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# DEVELOPMENT OF STRUCTURES IN THE STATES FAR FROM EQUILIBRIUM AT EXTERNAL REGULATION

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The present paper deals with the formation of new qualities in inorganic, biological and social systems that can be reduced to unbalanced thermodynamic systems. Thus, the possibility to apply laws describing the evolution of unbalanced inorganic systems is given. The subjective factor, corresponding with external regulation is represented through the member expressing changes in systems not resulting from internal dynamics. It has been demonstrated that the presence of this external regulation can either suppress the number of possible new qualities originating under conditions far from equilibrium, or on the other hand, increase their number and induce the formation of such structures not resulting from laws describing the internal dynamics of systems. In connection with the fact the problem of achieving optimal system management with respect to minimal system entropy is involved.

## РАЗВИТИЕ СТРУКТУР С ВНЕШНЫМИ РЕГУЛИРОВАНИЕМ В СОСТОЯНИЯХ, ДАЛЕКИХ ОТ РАВНОВЕСИЯ

В статье рассматриваются образования новых характеристик в неорганических, биологических и общественных системах, которые могут быть сведены к неуравновешенным термодинамическим системам. Это дает возможность применить в этом случае законы, описывающие развитие неуравновешенных неорганических систем. Субщективный фактор, соответствующий внешнему регулированию, представлен посредством члена, который выражает изменения в системе, не являющиеся резултьтатом внутренней динамики. Показано, что присутствие этого внешнего регулирования может или подавить спектр возможных новых характеристик, возникающих в условиях, далеких от состояния равновесия, или увеличить их число и вызвать образование таких структур, которые не вытекают из законов, описывающих внутреннюю динамику системы. В связи с этим факторм сформулирована проблема достижения оптималшного управления системой с учетом минимальной энтропии системы.

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## I. INTRODUCTION

Practically in the whole discipline of physics (e. g. in mechanics, thermics, electricity, magnetism, in atomic and molecular physics, in chemistry etc.) series of phenomena are known at present which may be sufficiently qualified as a new quality formation under conditions far from equalibrium. Prigogine and his coworkers [1—4] as well as other authors [5—7] elucidated the course of those processes and assessed criteria for evolution connected with the qualitative system change. It was found out that all processes of this type occurring in inorganic as well as in biological and social systems have common features consisting of the so-called subsystem cooperation arising on a certain level that can be examined by using a uniform formalism. This knowledge stimulated the formation of synergetics that has been developing very intensively thanks mainly to Haken and others [8—12].

The aim of synergetics is to yield a general phenomenological theory of the formation of new qualities in the systems irrespective of nonliving and living ones. Within synergetics the dynamics of the most perfect systems — systems consisting of members and characterized by their own intelligence, i. e. social systems — has been examined very intensively. The particularity of the systems is the possibility of the direct interference into the system evolution due to the ability of subsystems to produce negative or positive entropy directly, i. e. owing to the ability to manage the system.

Though certain conscious interferences within the evolution of the system can be considered in dynamical equations without dealing with "regulation" is seems to be useful to analyse processes with evidently discrete factors connected with objectively valid laws describing system dynamics and the factors resulting from the thinking subject activity. The problem formulated in such a way can immediately lead to the solution of basically new specific problems, e. g. the problem of optimal management with the aim to obtain the minimal entropy production, and the most perfect structure.

Through the indicated problem is topical especially in connection with the examination of social systems, models of such systems can be created also in the realm of unviable or biological systems, since also here the question may arise what structures the system would tend towards due to its internal dynamics if an intelligent being tried to control this process. In the course of the substance synthesis on the basis of valid chemical laws the process may be influenced by the supply or withdrawal of certain substances (catalysts). In case of biological systems the control may consist, for instance, in shooting predatory animals, increasing required species of animals, etc. It is evident that such artificially induced alteration in the concentration of given atoms, molecules, animals, etc. can immediately bring about the change of appropriate characteristic potentials, entropy production etc., thus producing an alteration in system dynamics. Thus, a new quality formation becomes the function of objective and subjective factors.



