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DYNAMICAL MASS SCALE IN MASSLESS SCALAR THEORY

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The dependence of the dynamic mass scale in scalar theory on the parameters of the theory is investigated using the Renormalization Group. We find by two simple models that it is possible to do the tunning of the parameters to get the specific mass ratios. Such a type of investigation is useful for the construction of the Gildener-Weinberg type of the symmetry breaking pattern in the Grand Unified theories with no gauge hierarchy problem.

ШКАЛА ДИНАМИЧЕСКИХ МАСС В БЕЗМАССОВОЙ СКАЛЯРНОЙ ТЕОРИИ

В работе на основе ренормализационной группы исследована зависимость шкалы динамических масс от параметров в скалярной теории. Показано, что в двух простых моделях возможно подобрать параметры таким образом, чтобы, получить характерное отношение масс. Такой результат полезен с точки зрения построения нарушения симметрии типа Гильденера–Вайнберга в теориях великого объединения, в которых неприсуствует проблема калибровачной иерархии.

I. INTRODUCTION

Today's theories of almost all interactions are the gauge theories with a spontaneously broken symmetry, what gives masses of gauge bosons and fermions in such a way which does not destroy renormalizability. The only known possibility to do this in such a way which gives perturbatively calculable predictions is to introduce elementary scalar fields. The whole business of the symmetry breaking then depends on the behaviour of the scalar potential, or more precisely (including also radiative corrections), on the behaviour of the effective scalar potential.

It is well known that the existence of a nontrivial minimum of the effective potential may depend crudially on radiative ocrrections [1 . Perturbative calcula-

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tions of these corrections are not always trivial and even not possible. In this paper we want to show how the knowledge of the dependence of the parameters of the theory on the renormalization point may help to decide about the presence of a nontrivial minimum in the effective potential (i.e. about the presence of the spontaneous symmetry breaking in the theory) using only perturbative methods [2].

If one starts with the Lagrangian with renormalized parameters defined at some arbitrarily chosen renormalization point, then it usually happens that the radiative correction to the effective potential cannot be calculated (perturbatively) directly. However, in some cases it is possible to find such a new renormalization point that the perturbative calculations can be done directly and reliably. Then one can (looking at the effective potential) recognize where the true position of the ground state is and how far the symmetry is broken.

If the symmetry breaking is stronger than it can be seen in the tree approximation, we have a further symmetry breaking, due to the dynamics of the scalar sector. Such an effect of the dynamic symmetry breaking may be of great importance [3] in the Grand Unified Theories [4]. It was suggested [2, 3] that such a symmetry breaking pattern may provide a natural explanation of the gauge hierarchies problem in GUT's.

The aim of the paper is to show explicitly on two simple pure scalar models how a computation of a dynamic symmetry breaking works and to investigate the dependence of the ratio of an initial and the new renormalization mass scales on the parameters of the Lagrangian defined at the initial renormalization point.

The paper is arranged as follows. In Sect. II we comment in more detail the connection of the investigated problems with the questions of gauge hierarchies following closely E. Gildener, S. Weinberg [3] and S. Weinberg [2]. In Sect. III and IV the description of the models and solutions of the renormalization group equations (RGE) are presented. The conclusions are given in Sect. V and the renormalization of the scalar theory and the derivation of RGE is briefly discussed in the Appendix.

II. HIERARCHIES OF MASS SCALES

It is well known that in GUT's the hierarchy of the masses of the superheavy and intermediate vector bosons is of the order $10^{13}-10^{15}$ [5]. To get such a hierarchy using Higgs phenomenon is not easy because of the radiative corrections. If one wants to compensate the effect of the radiative corrections on the hieratrchy one must define the renormalized parameters in the Lagrangian with an extreme accuracy [6], which seems to be highly unnatural.

There is an interesting possibility of the sequential symmetry breaking pattern which may naturally lead to the appropriate hierarchies [2, 3]. S. Weinberg

noticed [2] that if the parameters of the theory defined at the renormalization point M -of the order of the Grand Unification Mass- are constrained in such a way that after the spontaneous symmetry breaking as given by the tree approximation at least one scalar field is left massless or with a small mass, then there is a deeper minimum of the effective potential and the symmetry is consequently further broken. This statement follows from the decoupling theorem [7] and from the renormalization group considerations.

