

## THE CHARACTERISTICS OF A SF<sub>6</sub> ARC STABILIZED BY AXIAL GAS FLOW WITH DOMINATING MOLECULAR AND TURBULENT HEAT CONDUCTION<sup>1)</sup>

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Analytical expressions were formulated giving basic macroscopic parameters of a low current arc in axial gas flow. Characteristics of arc in laminar flow are compared with characteristics of a turbulence dominated arc for various values of the turbulence parameter.

### ХАРАКТЕРИСТИКИ ДУГИ SF<sub>6</sub>, СТАБИЛИЗИРОВАННОЙ АКСИАЛЬНЫМ ГАЗОВЫМ ПОТОКОМ С ПРЕОБЛАДАЮЩЕЙ МОЛЕКУЛЯРНОЙ И ТУРБУЛЕНТНОЙ ТЕПЛОПРОВОДНОСТЬЮ

В работе выведены аналитические выражения, определяющие основные макроскопические параметры низкотоковой дуги в аксиальном газовом потоке. Характеристики дуги в ламинарном потоке сравниваются с характеристиками дуги с преобладающей турбулентной для различных значений параметра турбулентности.

#### 1. INTRODUCTION

An electric arc in a supersonic nozzle flow has attracted much attention because of its practical importance in technical applications. Convection energy losses due to axial gas flow together with radiation are controlling mechanisms for a high current arc. At low currents the temperature of the arc plasma is lower and the radiative energy exchange has to be replaced by another efficient mechanisms of energy transport. Recent theoretical and experimental studies have emphasized the role of turbulence enhanced heat conduction as a dominant mechanism [1, 2]. Although the existence of turbulences in the arc was well confirmed by experiments, it is still not clear whether the turbulence or the laminar thermal conductivity is the principal mechanism determining the arc properties [3].

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In this paper analytical expressions are derived giving basic macroscopic arc characteristics of the laminar and the turbulent arc in dependence on the material coefficients of gas. These characteristics are compared to get some insight into the effect of the two principal mechanisms of energy transport.

## II. EQUATIONS OF THE ARC

The energy balance equation of the rotationally symmetric low current arc can be written in the form

$$\sigma E^2 = \rho v_z \frac{\partial h}{\partial z} - \frac{1}{r} \frac{\partial}{\partial r} r(k + k_t) \frac{\partial T}{\partial r}, \quad (1)$$

where  $\sigma$  is electrical conductivity,  $E$  intensity of the electric field,  $\rho$  density,  $v_z$  axial velocity,  $h$  specific enthalpy,  $T$  temperature,  $k$  thermal conductivity. The effect of turbulence is expressed by the coefficient of turbulent thermal conductivity given by the equation [4]

$$k_t = \lambda \rho c_p v_z, \quad (2)$$

where  $\lambda$  is constant with the dimension of length depending on the character of the turbulent flow and  $c_p$  is specific heat.

By integration of (1) radially from 0 to  $R$  (radius of the gas channel heated by the arc) and axially from 0 to  $z$  we obtain the integral power balance

$$IV(z) = \int_0^R 2\pi r \rho v_z h \, dr, \quad (3)$$

where  $I$  is the arc current and  $V(z)$  the potential at the axial position  $z$ . The continuity equation  $\frac{\partial}{\partial z} \rho v_z = 0$  and the assumption  $(\partial T / \partial r)_{r=R} = 0$  were used to obtain (3). Equation (3) is equivalent to the assumption that all power generated by Joule heating is removed by the gas flow, and the heat flux to the wall of the nozzle is negligible. This assumption is true for a low current gas blasted arc, the radius of which is substantially lower than the radius of the nozzle throat.

The arc current is given by Ohm's law as

$$I = \int_0^R 2\pi r E \sigma \, dr. \quad (4)$$

In the narrow channel of the arc radial temperature gradients are substantially higher than axial gradients. We can therefore neglect the term  $\rho v_z \frac{\partial h}{\partial z} = \rho v_z c_p \frac{\partial T}{\partial z}$  in (1) in favour of the thermal conductivity term. This is fully true within the conductive arc core. In the outer nonconductive thermal zone radial gradients are

lower and our assumption is strictly valid only in the region of the nozzle throat, where axial temperature distribution has its maximum. As this region decisively determines the arc properties, we can neglect the enthalpy flow term also in the thermal zone. In the nozzle throat, axial gas velocity is equal to the velocity of the sound  $c$ , in (1)–(3) we can therefore substitute  $v_z = c$ . Turbulent conductivity  $k_t$  is then the function of pressure and temperature only and the heat flux potential  $S$  can be introduced by the usual equation

$$S = \int_{T_0}^T (k + k_t) \, dT, \quad (5)$$

where  $T_0$  is the temperature of cold gas. From (1) we then obtain

$$\sigma E^2 = -\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial S}{\partial r}. \quad (6)$$