This article aims at examining the behaviour of certain systems in the presence of external regulation (especially the so-called constant regulation) and to apply formal theory to actual inorganic, biological and social regulated systems.

## II. DYNAMICAL EQUATIONS AND PRIGOGINE'S PRINCIP

The time evolution of every physical system can be described by a system of evolutional equations of the type

$$\dot{x} = F(x, \lambda) \tag{1}$$

where $x$ is a properly chosen characteristic and $\lambda$ is the characteristic parameter. When after some time of evolution a new quality formation should occur, the functions $F(x, \lambda)$ have to be nonlinear. Recently the very frequently examined so-called generalized Volterra—Lotka systems have been described by the equation system

$$\dot{x}_i = (a_i + \Sigma\, b_{ij} x_j) x_i \tag{2}$$

where $a_i$ is the characteristic constant, $b_{ij}$ are coefficients of the antisymmetric matrix with zero-diagonal members ($b_{ij} = -b_{ji}$). This quality of the coefficients $b_{ij}$ results in the fact that the decrease of one system component during interaction with the other means an increase of the other component.

The system of equations (2) describes a certain class of biochemical reactions, simultaneously it describes also the evolution of the viable systems in an "annihilation-generation" interaction (e. g. predatory animals and victims system). A certain class of social systems can be described by equations (2). All systems described by equations (2) with an even number of components can be qualified as physical systems for which general laws of non-equilibrium thermodynamics are valid (e. g. Prigogine's principle of minimal entropy production). Introducing new variables (e. g. a two-component system) we obtain

$$p = \ln \frac{x_1}{x_{1s}} \quad q = \ln \frac{x_2}{x_{2s}} \tag{3}$$

with $x_{1s}$ and $x_{2s}$ as stationary solutions, since it can be demonstrated [13, 14] that such a system has at least one kinetic integral ("energy", it has its Lagrangian, and that the new variables $p$, $q$ represent the solution of Hamilton's equations so that the mentioned chemical processes and systems of mutual interacting biological groups are isomorphic with non-equilibrium thermodynamic systems.

The systems described by equations (2) have an interesting feature in that they exhibit stationary states only for even numbers of components, because in case of their odd numbers the determinant created from the coefficients $b_{ij}$ is zero. But we shall prove that this limitation can be avoied by external regulation.

Let us introduce a regulation into systems described by equations (2) arranging the concentration of system components to be changed by external interference (e. g. material supply, shooting predatory animals etc.) in the course of time. If we indicate this induced alteration in a time unit as $z_i$, then the system is described by equations

$$\dot{x}_i = F(x_i, \lambda) + z_i \tag{4}$$

or in case of Volterra—Lotka systems by equations

$$\dot{x}_i = (a_i + \Sigma b_{ij} x_j) x_i + z_i \tag{5}$$

where the quantities $z_i$ are generally the functions of $x_i$ and time. If they are constant, then it is possible by substituting $y_i = x_i + \alpha_i$ to transfer the quation system (5) into the form of equations not containing explicitly the regulation

$$\dot{y}_i = (a'_i + \Sigma b_{ij} y_j) y_i + \alpha_i \Sigma b_{ij} y_j \tag{6}$$

while the numbers $\alpha_i$ have to fulfil the equations

$$a_i \alpha_i + b_{ij} \alpha_i \alpha_j + z_i = 0 .$$

It would be possible to demonstrate by an analytical method that also the system described by equation (6) is a "physical" one, i.e. it is possible to apply also here the formalism of analytical mechanics, though this statement results also from the fact that the presence of (constant) components $z_i$ in equations (5) changes only the position of stationary solutions and does not influence "the response" of the system to small disturbances from these stationary states. Then it follows that also constantly regulated Volterra—Lotka systems are isomorphic with the non-equilibrium thermodynamic systems — showing also a minimum entropy production in steady states. This knowledge follows also directly from the equations (5). If we indicate the production of entropy connected with the internal dynamics of the system as $\sigma_i$ and the production of entropy connected with external regulation as $\sigma_e$, the total entropy production $\sigma = \sigma_i + \sigma_e$ fulfils the condition

$$\frac{d\sigma}{dt} = \frac{d\sigma_i}{dt} + \frac{d\sigma_e}{dt} \leqslant 0 \tag{7}$$

because $d\sigma_i/dt \leqslant 0$ and $d\sigma_e/dt = 0$. However, at the minimum

$$\sigma_{\min} = \sigma_{i\,\min} + \sigma_e$$

so that the minimum of entropy production connected with the internal system dynamics is modified by entropy production connected with regulation that could be positive or negative (Fig. 1).



