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TUNNELLING BETWEEN TWO SUPERCONDUCTORS SEPARATED BY A DOUBLE SCHOTTKY BARRIER

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The tunnelling probability through tunnel junctions between two superconductors or metals separated by a double Schottky barrier is calculated in the WKB approximation. Further, the expression for the tunnel current is derived and its dependence on the applied voltage is numerically calculated for a Pb—CdS—Pb system at various temperatures.

It is shown that the energy gap of superconductors (and mainly its changes, e. g., by temperature) can be measured not only from the steep increase of current at $V=2\Delta/e$, but also at larger voltages, determined by the appearance of the resonance tunnelling through the double barrier.

ТУННЕЛЬНЫЙ ЭФФЕКТ ДЛЯ СЛУЧАЯ ДВУХ СВЕРХПРОВОДНИКОВ, РАЗДЕЛЕННЫХ ДВОЙНЫМ БАРЬЕРОМ ШОТКИ

В работе на основе метода ВКБ вычислен коэффициент прозрачности в случае туннельного перехода между двумя сверхпроводниками или металлами, разделенными двойным барьером Шотки. Выведено также выражение для туннельного тока и расчитано численое значение зависимости туннельного перехода в системе Pb—CdS—Pb от величины приложенного напряжения при различных температурах. Показано, что энергетическую щель сверхпороводников (и главными образом ее зависимость, например, от температуры) можно измерять не только по скачкообразному повышению тока при $V=2\Delta/e$, но также при высших напряжениях, которые определяются появлением резонансного туннельного эффекта в случае двойного барьера.

I. INTRODUCTION

Electron tunnelling is one of the most useful research methods in solid state physics. Since Giaever [1] first measured the energy gap of superconductors using this method, tunnelling has become an important instrument in superconductivity research, too.

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If a metal is placed in contact with a semiconductor, a potential barrier is created at the interface between them. If the barrier is thin enough and the temperature low enough, the current can flow through the barrier due to electron tunnelling, when a difference in electrical potential between the metal and the semiconductor is applied. The same is true for metals in the superconducting state, but the current will be influenced in an important manner by the modified density of the electron states.

The current flowing between two superconductors separated by a thin barrier due to the tunnelling quasiparticle can be derived in a simple way, only instead of the density of states in the normal state (usually $\varrho = \varrho(0)$ on the Fermi surface) we have to substitute the modified density of the one particle states in the superconductor (e. g. in the BCS form) [2]. Provided that the barrier is not very thin, we can neglect the Josephson contribution to the current.

The shape of the potential barrier is usually not very important for the tunnelling characteristics. Therefore, the main results can be obtained, e. g., by "edge" barriers, too [3, 4]. Although the transmission probabilities of particles are modified mainly if a potential well exists between the barriers (although only relatively to the barriers), most of the phenomena can be described by "edge" potentials with effective widths and heights also in this case. In [3, 4] the resonance tunnelling was shown to play an important role in some phenomena in solids and generally in superconductors in magnetic fields [5]. Similar fundamental changes can be also expected in the tunnel current of a system of two superconductors separated by double barrier (e. g. by the double Schottky barrier [6]). However, in this case the applied voltage influences the parameters of the system (barrier widths, barrier heights). The method taking into account these changes was worked out in [6, 7] but without including the resonance tunnelling.

In the present paper, the influence of resonance tunnelling on the tunnel current between normal metals and superconductors is also taken into consideration.

In Sect. II, the shape of the potential barrier and its dependence on the voltage for a metal-semiconductor-metal junction is suggested and the tunnelling probability in the WKB approximation is calculated. In Sect. III, the expression for the tunnel current is derived and the dependence of the tunnel current on the voltage for a Pb—CdC—PB junction is calculated numerically.