The effect of the gas properties on the arc behaviour is given by the material functions  $S(T)$ ,  $\sigma(T)$ , and  $F(T) = \rho c h$ . The dependence of  $\sigma$  and the enthalpy flow  $F$  on  $S$  for the SF<sub>6</sub> plasma is given in Figs. 1 and 2. In our calculations, we approximate the functions  $\sigma = \sigma(S)$  by straight lines  $\sigma = B(S - S_c)$  for  $S \geq S_c$  and  $\sigma = 0$  for  $S < S_c$ , where  $S_c$  is the value of  $S$  at the boundary of the conductive core ( $T \sim 4000$  K). The functions  $F(S)$  were approximated by  $F = A \cdot S$  for  $S < S_c$  and  $F = F(S_c) = \text{const.}$  for  $S \geq S_c$  (dash-and-dot lines in Figs. 1 and 2). The effect of this approximation on the results of calculations will be discussed later.

The solution of (6) gives the radial profile of  $S$ , which is equal to the Bessel function in the conductive arc core and to the logarithmic function in the outer thermal region. By evaluating the integrals in (3) and (4) and substituting  $dV/dz = E$  we obtain

$$V(z) \frac{dV}{dz} = \frac{\delta_1}{4J_1(\delta_1)} \frac{F[1 - (R/R_c)^2]}{\sigma \ln(R_c/R)}, \quad (7)$$

where  $R_c$  is the radius of the conductive core,  $J_1$  is the Bessel function of the first order and  $\delta_1$  is the root of the Bessel function  $J_0$ .

The integration of (7) by the method of Perkins and Frost [5] together with the solution of (6) gives for the total arc voltage

$$U = \frac{\delta_1}{2} \left( V_1 + \frac{1}{V_1} \lambda_u \right) \left[ \frac{L_u F(S_u - S_c)}{\sigma S_c} \left( \exp \frac{2S_c}{J_1(\delta_1) \delta_1 (S_u - S_c)} - 1 \right) \right]^{1/2} \quad (8)$$

where  $S_u$  is the value of  $S$  at the arc axis,  $L_u$  is the length of the upstream arc section to the nozzle throat,  $\lambda_u = L_u/l_u$ ,  $l_u$  is the length of the downstream section.  $V_1$  is the dimensionless factor depending on the gas properties [5]. The intensity of the electric field  $E$ , the arc current  $I$  and the radius  $R_c$  in the nozzle throat are then given by

$$E = \frac{\delta_1}{2V_1} \left[ \frac{F(S_0 - S_c)}{L_0 \delta S_c} \left( \exp \frac{2S_c}{J_1(\delta_1) \delta_1 (S_0 - S_c)} - 1 \right) \right]^{1/2} \quad (9)$$

$$I = 4\pi J_1(\delta_1) \delta_1 V_1 \left[ \frac{L_0 \delta S_c (S_0 - S_c)}{2S_c} \left( \exp \frac{2S_c}{J_1(\delta_1) \delta_1 (S_0 - S_c)} - 1 \right) \right]^{1/2} \quad (10)$$

$$R_c = 2V_1 \left[ \frac{L_0 S_c}{F \left( \exp \frac{2S_c}{J_1(\delta_1) \delta_1 (S_0 - S_c)} - 1 \right)} \right]^{1/2} \quad (11)$$

### III. RESULTS OF CALCULATIONS AND DISCUSSION

In this chapter, parameters of the arc calculated from (8)–(11) are presented for the SF<sub>6</sub> arc,  $p = 0.4$  MPa,  $L_0 = 5 \times 10^{-2}$  m,  $\lambda_d = 1$ .

In Fig. 3 the temperature at the arc axis is plotted against the arc current for the case of the laminar arc ( $k_0 = 0$ ) and for the turbulence controlled arc ( $k = 0$ ) for several values of turbulence length  $\lambda$ . The full lines correspond to the assumption of the constant enthalpy flow  $F = F(S_c)$  in the conductive arc core. The broken lines were obtained for the case when for each temperature  $T$  the function  $F(S)$  for  $S > S_c$  was approximated by the constant  $F = F(S_0)$ , as it is demonstrated by the broken line in Fig. 1. Curves representing the actual temperature dependence of  $F$  would lie between these two curves.

The strong dependence of the arc temperature on the turbulence length can be seen in Fig. 3. Temperatures corresponding to the experimental values would be obtained for  $\lambda$  in the interval  $5 \times 10^{-5} - 10^{-4}$  m. Lower values of  $\lambda$  as well as the assumption of pure laminar heat conduction result in very high temperatures at low

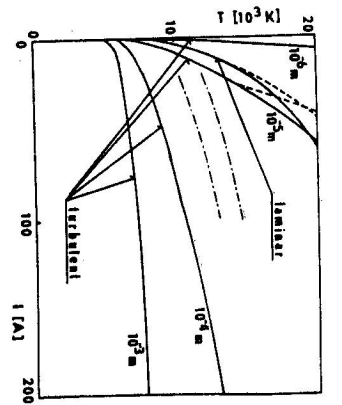


Fig. 3. Axial arc temperature  $T$  as a function of current  $I$  for laminar and turbulence dominated arc. SF<sub>6</sub>,  $p = 0.4$  MPa,  $L_0 = 5 \times 10^{-2}$  m,  $\lambda_d = 1$ , turbulence length  $\lambda$  as parameter. Dash-and-dot lines — experimental values [8].