When in general $z_i = z_i(x_i, t)$ it can be written

$$\frac{d\sigma}{dt} = \frac{d\sigma_i}{dt} + \sum_i \frac{\partial \sigma_e}{\partial x_i} \frac{dx_i}{dt} + \frac{\partial \sigma_e}{\partial t} . \tag{8}$$

It is evident that in general $d\sigma/dt \lesseqgtr 0$ so that the state with the minimum of entropy production need not be realized at all.

It is obvious that the biochemical, biological and social systems which are not described by equation system (2) or (5) cannot be a priori qualified as "physical" systems, though it is possible to discuss the analogy at the "entropy" level. For all the mentioned systems it is possible to write the equilibrium equation formally for entropy (if it can be defined for the examined systems)

$$\frac{dS}{dt} = -\text{div}\, \boldsymbol{i} + \sigma_i + \sigma_e \tag{9}$$

where $\boldsymbol{i}$ are entropy flows, $\sigma_i$ is the entropy production owing to the internal system dynamics and $\sigma_e$ is the entropy production due to a regulation. We can draw the conclusion that due to the objective validity of laws in biological and social systems as well as in inorganic systems the $\sigma_i$ must always be larger than 0. If insufficient outflows of entropy into the environment take place as well as "imperfect" management, $dS/dt > 0$ can occur, that means, the system develops spontaneously towards the states with greater entropy and by reaching a bifurcation point (characterized by the condition $\delta\sigma = 0$) it changes quantitatively. The evolution process in those states is then similar to that of inorganic ("physical") systems. Then we can suppose that there are stationary states also in such systems, if they exist at all, characterized by a minimal entropy production.

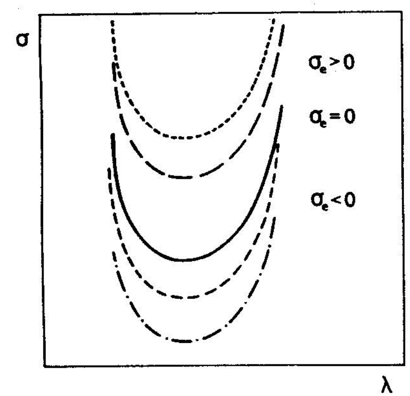



Fig. 1.

By the system regulation, structures can be created in it, as, for instance, with regard to age, sex, various dispositions of system components etc., creating certain subsystems. In this case the entropy production becomes the function of system structure, i. e. $\sigma = \sigma_i + \sigma_e = \sigma(\lambda)$, where the parameter $\lambda$ characterizes this structure. In this connection quite naturally the question of "the best" and "the worst" regulations arises, corresponding to the absolute minimum or the highest minimum of entropy production (Fig. 1). These critical states are determined by extremes of the function $\sigma = \sigma(\lambda)$, i. e. by the condition

$$\frac{d\sigma}{d\lambda} = 0. \tag{10}$$

In the conclusion of the article it will be shown that the above mentioned problem has a nontrivial solution under certain conditions.

### III. THE STRUCTURES IN VOLTERRA—LOTKA SYSTEMS WITH REGULATION

It is known that after the deviation from the stationary state unregulated Volterra—Lotka systems show the so-called oscillation around the stable centre [15]. It is a time-periodical concentration change of one component and a similar change with a certain phase shift of the second component concentration. Equation (2) for two components rewritten after the introduction of the non-dimensional quantities ($X_1 = bx_1/a_2$, $X_2 = bx_2/a_1$, $A = a_1/a_2$, $\tau = a_2 t$, $b_{12} = b_{21} = b$) in the form

$$\dot{X}_1 = A(1 - X_2)X_1 \tag{11. a}$$

$$\dot{X}_2 = (X_1 - 1)X_2 \tag{11. b}$$

has the non-trivial stationary solution $X_{1s} = 1$ a $X_{2s} = 1$. At a small perturbation of the stationary state ($y_1 = X_1 - X_{1s}$) equation (11. a) has the solution $y_1 = y_{10} \exp(pt)$, where $p = \pm i\sqrt{A}$ so that the already mentioned "oscillation" arises around the stable centre.