II. TUNNELLING THROUGH A DOUBLE BARRIER

Let us consider the case of two metals separated by a thin layer of heavily doped semiconductor forming Schottky barriers in the neighbourhood of the surface of both metals [6, 7] (Fig. 1). The distribution of impurities in the barrier (which is assumed to be homogeneous for a Schottky barrier), as well as the surface states (which are between the metal and the semiconductor) determine the shape of the

potential barriers. We shall assume that the height of the "left" barrier E_b remains constant, that the applied voltage $V = U/e$ is low compared to E_b/e i. e. $U < E_b$) and that the course of the potential barrier from the interface is parabolic (ideal Schottky barrier). The potential energy in the junction is then given by the expressions (the energy is measured from the Fermi energy ζ_A of the "left" metal)

$$\Phi(x) = \begin{cases} E_b + \dfrac{Ne^2}{2\varepsilon}[(x-\lambda_1)^2 - \lambda_1^2] & \text{for } 0 \leq x \leq \lambda_1 \\ E_b - \dfrac{Ne^2}{2\varepsilon}\lambda_1^2 & \text{for } \lambda_1 \leq x \leq L - \lambda_2 \\ E_b + \dfrac{Ne^2}{2\varepsilon}[(x-L+\lambda_2)^2 - \lambda_1^2] & \text{for } L-\lambda_2 \leq x \leq L \end{cases} \tag{1}$$

where λ_1, λ_2 are the widths of the depletion zones defined by

$$\lambda_1 = \left[\frac{2\varepsilon}{Ne^2}(E_b + \zeta + U_1)\right]^{1/2}$$
$$\lambda_2 = \left[\frac{2\varepsilon}{Ne^2}(E_b + \zeta - U_2)\right]^{1/2}, \tag{2}$$

E_b is the value of the potential energy on the "left" metal-semiconductor interface, e is the absolute value of the electron charge, ε is the static dielectric constant of the semiconductor, ζ is the Fermi degeneracy energy of the semiconductor, N is the donor or acceptor density in the semiconductor, L is the thickness of the

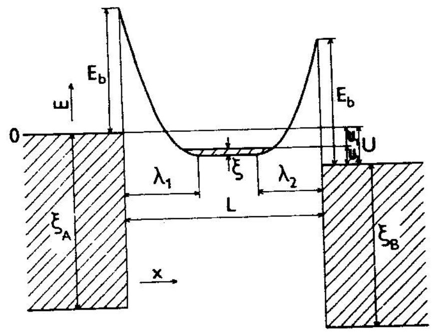

Fig. 1. The distribution of potential energy in a metal-semiconductor-metal junction.

semiconductor and x is the distance perpendicular to and measured from the "left" metal-semiconductor interface, $U_1 = eV_1$, $U_2 = eV_2$, where V_1, V_2 are the differences in electrical potential between the metal and the semiconductor (therefore U_1, U_2 express the changes of electron energy, whereby $U_1 + U_2 = U = eV$, V is the difference in the electrical potential between the metals. In our calculation we restrict ourselves to the case $L > \lambda_1 + \lambda_2$, where the whidths of the depletion zones λ_1, λ_2 are smaller than the total thickness of the barrier L.

To calculate the tunnel current through the barrier it is necessary to know the tunnelling probability P through the potential barrier $\Phi(x)$. Mehbod et al. [6] dealt with the tunnelling probability through a double Schottky barrier. They calculated the tunnelling probability as a product of the tunnelling probabilities through the single Schottky barriers and neglected the reflection of the waves inside the potential well leading to resonance tunnelling (the essential increasing of the transmission probability for certain energies). For the calculation of the tunnelling probability we have used the WKB method taking into account the existence of resonance tunnelling. The tunnelling through a symmetric double barrier was calculated in the WKB approximation in [8]. That procedure can be easily generalized for an asymmetric double barrier.

