), or even a constantly maintained irregular sequence of amplitudes may occur. The latter case corresponds to a strange attractor of the nonlinear oscillator, and could be connected with a direct onset of a turbulent wave motion. But the mere appearance of the second subharmonic modulation is in itself sufficient to cause the (originally periodic and coherent) wave to become turbulent at some distance from the source due to changes in the local group velocity [4] caused by the spontaneous frequency modulation.

Though the whole chain of processes causing the propagation and instability of an ionization wave in a nonequilibrium plasma is well known, and the nonlinear behaviour is chiefly due to the strongly nonlinear dependence of the ionization rate on the mean electron energy, we did not succeed in attempts to find a single decisive process responsible for the tendency to subharmonic modulation. The relaxation time or length is, on the other hand, simply and directly determined by the ion life time and the amplitude.

Some complication, which we have not overcome yet, arose from the necessity to solve numerically the wave equations (see [2]) for decreasing instead for increasing time. This was connected with the above mentioned and rather artificial restriction on solution in the form of a constant profile wave. In a nonlinear system, the time reversal in the solution is not fully correct. It can be seen, e.g., from the fact that for a single amplitude $a_{n+1}$ there may exist in Fig. 3 even three different amplitudes $a_n$ and there is no rule to decide which the correct one is. Nevertheless we believe that an ordinary solution of the dynamic wave equations not restricted to a constant profile wave would have properties close to those referred to in this contribution.

Anyway, the simple dependence between the amplitude and the period found here shows that in a strongly nonlinear wave some plasma parameters can determine the wave parameters directly, which seems promissing in plasma diagnostics.

## REFERENCES

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