As we proceed to lower energies the physics is described by the effective Lagrangian, all the superheavy fields droping out and all the coupling constants appropriately renormalized. The relevant effective potential is then [2]

$$U(\eta)=U(0)+\frac{1}{4!}f_{ijkl}(M)\eta_i\eta_j\eta_k\eta_l+ \qquad (1)$$
$$+\text{radiative corrections}$$

where the f's are coupling constants, the M is the renormalization point and the η_i are the massless scalar fields. If we succeed in finding such a renormalization mass scale k_0 that

$$\min_{n_i n_i=1}\,[f_{ijkl}(k_0)n_in_jn_kn_l]=0 \qquad (2)$$

with n_i being the unit vector in the η-field space, then the minimum of the effective potential (1) is given by the radiative corrections only [3]. In the one loop approximation these radiative corrections become [1]

$$A\eta^4\left(\ln\frac{\eta}{k_0}-25/12\sigma\right) \qquad (3)$$

with $A\sim 0(f^2)$ and positive, the η being the length of the field vector η_i. Thus (1) has the nontrivial minimum at $\eta_i=\eta^*n_i^*$ with the n_i^* given by (2) and $\eta^*=k_0e^{11/6}$, which can be found by minimization of (3).

The important point is that the perturbative methods for such fields are still permissible and the whole scheme is consistent [1]. The relevant quantity for the hierarchy of the mass scales in the theory is the ratio M/k_0, which is here completely governed by the RGE for the running coupling constants and its implicit definition through the constraint (2). A slow logarithmic dependence of the running coupling constants on the renormalization mass scale may provide an appropriate ratio of the mass scales of the fields in the theory without any unnatural restrictions on the parameters.

III. MODELS

The most simple way of investigating the ratio of the mass scales due to the dynamic effects is to have a look at the pure scalar theory with more massless fields.

The dependence of the coupling constants on the renormalization point is given in the one loop approximation by a system of coupled first order differential eqs. (RGE, see Appendix)

$$16\pi^2 \frac{\mathrm{d}f_{ijkl}(t)}{\mathrm{d}t} = f_{ijmn}f_{mnkl} + f_{ikmn}f_{mnjl} + f_{ilmn}f_{mnkj} \qquad (4)$$

with $t = \ln M$. According to Sect. II the presence of the symmetry breaking in such a theory depends only on the existence of the new renormalization scale k_0, which is implicity defined by eq. (2) and RGE (4). We shall call condition (2) Gildener's criterion. This criterion of the presence of the dynamic symmetry breaking can be also formulated geometrically: there exist stationary directions of the tree potential in the space of the scalar fields, with the positive matrix of the second derivatives (i.e. minimum) and in which moreover the potential has the value zero. The radiative corrections are able (see (3)) to produce a dip in this (in general not only one dimensional) „valley".

In the following we shall investigate two SO (2) invariant models, the first with a vector and scalar representation and the second with N vectors.

1. In the model with the $\{2\}+\{1\}$ representation the most general potential has three independent coupling constants

$$V(\Phi) = \frac{1}{4!}\left[f_1(\Phi_1^2+\Phi_2^2)^2 + f_2\Phi_3^4 + f_{12}(\Phi_1^2+\Phi_2^2)\,\Phi_3^2\right].$$

The RGE for $f_i(t)$ $i = 1, 2, 12$ are according to (4)

$$16\pi^2 \frac{\mathrm{d}f_1(t)}{\mathrm{d}t} = \frac{10}{3} f_1^2 + \frac{1}{12} f_{12}^2$$

$$16\pi^2 \frac{\mathrm{d}f_2(t)}{\mathrm{d}t} = 3f_2^2 + \frac{1}{6} f_{12}^2$$

$$16\pi^2 \frac{\mathrm{d}f_{12}(t)}{\mathrm{d}t} = \frac{1}{3}(4f_1 + 3f_2 + 2f_{12})\, f_{12}.$$

The problem is completely specified if the initial values $f_i(M) \equiv f_i^M$ are given. Gildener's criterion (2) can be written as

$$\left.\frac{\partial V}{\partial \Phi_i}\right|_{\Phi_i = \Phi n_i^*} = 0, \quad V(\Phi n^*) = 0 \quad V(\Phi n_i) \geq 0$$

for all n_i; $n_i n_i = 1$. The criterion is satisfield if such a k_0 exist that for the $f_i(k_0) \equiv f_i^G$ $i = 1, 2, 12$ relations $f_{12}^G < 0$, and $4f_1^G f_2^G = (f_{12}^G)^2$ holds.