Let us decompose the energy of electrons (measured from the Fermi energy of the "left" metal) into the part corresponding to the motion of electrons in the direction parallel to the barrier $E_{\parallel}$ and into the part corresponding to the motion of electrons perpendicular to the barrier $E_\perp$, i. e. $E = E_{\parallel} + E_\perp$. We introduce the following notations

$$E_0 = \frac{e\hbar}{2}\left(\frac{N}{m\varepsilon}\right)^{1/2}, \quad \varepsilon_b = \frac{Ne^2L^2}{2E_0\varepsilon}, \quad \varepsilon_\perp = \frac{E_b - E_\perp}{E_0} \tag{3}$$

$$u_1 = \frac{E_b + \zeta + U_1}{E_0}, \quad u_2 = \frac{E_b + \zeta - U_2}{E_0},$$

where m is the electron mass.

In the calculation of the tunnelling probability, we shall distinguish the following cases

1) $u_2 - u_1 > -\varepsilon_\perp > -u_1$.

This case corresponds to the situation, where the energy $E_\perp$ is smaller than the height of the "right" potential barrier and larger than the potential energy in the potential well. In such a case the tunnelling probability is given by

$$P = \left\{\frac{1}{4}\left(\frac{\Theta_1}{\Theta_2} + \frac{\Theta_2}{\Theta_1}\right)^2 + \frac{1}{4}\left(\frac{1}{4\Theta_1^2} - 4\Theta_1^2\right)\left(\frac{1}{4\Theta_2^2} - 4\Theta_2^2\right)\cos^2 J\right]\right\}^{-1} \tag{4}$$

where

$$\Theta_1 = \exp(\varphi_1) = \exp\left(\int_0^{x_1} \frac{\varkappa_\perp}{\hbar}\,dx\right), \tag{5}$$

$$\Theta_2 = \exp(\varphi_2) = \exp\left(\int_{x_2}^{L} \frac{\varkappa_\perp}{\hbar}\,dx\right),$$

$$J = \int_{x_1}^{x_2} \frac{p_\perp}{\hbar}\,dx$$

$$\varkappa_\perp\{2m[\Phi(x) - E_\perp]\}^{1/2}, \quad p_\perp = \{2m[E_\perp - \Phi(x)]\}^{1/2}.$$

The limits of integrations are the turning points of the classical motion of electrons. For the double Schottky barrier one obtains

$$\varphi_1 = \frac{1}{2}\left\{u_1^{1/2}\varepsilon_\perp^{1/2} + (\varepsilon_\perp - u_1)\ln\left[\frac{u_1^{1/2} + \varepsilon_\perp^{1/2}}{(u_1 - \varepsilon_\perp)^{1/2}}\right]\right\}, \tag{6}$$

$$\varphi_2 = \frac{1}{2}\left\{u_2^{1/2}(u_2 - u_1 + \varepsilon_\perp)^{1/2} + (\varepsilon_\perp - u_1)\ln\left[\frac{u_2^{1/2} + (u_2 - u_1 + \varepsilon_\perp)^{1/2}}{(u_1 - \varepsilon_\perp)^{1/2}}\right]\right\},$$

$$J = \frac{\pi}{2}(u_1 - \varepsilon_\perp) + (u_1 - \varepsilon_\perp)^{1/2}(\varepsilon_b^{1/2} - u_1^{1/2} - u_2^{1/2}).$$

2) $-u_1 > -\varepsilon_\perp$

This case corresponds to the situation, where the energy $E_\perp$ is smaller than the potential energy on the bottom of the potential well. Tunnelling probability is then given by

$$P = \left[\Theta + \frac{1}{4\Theta}\right]^{-2},$$

where

$$\Theta = \exp(\varphi) = \exp\left(\int_0^L \frac{\varkappa_\perp}{\hbar}\,dx\right). \tag{8}$$