The two component Volterra—Lotka system with constant external regulation is described by equations ($R = bz_1/a_2^2$, $S = bz_2/a_1 a_2$)

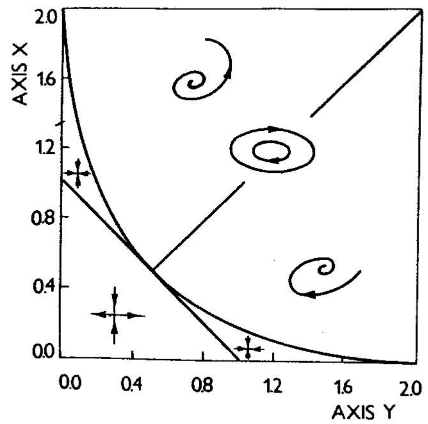
$$\dot{X}_1 = A(1 - X_2)X_1 + R \tag{12. a}$$

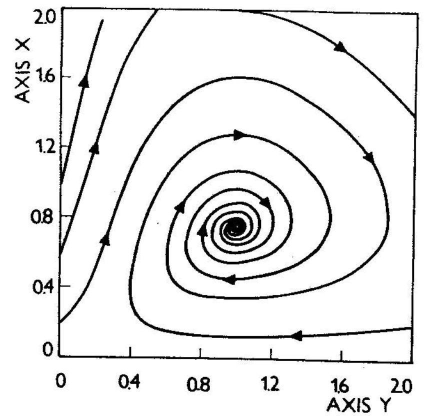
$$\dot{X}_2 = (X_1 - 1)X_2 + S. \tag{12. b}$$

The stationary solutions of this equation system differ from the values $X_1 = 1$ and $X_2 = 1$, that is why also the parameter $p$ characterizing the time evolution of the system after shifting from the balance state will not be entirely imaginary. It is determined by the relations

$$p_{1,2} = -\alpha \pm (\alpha^2 - \beta)^{1/2}, \quad \alpha = \frac{1}{2}[(1 - X_{1s}) - A(1 - X_{2s})],$$
$$\beta = A[1 - (X_{1s} + X_{2s})].$$





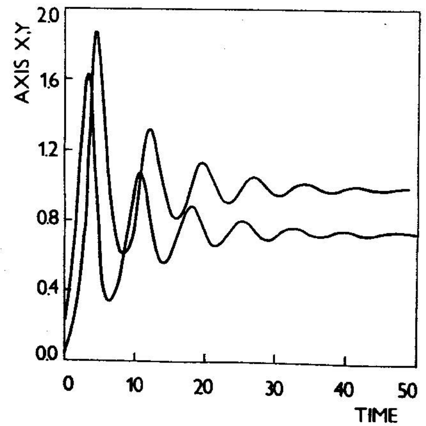




Fig. 2. Parameter: $A = 1$

Fig. 3. Parameters: $A = 1$, $R = 0$, $S = 0.25$

Fig. 4. Parameters: $A = 1$, $R = 0$, $S = 0.25$

The pair of solutions $p_1$ and $p_2$ can obtain the values characteristic for all the types of structures arising generally in two-component systems in dependence on the external-regulation (the parameters $R$ and $S$), i. e. stable and unstable nodes, stable and unstable foci, unstable saddle and oscillation around the stable centre. In dependence on stationary solution values the respective cases are illustrated in the phase plane in Fig. 2.

In Figs. 3 and 4 the oscillation alternation around the stable centre on the stable node is illustrated, in the first case in the phase plane, in the other by means of evolutionary curves.