2. The most general SO (2) invariant potential with $N \times \{2\}$ representation is

$$V(\Phi_i) = \frac{1}{4!}\left[f_{self}^a(\Phi_i^a\Phi_i^a)^2 + f_{mix}^{ab}(\Phi_i^a\Phi_i^a)(\Phi_j^b\Phi_j^b) + f_{mix}^{ab}(\Phi_i^a\Phi_i^b)^2\right],$$

where a is the vector index with N values and $i, j = 1, 2$. We restricted our investigation to the simplified symmetrized model with $f_{mix}^{ab} \equiv f_{mix}$, $f_{self}^a \equiv f_{self}$ and $f^{ab} \equiv 0$ for all a, b. Then the RGE become

$$16\pi^2 \frac{\mathrm{d}f_{self}(t)}{\mathrm{d}t} = \frac{10}{3} f_{self}^2 + \frac{N-1}{6} f_{mix}^2$$

$$16\pi^2 \frac{\mathrm{d}f_{mix}(t)}{\mathrm{d}t} = \frac{1}{3}(8f_{self} + Nf_{mix})\, f_{mix}$$

with the initial values $f_i(M) \equiv f_i^M$ $i = self, mix$. To satisfy Gildener's criterion one needs such a k_0 that $f_i(k_0) \equiv f_i^G$ obey the relations $f_{self}^G > 0$ and $f_{self}^G = -\frac{1}{2}(N-1)\, f_{mix}^G$.

IV. SOLUTION OF THE RGE

In the context of the investigation of the symmetry breaking there arose three questions:

What are the restrictions which have to be imposed on the initial values of the coupling constants f_i^M in order to make it possible to find such a renormalization s ale k_0 that Gildener's criterion holds and the symmetry breaking occurs.

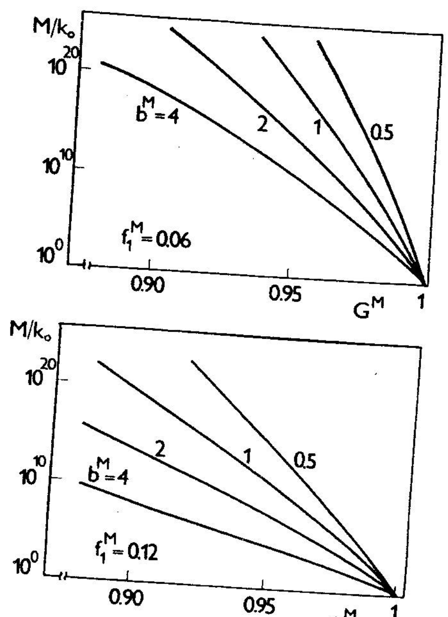

Fig. 1. The model with vector and scalar. The dependence of the hierarchy of the mass scales M/k_0 on the parameters of the potential is shown. f_1^M, f_2^M, f_{12}^M are reparametrized by f_1^M, $b^M = f_2^M/f_1^M$ and $G^M = \sqrt{4f_1^M f_2^M}/f_{12}^M$.

What can be said about the dependence of the ratio M/k_0 on the initial parameters and in particular whether the obtained ratio is stable with respect to the small changes of these initial parameters.

Whether there is a possibility to choose the initial values f_i^M in order to obtain some specific ratio M/k_0 (this is relevant to the problem of the hierarchies).

In the model with the $\{2\} + \{1\}$ representation the RGE for the running coupling constants were solved numerically on a computer. The results are given in Fig. 1. We can see that for the investigated part of the space of the initial coupling constants f_i^M one can always find such a k_0 that Gildener's criterion holds and the dynamic symmetry breaking evidently occurs. There is in general no problem to fit the parameters in such a way that the ratio M/k_0 of the desired order 10^{10}—10^{20} is obtained.

In the model with the $N\times\{2\}$ representation we succeeded in finding the solution of the RGE in the parametric form:

$$f_{self}(w;f_{self}^M)=A^M f_{self}^M(w-2/(N-1))^{-1}(w-2)^{1/5}w^{-1/5}$$

with $A^M=(w_M-2/(N-1))(w_M-2)^{-1/5}w_M^{1/5}$
and for the renormalization mass

$$k(w;w_M,f_{self}^M)=M\exp(6\times16\pi^2 I(w,w_M)/A^M(N-1)f_{self}^M),$$

where $I(w;w_M)=\int_{|w_M|}^{|w|}dx(x-2)^{-6/5}x^{-4/5}$.

The parameter w is defined by $w\equiv f_{mix}(k)/f_{self}(k)$. Gildener's criterion becomes

$$w=-2/(N-1)\equiv w_N^G.$$

A new renormalization mass scale resulting from Gildener's criterion is then given by

$$k_0=k(w_N^G;w^M,f_{self}),$$

where the w^M and f_{self}^M are the initial values of the parameters defined at the renormalization point M. The results are presented in Fig. 2.