The meaning of the expression $\varkappa_\perp$ is the same as in the previous case. For the double Schottky barrier one has in this case

$$\varphi = \frac{1}{2}\left\{u_1^{1/2}\varepsilon_\perp^{1/2} + (\varepsilon_\perp - u_1)\ln\left[\frac{u_1^{1/2} + \varepsilon_\perp^{1/2}}{(\varepsilon_\perp - u_1)^{1/2}}\right] + \right. \tag{9}$$

$$+ u_2^{1/2}(u_2 - u_1 + \varepsilon_\perp)^{1/2} + (\varepsilon_\perp - u_1)\ln\left[\frac{u_2^{1/2} + (u_2 - u_1 + \varepsilon_\perp)^{1/2}}{(\varepsilon_\perp - u_1)^{1/2}}\right] +$$

$$\left. + (\varepsilon_\perp - u_1)^{1/2}(\varepsilon_b^{1/2} - u_1^{1/2} - u_2^{1/2})\right\}.$$

We shall not give the expression of the tunnelling probability for the energies $E_\perp$ larger than the height of the "right" potential barrier, because the contribution to the tunnel current from these values of energies is negligible. For the calculation of the tunnel current the most important values of energies are those close to the Fermi energy, $E \sim 0$, as the product of the probability of the state in one metal being occupied and the probability of the state in the other metal beaing free is the greatest for these values of energies (we consider the case of low temperatures, $E_h \gg kT$).

For the value of energy corresponding to the bottom of the potential well, $\varepsilon_\perp = u_1$, the introduced expression of the tunnelling probability is not continuous (as the WKB approximation for $\varepsilon_\perp$ near u_1 is not valid). To determine the tunnelling probability for the energy domain $\varepsilon_\perp \sim u_1$ we had to solve Schrödinger equation numerically. However, this domain is a very narrow one. The tunnelling probability appears in the expression for the tunnel current (eq. 17) as an integrand. For the calculation of the tunnel current, we therefore use expression (4) also for energies $\varepsilon_\perp \sim u_1$. The error caused by using expression (4) throughout the calculation of the tunnel current is negligible.

The WKB approximation is not valid in the neighbourhood of the points $x = 0$, $x = L$ (Fig. 1), because in these points the potential is sharp-edged. But the ignorance of this fact does not lead to errors in the tunnel current, as the contribution of these domains to the tunnelling probability are very small.

In Fig. 2 the function log P is shown for the values of energies $\varepsilon_\perp$ from the interval $(u_1 - u_2, u_1)$. In this interval the total energy of the tunnelling electron is larger than the potential energy on the bottom of the potential well and is smaller than the height of the potential barrier. It can be seen from this figure, that sharp resonance peaks arise for energies near the bottom of the potential well.

III. CALCULATION OF THE TUNNEL CURRENT

III. 1. Derivation of the formula

If we use the methods of the tunnelling Hamiltonian and of the perturbation theory, the following formula, describing the amount of electrons which cross from the "left" superconductor (label A) to the "right" superconductor (label B) per time unit, can be easily derived [9]

$$R_{A,B} = \frac{4\pi}{\hbar}\sum_{p,k} |T_{p,k}|^2 \{ v_{p,A}^2 u_{k,B}^2 (1 - f_{p,A} - f_{k,B}) \delta(E_{p,A} + E_{k,B} - U) + \tag{10}$$
$$+ u_{p,A}^2 u_{k,B}^2 (f_{p,A} - f_{k,B}) \delta(E_{p,A} - E_{k,B} + U) +$$
$$+ v_{p,A}^2 v_{k,B}^2 (f_{k,B} - f_{p,A}) \delta(E_{p,A} - E_{k,B} - U) +$$
$$+ u_{p,A}^2 v_{k,B}^2 (f_{p,A} + f_{k,B} - 1) \delta(E_{p,A} + E_{k,B} + U) \},$$

where $u_{p,A}^2$, $v_{p,A}^2$ are the well-known expressions of the BCS theory of superconductivity expressing the probability that the state is occupied by a Cooper pair or free, respectively, $E_{p,A} = [(P^2/2m - \zeta_A)^2 + \Delta_{p,A}^2]^{1/2}$ is the excitation energy of electrons in the superconductor, $\Delta_{p,A}$ is the energy gap parameter of the