In this connection a remarkable fact has been found namely that the oscillation process could be suppressed or eventually induced also by a mere constant regulation (i. e. by constant supply or withdrawal of suitable substances) in the system of the Volterra—Lotka type with frequencies that are functions of regulation parameters $(R, S)$. The above mentioned simple analyses provides a very simple model for understanding many interesting qualities of the viable world (e. g. an increase of the heartbeat on receiving certain substances, the beginning or end of characteristic periodical cycles ets.).

It has already been mentioned that the Volterra—Lotka systems with the odd number of components have not a stationary solution, i. e. they cannot be characterized by a time stable "structure". However, it can easily be proved that the situation changes after introducing regulation into such a system. It is, for instance, possible to choose all regulating components zero except one of them and we shall find out immediately that the system has a stationary solution. Thus it follows that all constantly managed Volterra—Lotka systems can be characterized by a time stabilized structure. Some very interesting results can be obtained using a "periodic" regulation. Let us suppose that the regulation parameters $R$ and $S$ change with time according to the real parts of the functions

$$R = Ae^{i\omega\tau} \qquad (13.\ a)$$

$$S = Be^{i(\omega\tau+\varphi)}. \qquad (13.\ b)$$

Then the solutions of equation (12) for small perturbations are

$$x_1 = 1 + \frac{(A^2+B^2\omega^2)^{1/2}}{\omega_0^2-\omega^2}\cos(\omega\tau+\varphi_{11}) + \frac{\omega\omega_0^2(1+4\omega^2)^{1/2}}{(\omega_0^2-\omega^2)} \times \qquad (14.\ a)$$

$$\times \frac{(A^2+B^2\omega^2)}{(\omega_0^2-4\omega^2)}\cos(2\omega\tau+\varphi_{12}) + \ldots$$

$$x_2 = 1 + \frac{(A^2+B^2\omega^2(^{1/2}}{\omega_0^2-\omega^2}\cos(\omega\tau+\varphi_{21}+ \qquad (14.\ b)$$

$$+ \frac{\omega(\omega_0^4+4\omega^2)^{1/2}(A^2+B^2\omega^2)}{(\omega_0^2-\omega^2)(\omega_0^2-4\omega^2)}\cos(2\omega\tau+\varphi_{22}) + \ldots$$

where $\omega_0 = (A)^{1/2}$ and $\varphi_{11}, \varphi_{12}, \ldots, \varphi_{21}, \varphi_{22}$, are phases.

It is seen that two interesting effects arise in this case, i. e. the generation of higher harmonics and the resonance effect when the angle frequency $\omega$ is equal to the basic angle frequency of the system $\omega_0 = (A)^{1/2}$ and the same phenomena at higher harmonics.

This formalism allows to solve also the reciprocal problem: how to choose the regulation parameters if the desired structures have to be realized.

### IV. THE STRUCTURES IN SOCIAL SYSTEMS

The applications of physical formalism used so far for the investigation of social processes have been based upon the process analogy in inorganic and living



systems. It can be shown that considering a certain class of social processes not only analogy but also isomorphism concerned. A social system from the point of view of its members belongs to certain groups according to opinions (e. g. political, religious, etc.) The increase of members in a given group having uniform opinions is determined partly by a natural population growth and partly by the influx of members from the other groups resulting from the interaction $n_i$ of the $i$-group members with the members $n_j$ of the $j$-group. Then the increase is proportional to the product of appropriate concentrations while the $i$-group increase caused by this interaction means a $j$-group decrease. The dynamics of such a social system is described by equation system (2) so that there are Volterra—Lotka systems in question.

The social system belongs to the category of the managed systems. In our case it can be expressed through the choice of certain measures by which additional group members can be obtained from the ranks of other groups or the rank of the "nonorganized". The development of such systems is determined by equation system (5) and the conclusions we have found in the preceding paragraph are valid for them.

Weidlich and Haag [16] examine social systems described by the "master equation" having the following form

$$\dot{n}_{\alpha i} = \sum_j^s n_{\alpha j} w_{ij}^{\alpha} - \sum_j^s n_{\alpha i} w_{ji}^{\alpha} \qquad (15)$$

where $n_{\alpha i}$ is the mean value of individuals of the subsystem $\alpha$, belonging to the $i$-group. The parameters $w_{ij}$ represent the transit probability of individuals from the $j$-group into the $i$-group within a time unit. This probability can be the function of the proper numbers $n_{\alpha i}$ or other parameters.

