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INFLUENCE OF THE TWO-STAGE IONIZATION ON RADIAL PROFILES OF CHARGED AND METASTABLE PARTICLES[1])

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A steady-state diffusion of charged and metastable particles controlled by two-stage ionization and destruction of metastable atoms by electron impact is studied. A preliminary analysis of the diffusion equations by the approximation of the first fundamental diffusion mode has shown that for a given fraction of the ionizing rate due to two-stage processes and for given values of other rate coefficients, two different values of particle concentration are possible. Only the higher of them is stable against small perturbations. As a result a minimum value for the electron concentration is necessary to obtain a stable plasma column. The numerical solution has confirmed this result and a family of radial profiles of electrons and metastables was obtained.

ВЛИЯНИЕ ДВУХСТУПЕНЧАТОЙ ИОНИЗАЦИИ НА РАДИАЛЬНЫЕ РАСПРЕДЕЛЕНИЯ ЗАРЯЖЕННЫХ И МЕТАСТАБИЛЬНЫХ ЧАСТИЦ

В работе исследована установившаяся диффузия заряженных и метастабильных частиц, сопровожданная двухступенчатой ионизацией и распадом метастабильных атомов посредством соударений с электронами. Предварительный анализ уравнений диффузии в приближении первой основной моды показывает, что для данной относительной скорости ионизации, обусловленной двухступенчатыми процессами, и для данных значений других коэффициентов возможны два разных значения концентрации частиц. Только большее из этих значений обладает стабильностью относительно малых возмущений. В связи с этим для достижения устойчивого столба плазмы необходимо добиться минимального значения концентрации электронов. Это подтверждают также приведенные численные расчеты. Кроме того, получены радиальные распределения электронов и метастабильных частиц.

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I. INTRODUCTION

The two-stage (stepwise) ionization is an important elementary process in explaining some phenomena in the gas discharge (see for example [1, 2]). In spite of this there is no steady-state solution of the coupled diffusion equation describing a transport of charged and metastable particles to the discharge tube wall. This situation can be explained by difficulties in the numerical solution of these nonlinear equations for given boundary conditions. In the presented paper we shall give a preliminary information about the results obtained.

II. THE MODEL

We are concerned here with the following processes leading to the production and the destruction of charged and metastable particles:

$$X+e \xrightarrow{\alpha_i} X^+ + 2e; \quad X+e \xrightarrow{\alpha_m} X_m + e;$$

$$X^+ + e \xrightarrow{\varepsilon} X; \quad X_m + e \begin{smallmatrix} \xrightarrow{\beta_m} X+e \\ \xrightarrow{\beta_i} X^+ + 2e \end{smallmatrix}$$

The first two processes describe single-stage ionization and direct excitation by electron impact or excitation through the states optically connected with the metastable states. The third process is due to volume recombination between electrons and positive ions, and the last process corresponds to the destruction of metastables by electron impact or to the two-stage ionization.

If n_e, n_m, n_g are the respective concentrations of electrons, metastable and ground state atoms, the diffusion equations become

$$-\frac{1}{r} D_a \frac{\mathrm{d}}{\mathrm{d}r}\left(r \frac{\mathrm{d}n_e}{\mathrm{d}r}\right) = \alpha_i n_g n_e + \beta_i n_m n_e - \varepsilon n_e^2 \tag{1}$$

$$-\frac{1}{r} D_m \frac{\mathrm{d}}{\mathrm{d}r}\left(r \frac{\mathrm{d}n_m}{\mathrm{d}r}\right) = \alpha_m n_g n_e - \beta_m n_m n_e, \tag{2}$$

where D_a, D_m are the ambipolar and metastable atom duffusion coefficients. The boundary conditions $\mathrm{d}n_e/\mathrm{d}r = \mathrm{d}n_m/\mathrm{d}r = 0$ at $r=0$ and $n_e = n_m = 0$ at the wall were used.

III. A PRELIMINARY ANALYSIS OF THE MODEL

As the excitation and ionization rates depend strongly on the energy distribution of the electrons, we try to eliminate them by introducing the central electron concentration n_{e0} and the fraction of ionizing rate due to the two-stage process $s = \beta_i n_{m0} n_{e0} / \alpha_i n_g n_{e0} = \beta_i n_{m0} / \alpha_i n_g$ (n_{m0} is the metastable concentration on the axis).