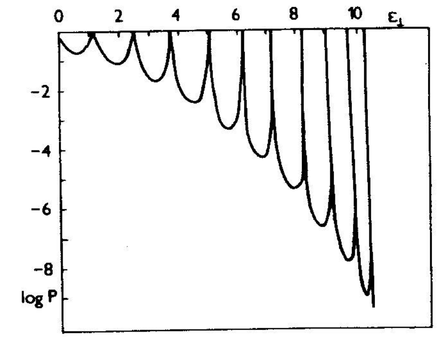

Fig. 2. The dependence of the tunnelling probability on the energy of the electron ($V = 0$, $E_h = 20$ meV, $E_0 = 2$ meV, $\zeta = 1$ meV, $L = 50$ nm, $\varepsilon_\perp = (E_h - E_\perp)/E_0$).

superconductor, ζ_A is the Fermi energy of the "left" superconductor, $f_{p,A} = f(E_{p,A})$ is the Fermi distribution function, $T_{p,k}$ are tunnelling matrix elements. The form of the expression with label B is obvious. The summations go through all the excitation states of both superconductors.

The following form of the tunnelling matrix element is assumed to be valid:

$$|T_{p,k}|^2 = \frac{P(E_\perp)}{4\pi^2 \varrho_{\perp,A} \varrho_{\perp,B}} \delta_{k_\parallel, p_\parallel}, \tag{11}$$

where $\varrho_\perp$ is the one-dimensional density of states in the direction perpendicular to the potential barrier, P is the tunnelling probability (which we have determined in the WKB approximation). Kronecker's delta symbol for the components of momenta parallel to the potential barrier expresses the specular transformation.

Substituting expression (11) for $|T_{p,k}|^2$ into formula (10) and replacing the summation through the discrete values of momenta by the integration through the quasi-continuous spectrum of energies, we find

$$R_{A,B} = \frac{4\pi m S}{(2\pi\hbar)^3} \int dk_\parallel \int dk_\perp \int dp_\perp \frac{k_\parallel}{m} \frac{k_\perp}{m} \frac{p_\perp}{m} \{ P(-E_{p,A} - E_\parallel) \times \tag{12}$$
$$\times v_{p,A}^2 u_{k,B}^2 (1 - f_{p,A} - f_{k,B}) \delta(E_{p,A} + E_{k,B} - U) +$$
$$+ P(E_{p,A} - E_\parallel) u_{p,A}^2 u_{k,B}^2 (f_{p,A} - f_{k,B}) \delta(E_{k,B} - E_{p,A} - U) +$$
$$+ P(-E_{p,A} - E_\parallel) v_{p,A}^2 v_{k,B}^2 (f_{k,B} - f_{p,A}) \delta(E_{p,A} - E_{k,B} - U) \} =$$
$$= \frac{4\pi m S}{(2\pi\hbar)^3} \int_0^\infty dE_\parallel \int_{-\infty}^\infty dE_{\perp,A} \int_{-\infty}^\infty dE_{\perp,B} \{ P(-E_A - E_\parallel) v_A^2 u_B^2 (1 - f_A - f_B) \times$$
$$\times \delta(E_A + E_B - U) + P(E_A - E_\parallel) u_A^2 u_B^2 (f_A - f_B) \delta(E_B - E_A - U) +$$
$$+ P(-E_A - E_\parallel) v_A^2 v_B^2 (f_B - f_A) \delta(E_A - E_B - U) \}, \tag{12}$$