In [16] this formalism is applied to the investigation of a two-types population migration in two parts of a big city. The probability $w$ is postulated in the form

$$w_{ij} = A\ \exp\,(a_\alpha + b_\alpha n_i + c_\alpha n_j) \qquad (16.\ a)$$

$$w_{ji} = A\ \exp\,(-a_\alpha - b_\alpha n_i - c_\alpha n_j) \qquad (16.\ b)$$

where $a_\alpha$ is a natural preferential factor, $b_\alpha$ is an internal parameter of sympathy (expressing the effort of a given population group to live together in a certain part of the town) and $c_\alpha$ is an external sympathy parameter (expressing the effort of a given population group to live together with the other group in the same part of the town).

When we develop the functions (16) into series and concentrate upon the first approximation, we shall obtain again equations describing Volterra—Lotka systems.

After inducing the notation $u = a_1 + b_1 m_0 x + c_1 n_0 y$, $v = a_2 + b_2 m_0 x + c_2 n_0 y$, where $x = m/m_0 = (m_1 - m_0)/m_0$, $y = n/n_0 = (n_1 - n_0)/n_0$, the equation system can be written with regard to function form (16) in the form ($m_1 + m_2 = 2m_0$, $n_1 + n_2 = 2n_0$)

$$\dot{x} = 2A[\text{sh}(u) - x\,\text{ch}(u)] \qquad (17.\,a)$$

$$\dot{y} = 2A[\text{sh}(v) - y\,\text{ch}(v)]. \qquad (17.\,b)$$

The quantities $m$ and $n$ represent population numbers in respective parts of the town. It can be proved by a numerical solution of equations (17) that all basic types of structures characteristic for two-component Volterra—Lotka systems can exist in the examined systems. Now we shall examine how the situation will change after introducing regulation into such systems. The dynamics of these systems upon external regulation will be described by equations

$$\dot{x} = 2A[\text{sh}(u) - x\,\text{ch}(u)] + R \qquad (18.\,a)$$

$$\dot{y} = 2A[\text{sh}(v) - y\,\text{ch}(v)] + S. \qquad (18.\,b)$$

The mathematical analysis of equations without regulating components shows the position of particular stationary solutions in the phase plane $(x, y)$ which corresponds to points in Fig. 5. Introducing regulation the position of these points,

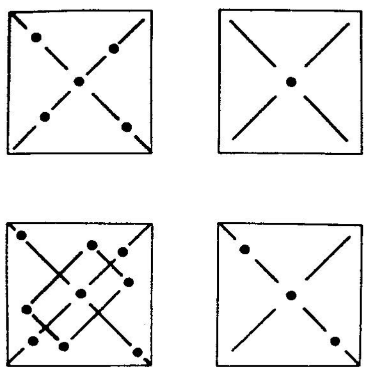


Fig. 5.

their number and character (i. e. if the stable or unstable solution is in question) generally change. In Figs. 6 and 7 we have presented a case of the alteration of three stable points in one excentrically situated stable centre, and in Figs. 8 and 9 "the deformation" of solutions with five points induced by regulation is illustrated.



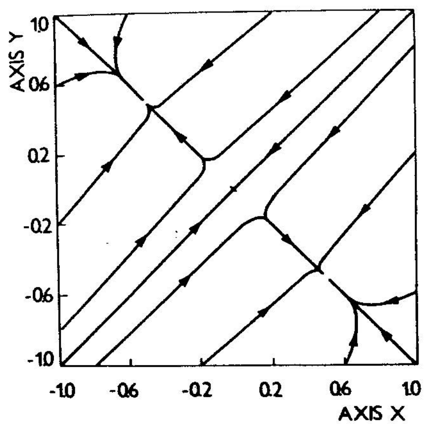



Fig. 6. Parameters: $A = 0.5$, $a_1 = a_2 = 0$, $b_1 = b_2 = 0.3$, $c_1 = c_2 = -0.8$, $R = S = 0$