Equations (1) and (2) can be approximated in terms of the first fundamental diffusion mode of the characteristic length $\Lambda = R/\mu$ (μ is the first root of the Bessel function and R the inner radius of the tube) to give $D_a/\Lambda^2 = \alpha_i n_g + \beta_i n_{m0} - \varepsilon n_{e0}$ and $(D_m/\Lambda^2) n_{m0} = (\alpha_m n_g - \beta_m n_{m0}) n_{e0}$. By introducing $\varrho = r/\Lambda$; $\gamma_0 = \beta_m \Lambda^2 / D_m$; $\lambda_0 = \varepsilon \Lambda^2 / D_a$ we have $\frac{\alpha_i n_g \Lambda^2}{D_a} = \frac{1+\lambda_0 n_{e0}}{1+s}$; $\frac{\beta_i \Lambda^2}{D_a} = \frac{s}{1+s} \frac{1+\lambda_0 n_{e0}}{n_{m0}}$; $n_{m0} = C_m \frac{\gamma_0 n_{e0}}{1+\gamma_0 n_{e0}}$. Here $C_m = \alpha_m n_g / \beta_m$ is an equilibrium number density of metastables. The diffusion equations then become

$$-\frac{1}{\varrho} \frac{\mathrm{d}}{\mathrm{d}\varrho}\left(\varrho \frac{\mathrm{d}n_e}{\mathrm{d}\varrho}\right) = \left[1 + \lambda_0 (n_{e0} - n_e) + s \frac{1+\lambda_0 n_{e0}}{1+s} \left(\frac{n_m}{n_{m0}} - 1\right)\right] n_e \tag{3}$$

$$-\frac{1}{\varrho} \frac{\mathrm{d}}{\mathrm{d}\varrho}\left(\varrho \frac{\mathrm{d}n_m}{\mathrm{d}\varrho}\right) = \gamma_0 (C_m - n_m) n_e; \quad 0 \leqslant \varrho \leqslant \mu. \tag{4}$$

It can be easily verified in terms of the first diffusion mode that these equations admit steady-state solutions: besides the solution $n_{e1} = n_{e0}$ there is another solution, i.e. $n_{e2} = (s - \lambda_0 n_{e0})/(1+s)\,\gamma_0 \lambda_0 n_{e0}$.

IV. SOLUTION STABILITY

The next step in the analysis is to determine the stability properties of the stationary states. Let us investigate the response of the plasma to the infinitesimal perturbation δn_e and δn_m from the concentrations n_{e0} and n_{m0} by means of time varying diffusion equations. The linearization of these equations leads to the following system

$$\frac{\partial \delta n_e}{\partial \tau} - \frac{1}{\varrho} \frac{\partial}{\partial \varrho}\left(\varrho \frac{\partial \delta n_e}{\partial \varrho}\right) = A \delta n_e + B \delta n_m$$

$$\frac{\partial \delta n_m}{\partial \tau} - q \frac{1}{\varrho} \frac{\partial}{\partial \varrho}\left(\varrho \frac{\partial \delta n_m}{\partial \varrho}\right) = C \delta n_e + D \delta n_m,$$

where $q = D_m/D_a$; $\tau = t D_a / \Lambda^2$; $A = 1 - \lambda_0 n_e$; $B = s \frac{1+\lambda_0 n_{e0}}{1+s} \frac{n_e}{n_{e0}}$; $C = q n_m / n_e$ and $D = -q \gamma_0 n_e$. These equations admit solutions of the form

$$\delta n = [C_1 \exp(k_1 \tau) + C_2 \exp(k_2 \tau)] J_0(\varrho),$$

provided k_1 and k_2 satisfy the characteristic equation

$$k^2 + (1 + q - A - D) k = BC - AD + q(A-1) + D.$$

We obtain a stable solution when $k_1 < 0$ and $k_2 < 0$. After some manipulation one finds

$$n_e^2 > n_{e1} n_{e2}.$$

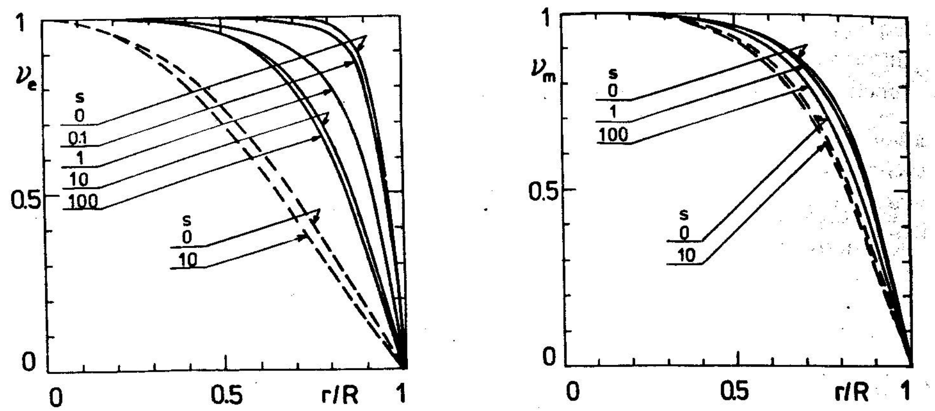

Fig. 1. The relative radial distribution of electrons (a) and metastables (b) for $\lambda = 100$, $\gamma = 10$ (solid line) and $\lambda = 1$, $\gamma = 10$ (dashed line) as a function of the parameter s (the fraction of ionizing rate due to two-stage processes). The stability criterion (6) is fulfilled.