where S is the area of the junction, $E_{\parallel}=P_{\parallel}^2/2m=k_{\parallel}^2/2m$, $E_A=[(E_{\parallel}+E_{\perp,A})^2+\Delta_A^2]^{1/2}$, $u_A^2=[1+(E_{\parallel}+E_{\perp,A})E_A]/2$, $v_A^2=[1-(E_{\parallel}+E_{\perp,A})/E_A]/2$, $f_A=f(E_A)=[\exp(E_A/kT)+1]^{-1}$. The form of the expression with label B is obvious. In arranging formula (12) we have taken into account that the Fermi energies of both metals are much larger than the factor kT (i. e. $\zeta_A,\ \zeta_B \gg kT$), therefore the limits of the integrations with respect to $E_{\perp,A}$, $E_{\perp,B}$ are taken as ∞. We have mentioned that the most important values of energies are close to the Fermi energy and the contribution to the tunnel current from energies much larger or much smaller than the Fermi energy is negligible. In the following arrangements we use the substitutions $\varepsilon_A=E_{\parallel}+E_{\perp,A}$, $\varepsilon_B = E_{\parallel}+E_{\perp,B}$:

$$R_{A,B}=\frac{4\pi mS}{(2\pi\hbar)^3}\int_0^\infty dE_{\parallel}\int_{-\infty}^{\infty} d\varepsilon_A\int_{-\infty}^{\infty} d\varepsilon_B\{P(-E_A-E_{\parallel})v_A^2u_B^2\times \tag{13}$$

$$\times[1-f_A-f_B]\delta(E_A+E_B-U)+P(E_A-E_{\parallel})u_A^2u_B^2[f_A-f_B]\delta(E_B-E_A-U)+$$

$$+P(-E_A-E_{\parallel})v_A^2v_B^2[f_B-f_A]\delta(E_A-E_B-U)\}.$$

Then we have $E_A=[\varepsilon_A^2+\Delta_A^2]^{1/2}$, $u_A^2=[1+\varepsilon_A/E_A]/2$, $v_B^2=[1-\varepsilon_A/E_A]/2$, etc. It can be seen from the dependence of the factors u^2, v^2 on variables of the integration ε_A, ε_B that they give the value 1/2 (the remaining functions are even)

$$R_{A,B}=\frac{4\pi mS}{(2\pi\hbar)^3}\int_0^\infty dE_{\parallel}\int_0^\infty d\varepsilon_A\int_0^\infty d\varepsilon_B\{P(-E_A-E_{\parallel})[1-f_A-f_B]\times \tag{14}$$

$$\times\,\delta(E_A+E_B-U)+P(E_A-E_{\parallel})[f_A-f_B]\delta(E_B-E_A-U)+$$

$$+P(-E_A-E_{\parallel})[f_B-f_A]\delta(E_A-E_B-U)\}$$

or

$$R_{A,B}=\frac{4\pi mS}{(2\pi\hbar)^3}\int_0^\infty dE_{\parallel}\int_0^\infty dE_A\int_0^\infty dE_B\varrho_A(E_A)\varrho_B(E_B)\times \tag{15}$$

$$\times\{P(-E_A-E_{\parallel})[1-f(E_A)-f(E_B)]\delta(E_A+E_B-U)+$$

$$+P(E_A-E_{\parallel})[f(E_A)-f(E_B)]\delta(E_B-E_A-U)+$$

$$+P(-E_A-E_{\parallel})[f(E_B)-f(E_A)]\delta(E_A-E_B-U)\},$$

where $\varrho(E)=|E|\Theta(E^2-\Delta^2)/[E^2-\Delta^2]^{1/2}$ is the well known expression for the density of states in the BCS theory. Using $f(-E)=1-f(E)$ and making some simple rearrangement one has

$$R_{A,B}=\frac{4\pi mS}{(2\pi\hbar)^3}\int_{-\infty}^\infty dE\int_0^\infty dE_{\parallel}P(E-E_{\parallel})\varrho_A(E)\varrho_B(E+U)\times \tag{16}$$