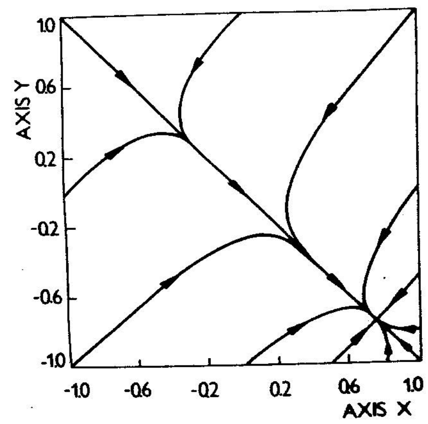



Fig. 7. Parameters: $A = 0.5$, $a_1 = a_2 = 0$, $b_1 = b_2 = 0.3$, $c_1 = c_2 = -0.8$, $R = 0.1$, $S = -0.1$

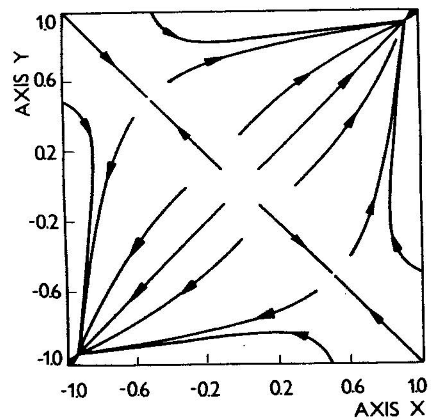



Fig. 8. Parameters: $A = 0.5$, $a_1 = a_2 = 0$, $b_1 = b_2 = 1.5$, $c_1 = c_2 = 0.4$, $R = S = 0$

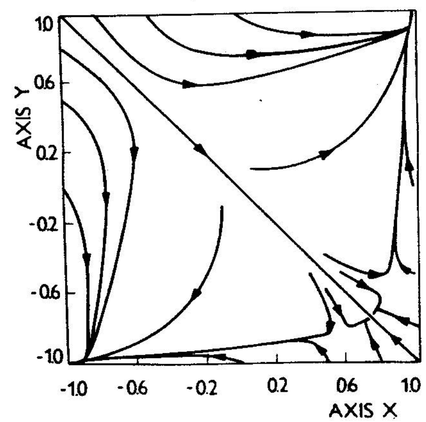



Fig. 9. Parameters: $A = 0.5$, $a_1 = a_2 = 0$, $b_1 = b_2 = 1.5$, $c_1 = c_2 = 0.4$, $R = 0.1$, $S = -0.1$

### V. THE OPTIMAL SCHEME OF REGULATION

We shall now show that the Prigogine principle, which is valid for external (constantly) regulated systems, can be used in the solution of the problem of an optimization of the regulation scheme.

For the sake of simplicity let us suppose a model homogeneous scheme, i. e. a system divided into subsystems according to a constant modulus $x$. Such a system is schematically drawn in Fig. 10 (for $x=3$). The interaction between the subsystems is accompanied by positive entropy production. We suppose that each subsystem from the level $n$ is in interaction with only $x$ subsystems in the $(n+1)$ level. (When considering social systems, this supposition is well understood). Let us designate the positive entropy production by a pair of interactions by the letter "a" and let us suppose it to be the same for all levels. In order to get an analytical

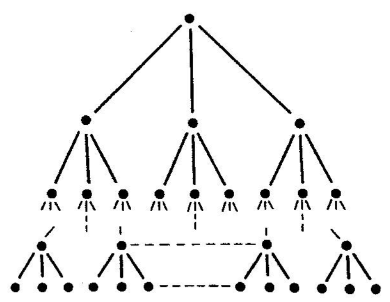



Fig. 10. Parameter: $x=3$

solution we express the entropy production in an elementary cell on the last level as "$ax$" instead of $ax(x-1)/2$. (This simplification is not necessary in a computer calculation). Then the total positive entropy production can be expressed in the form

$$\sigma_1 = a(x + x^2 + \ldots + x^{n+1}) = ax\,\frac{x^{n+1}-1}{x-1}. \qquad (19)$$