It is obvious that only the higher of the roots n_{e1} and n_{e2} is stable against small perturbations. Thus, a minimum value for the electron concentration $n_{e\,min}$ is necessary to obtain a stable plasma column:

$$n_{e0} > n_{e\,min} = \frac{2s/\lambda_0}{1+[1+4(1+s)\gamma_0 s/\lambda_0]^{1/2}}. \tag{5}$$

V. NUMERICAL SOLUTION

By introducing the dimensionless variables $\nu_e = n_e/n_{e0}$, $\nu_m = n_m/n_{m0}$, $\lambda = \lambda_0 n_{e0}$, $\gamma = \gamma_0 n_{e0}$ equations (3) and (4) can be written

$$-\frac{1}{\varrho}\frac{d}{d\varrho}\left(\varrho\frac{d\nu_e}{d\varrho}\right) = \left[1+\lambda(1-\nu_e)+s\frac{1+\lambda}{1+s}(\nu_m-1)\right]\nu_e = E(\nu_e, \nu_m)$$

$$-\frac{1}{\varrho}\frac{d}{d\varrho}\left(\varrho\frac{d\nu_m}{d\varrho}\right) = [1+\gamma(1-\nu_m)]\,\nu_e = M(\nu_e, \nu_m).$$

The equations were solved by an iterative technique according to the scheme

$$-\frac{1}{\varrho}\frac{d}{d\varrho}\left(\frac{\varrho d\nu_m^{(k+1)}}{d\varrho}\right) = M(\nu_e^{(k)}, \nu_m^{(k+1)})$$

$$-\frac{1}{\varrho}\frac{d}{d\varrho}\left(\varrho\frac{d\nu_e^{(k+2)}}{d\varrho}\right) = E(\nu_e^{(k+2)}, \nu_m^{(k+1)}).$$

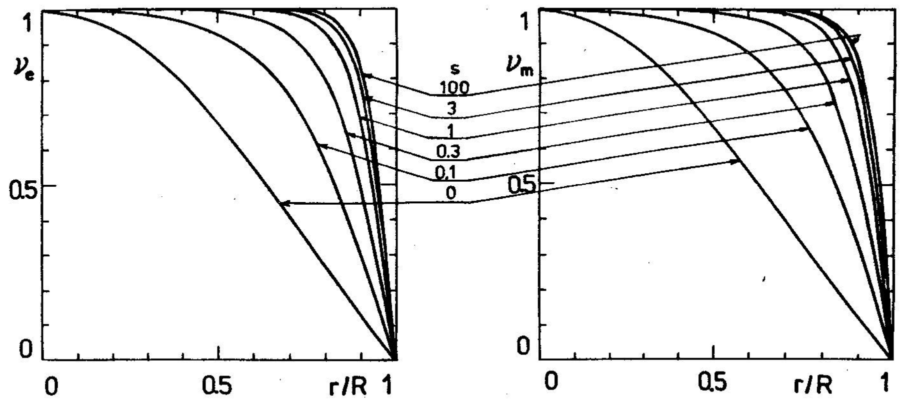

Fig. 2. The relative radial distribution of electrons (a) and metastables (b) for $\lambda = 0.01$, $\gamma = 0.001$ as a function of the parameter s. The stability criterion (6) is not fulfilled.

Each differential equation was linearized with respect to the unknown variable and numerically solved by the finite-difference method. The stability criterion (5) takes the form

$$\lambda - \frac{s}{1+\gamma(1+s)} > 0. \tag{6}$$

If this criterion is fulfilled, the normalized number densities ν_{e0} and ν_{m0} at the centre $\varrho = 0$ tend to unity. In an opposite case the values ν_{e0} and ν_{m0} are much greater than 1.

Table 1

The normalized number density ν_{e0} and ν_{m0} at the centre of the plasma column as a function of the parameters s for $\lambda = 0.01$ and $\gamma = 0.001$

s	0	0.1	0.3	1	3	100
ν_{e0}	1.41	9180	23400	50600	75900	100000
ν_{m0}	1.41	993	1000	1000	1000	1000

Fig. 1 illustrates the shapes of the radial distributions of electrons and metastables when the stability criterion (6) is true. The case when the criterion is not fulfilled is shown in Fig. 2. The respective values of ν_{e0} and ν_{m0} are presented in Table 1.

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