$$\times[f(E)-f(E+U)]$$

and for the density of the tunnel current

$$j=\frac{4\pi me}{(2\pi\hbar)^3}\int_{-\infty}^\infty dE\int_0^\infty dE_{\parallel}P(E-E_{\parallel})\varrho_A(E)\varrho_B(E+U)\times \tag{17}$$

$$\times[f(E)-f(E+U)]=\frac{4\pi me}{(2\pi\hbar)^3}\int_{-\infty}^\infty dE\int_{-\infty}^E dE_{\perp}P(E_{\perp})\varrho_A(E)\varrho_B(E+U)\times$$

$$\times[f(E)-f(E+U)].$$

III. 2. NUMERICAL CALCULATION

Formula (17) has been used for the numerical calculation of the tunnel current through a Pb—CdS—Pb junction at voltages V small compared to E_b/e (i. e. $V<E_b/e$). The technology of the preparation of the Pb—CdS—Pb junction was described in [6] and the expression for the tunnel current was derived for two limiting situations (low and at large voltages). In [6] the derived expression was verified experimentally, but only for the case of larger voltages, for which the junction is essentially equivalent to a single Schottky barrier. We have used the expression (17) for lower voltages, at which the junction is still essentially a double Schottky barrier. In this case a potential well exists between the barriers and therefore the tunnelling resonances appear at certain values of energy of the tunnelling electron (in [6], the existence of the tunnelling resonances was neglected).

The numerically calculated current-voltage characteristics are shown in Fig. 3 for a Pb—CdS—Pb tunnel junction at various temperatures. The following values of parameters have been used in our calculation $E_b=20$ meV, $E_0=2$ meV, $\zeta=1$ meV, $L=50$ nm. Furthermore $U_1=U_2=U/2$ has been assumed to be valid (both metals are identical) i.e. the potential difference at both interfaces are the same. The curve 1 in Fig. 3 corresponds to the current voltage characteristics at $T=0$ K, the energy gap parameter is $\Delta_0=1.33$ meV. At voltages lower than $2\Delta_0/e$, the tunnel current between two identical superconductors does not flow, the growth of the tunnel current begins when the voltage is larger than $2\Delta_0/e$. The curve 2 in Fig. 3 corresponds to the current voltage characteristic at the temperature $T=4.2$ K, the energy gap parameter being $\Delta=1.22$ meV. In this situation the tunnel current also flows at voltages lower than $2\Delta/e$, but the sharp increase of the tunnel current begins only at voltages above $2\Delta/e$. Both curves are similar at larger voltages and depend only little on temperature. The rapid growth of the tunnel current at certain voltages is connected with the resonance tunnelling. The energies, at which the resonance tunnelling appears, depend only on the shape of the potential barrier. For the double Schottky barrier with constant values of E_b, E_0, ζ, L, the resonance tunnelling depend only on the applied voltage. If

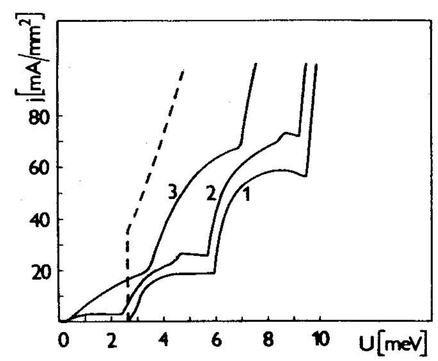

Fig. 3. The current-voltage characteristics of a Pb—CdS—Pb tunnel junction. Curve 1 corresponds to $T=0$K, curve 2 to $T=4.2$ K, curve 3 to the temperature $T=7.2$ K at which lead is in the normal state ($E_b=20$ meV, $E_0=2$ meV, $\zeta=1$ meV, $L=50$ nm). For comparison, the tunnelling current through a simple Schottky barrier is illustrated at $T=0$ K (dashed line).