The presence of a structure in a system can manifest itself in a better regulation, i.e. in the production of a negative entropy. Let us designate as "b" the negative entropy production on zero level (i. e. the "head" of the system produces "b" negative entropy within a time unit). We have $x$ subsystems on the first level, so we can suppose that the production of an "$xb$" negative entropy is connected with the subsystem. It seems to be more realistic (from the point of view of a possible application to social systems) to use an "ability" coefficient $k$ and express this negative entropy production as "$kxb$". If we do the same on the other levels (supposing $k$ being constant), we obtain a formula for the total negative entropy production in the form

$$\sigma_2 = -b(1 + kx + k^2x^2 + \ldots + k^n x^n) = -b\,\frac{(kx)^{n+1}-1}{kx-1}. \qquad (20)$$

The total (positive and negative) entropy production in the system thus is

$$\sigma = \sigma_1 + \sigma_2 = a\left(x\,\frac{x^{n+1}-1}{x-1} - \alpha\,\frac{(kx)^{n+1}-1}{kx-1}\right), \qquad (21)$$



where $\alpha = b/a$ is a new characteristic constant. An optimal scheme of regulation is determined by a minimum of entropy production, i. e. by a condition $d\sigma/dx = 0$. The solution must fulfil the condition

$$\frac{x^{n+1}-1}{x-1} = N \qquad (22)$$

where $N$ is the total number of members of a given system.

The solution can be simplified by supposing $x^{n+1} \gg 1$ and $(kx)^{n+1} \gg 1$, which is always correct in large systems. Thus the optimal modulus of a given system fulfils the equation

$$-\frac{x \ln^2 x}{k^{(\ln N/\ln x)} \ln N \ln k} = \alpha. \qquad (23)$$

We see that the solution of an optimal regulation of a system needs in a simple case two characteristic constants: $k$ and $\alpha$. It can be seen before the analysis of equation (23) that the system cannot by optimized at $k>1$. At the values $k<1$ the solution can be obtained as the intersection of straight lines $y=\alpha$ with the curves corresponding to the left-hand sides of expression (23). They are drawn in Fig. 11 (for $N=10^6$). The first intersection corresponds to the maximum, the second to the minimum of the entropy production. For some intervals of the values of the constant $k$ and $\alpha$ no solution exists.

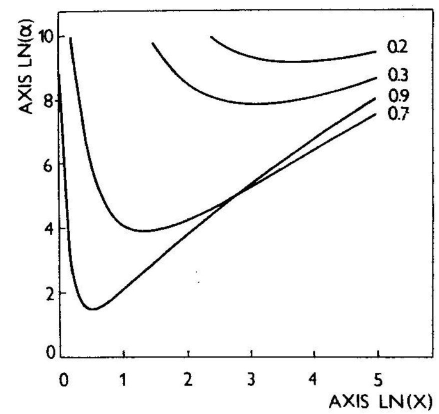



Fig. 11. Parameters: $N=10^6$, $k=0.2$, 0.3, 0.7, 0.9

These results can be applied to social systems. The positive entropy production is connected with every process of directives, planes and aims transfer as well as the process of a mutual coordination of individuals within the framework of one basic "cell". On the other hand, every "head" of the cell has the ability to produce negative entropy. This ability manifests itself through new ideas, aims, etc., which help to improve the organization of the system.

(It results from the observation of more social systems that the two basic constants $k$ and $\alpha$ *are approximately*: $k \approx 0.8$ and $\alpha \approx 50$. It follows then for the system $N = 10^6$ that the optimal modulus is $x \approx 8$—9, which is not a very unrealistic result).

## VI. CONCLUDING REMARKS

It was said that the more mathematics there is in a certain science, the more perfect the science. However, the mathematics as a science cannot solve by itself the problem of the mathematization of the systems of biological and social sciences, as it contains no basis for setting up equations indispensable for the description of the given phenomena. Physics, investigating the laws and principles of the generalized motion not only in inorganic, but also in biological and social systems, could become an integrating factor in this situation. Thus there arises the possibility of applying quantitative methods to the solution of problems also in the branches of sciences of living systems, including social systems.

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