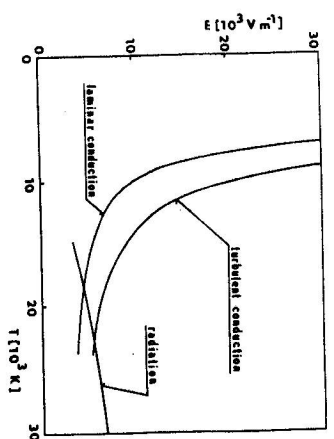


Fig. 4. Components of total electric field corresponding to various energy transport mechanisms as functions of arc axial temperature. SF<sub>6</sub>,  $p = 0.4$  MPa,  $L_0 = 5 \times 10^{-2}$  m,  $\lambda_d = 1$ .

currents. Values of turbulent parameters for the SF<sub>6</sub> arc as given in [1, 6] are equivalent to  $\lambda = 7 \times 10^{-6} - 2.5 \times 10^{-5}$  m, which seems to be rather small for the observed arc temperatures.

In Fig. 4 the electric field intensity  $E$  calculated from (11) for the laminar and the turbulent arcs is plotted against axial temperature. The third curve is for the arc controlled by radiative power losses only, an estimate for our conditions based on data of Liebermann and Lowke [7]. It can be seen that radiation noticeably influences the arc properties from a temperature of  $T \sim 15$  000 K, the effect of radiation predominates for  $T \sim 20$  000 K.

Fig. 5 gives the calculated dependence of the radius  $R_c$  on the current  $I$ . Turbulence and laminar models give similar resulting curves, also the value of turbulence length  $\lambda$  has little effect on the arc radius.

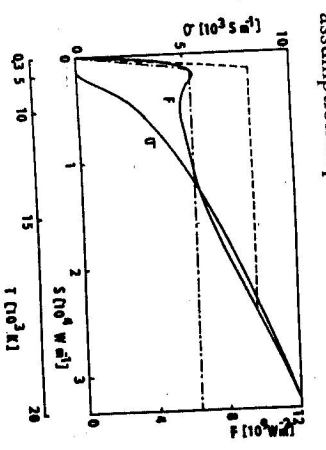


Fig. 1. Electrical conductivity  $\sigma$  and enthalpy flow  $F = gch$  as a function of laminar heat flux potential  $S$  ( $k = 0$ ). SF<sub>6</sub>,  $p = 0.4$  MPa (solid lines). Approximation (dash-and-dot lines) — see text.

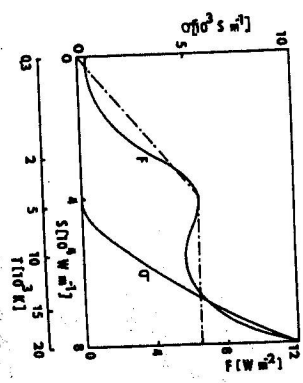


Fig. 2. Electrical conductivity  $\sigma$  and enthalpy flow  $F = gch$  as functions of turbulent heat flux potential  $S$  ( $k = 0$ ). SF<sub>6</sub>,  $p = 0.4$  MPa,  $\lambda = 10^{-5}$  m (solid lines). Approximation (dash-and-dot line) — see text.

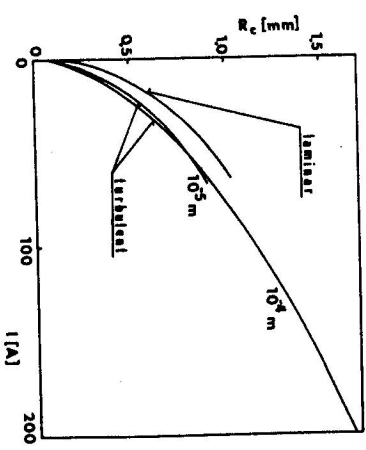


Fig. 5. Radius of conductive arc core  $R_c$  as a function of arc current  $I$  for laminar and turbulent arc. SF<sub>6</sub>,  $p = 0.4$  MPa,  $L_0 = 5 \times 10^{-2}$  m,  $\lambda_d = 1$ , turbulence length  $\lambda$  as a parameter.

#### IV. CONCLUSIONS

Calculations based on the simple model of turbulent and laminar arcs allow to compare the effect of possible mechanisms of heat transport on the arc properties. Results corresponding to the observed values of arc temperature were obtained for the model of a turbulent arc with a high value of turbulence length  $\lambda$ . An assumption of molecular thermal conductivity as the principal transport mechanism leads to unrealistic values of the arc temperature. Conductive energy transfer predominates over radiation up to relatively high temperatures, in our case approximately to 20 000 K.

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