a resonance appears at the energy $-\Delta$, the tunnel current increases sharply by next increasing the applied voltage, because at this energy the density of the occupied states in the "left" superconductor is the largest. The moderate drop of the tunnel current appearing before its increasing is connected with the drop of the tunnelling probability at the energy $\Delta-U$, i.e. at the energy, at which the density of the free states in the "right" superconductor is the largest. It is evident from Fig. 3, that at lower temperatures (or at larger values of the energy gap parameter) the growth of the tunnel current due do the resonance tunnelling appears at larger values of the voltage. From a detailed analysis of the expressions (4), (6), (17) it follows that the differences in the electrical potential V_1-V_2 (here, the subscripts 1, 2 refer to two different current-voltage characteristics), at which the sharp increase of the tunnel current appears, depend on the difference of the energy gap parameter. Neglecting a very small modification of the shape of the potential barrier by the applied voltage we can find the $V_1-V_2=2(\Delta_1-\Delta_2)/e$ expression to be valid.

By measuring the "steps" in the current-voltage characteristics even at large voltages than $V=2\Delta/e$ one has a useful and exact method for determining the energy gaps of superconductors, mainly their changes (e. g. the temperature or magnetic field dependence).

IV. CONSLUSION

Using the WKB approximation the tunnelling probability through a double barrier, separated by a potential well, has been calculated. The calculated transmission probability has been used in the calculation of the one-particle tunnel current between two metals (both in the superconductiong and in the normal state) separated by the double Schottky barrier. The resulting integral for the tunnel current has been calculated numerically for a Pb—CdS—Pb junction. We have shown that the tunnel current at voltages larger than $2\Delta/e$ is markedly determined by tunnelling resonances. The voltage at which the "steps" of the tunnel current appear depends on the energy gap parameter. This can be used for measuring the energy gap parameter and its temperature dependence.

The measurement of the tunnel current through a double (e. g. Schottky) barrier can have some advantages with respect to the single barrier. At first, the width of the barrier can be a few orders larger ("technological advantage"). Further, the jumps of the current-voltage characteristics are very sharp ("measuring advantage"). Besides the measurements of the energy gap parameters, the differences of the energy gap parameters can be exactly measured even from the jumps connected with the maxima of the tunnelling transmission through the double barrier.

An additional important information can be gained from the differences of voltage at which the current has a jump-like character. The resonances (maxima of the transmission through a double barrier) are determined in a very simple way by $\partial P/\partial E_\perp=0$. However, these expressions are not always very simple.

Therefore, we used an approximate calculation for the resonances from [3]. The sharp increase of the tunnel current in the normal state occurs if P has a maximum at $E_\perp=0$ (for electrons on the Fermi surface). This happens approximately for

$$\mathrm{tg}\left[\left(\frac{2mU_\alpha}{\hbar^2}\right)^{1/2}(L-\lambda_1-\lambda_2)/2\right]=\frac{\mathrm{th}\left[\left(\frac{2m}{\hbar^2}E_b\right)^{1/2}\lambda_1\right]}{\left(\frac{U_\alpha}{E_b}\right)^{1/2}}, \tag{18}$$

where step-like potentials have been assumed.

By inserting the considered parameters of the double barrier, we obtain the values of U_α at which the steps appear as $U_{\alpha 1}=0.6$ meV, $U_{\alpha 2}=3$ meV, $U_{\alpha 3}=6.2$ meV. These values are in good agreement with Fig. 3 (the step at $U_{\alpha 1}\approx 0.3$ meV is not very conspicious.

As in practical cases th $[(2mE_b/\hbar^2)^{1/2}\lambda_1]\approx 1$ the righthand side of (18) depends mainly on U_α/E_b, we can make some conclusions as to the two unknown parameters E_b and $(L-\lambda_1-\lambda_2)$, i. e. effective barrier height and effective well width, from measurements of the current steps.

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