V. CONCLUSIONS

We summarize here the main results of our investigation. They were obtained by solving two simple models, one may however expect similar results to hold in other similar situations as well.

Gildener's criterion holds and the symmetry is always broken when the initial parameters are from the appropriate region of the coupling constants space, which is quite broad. (E.g. in the model with N vectors if $f_{mix}^M<0$ and $-f_{mix}^M<\frac{1}{2}(N-1)f_{self}^M$).

Being in the appropriate region of the coupling constants space it is presumably always possible to arrange the desired ratio M/k_0 if one has the freedom to change continuously one parameter only.

The ratio M/k_0 is stable with respect to small changes of the parameters of the Lagrangian defined at the inital renormalization point M. This property might be important because of the problems with stability in other approaches to introducing the needed hierarchy of the mass scales in GUT's [6].

For the fixed f_{self}^M there exist an upper bound on the ratio M/k_0

$$M/k_0<\exp(-h(N)/f_{self}^M)$$

with $h(N)$ given in Fig. 2.

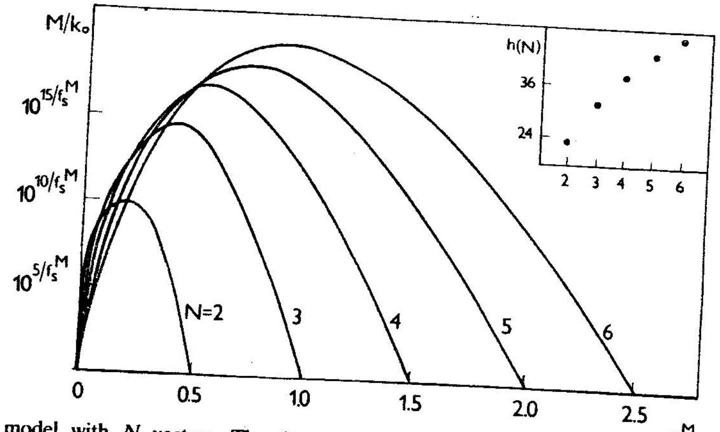

Fig. 2. The model with N vectors. The dependence of the M/k_0 expressed by the f_{self}^M on the $X^M=1/|w^M|=f_{self}^M/|f_{mix}^M|$ is shown. $h(N)$ enters in the upper bound the M/k_0 as given in Sect. V.

We got quite a wide range of quantitative information about the considered simplified model with N vectors and an SO(2) symmetry. In more realistic theories, like the SU(5), the knowledge of such information will be necessary to fit the parameters so as to get the given mass spectrum and so on. It must be, however, pointed out that it is impossible to arrive at any conclusions about the validity of similar results in more realistic theories, where further detailed investigations must be performed. We hope that the RGE will help to elucidate the gauge hierarchies problem also in these much more complicated situations [8].

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APPENDIX

There are no problems with the renormalization of the scalar theory and the RGE when one works only in the first nontrivial order. It is well known that there is no dependence on the finite renormalization and any renormalization scheme works [9].

We have the renormalized Lagrangian of massless theory

$$L=\frac{1}{2}\partial_\mu\Phi_i\partial^\mu\Phi_i-\frac{1}{4!}f_{ijkl}\Phi_i\Phi_j\Phi_k\Phi_l$$

$i=1, 2, \ldots, N$. Using the order by order minimal subtraction renormalization procedure with dimensional regularization, we may construct perturbatively the bare Lagrangian in terms of the renormalized quantities

$$L'=\frac{1}{2}Z^P_{ij}\partial_\mu\Phi_i\partial^\mu\Phi_j-\frac{1}{4!}[Z^V f]_{ijkl}\Phi_i\Phi_j\Phi_k\Phi_l$$

with the renormalization functions Z's dependent on the renormalization point M, renormalized coupling constants f's and the regularization parameter $\varepsilon=n-4$.

Using the fact that bare quantities cannot depend on the renormalization mass scale M, we get the RGE for running coupling constants in the form

$$M\frac{\partial f}{\partial M}=\frac{f(-M\partial\ln Z^V/\partial M+2M\partial\ln Z^P/\partial M)}{1-2f\partial\ln Z^P/\partial f+f\partial\ln Z^V/\partial f},$$

where the field indices are suppressed. After performing calculations of the divergent parts of the Green functions we get (4) [10].

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