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WHY THE ϱ-MESON POLE POSITION COULD NOT BE FOUND FROM THE SPACE-LIKE REGION DATA ON ELECTRIC OR MAGNETIC PROTON FORM FACTORS

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The analysis of the data on the isovector part of the electric nucleon form factor in a conformally mapped variable is carried out by the Padé type approximation. One stable pole in the place of the short branch cut on the unphysical sheet that comes from the partial wave projection of the nucleon Born term in the πN scattering amplitude is found. The results of the analysis reveal that the contribution of this short branch cut of the second Riemann sheet is comparable with the contribution of the $K\bar{K}$ inelastic channel and both prevent the ϱ-meson to be found from the data on the isovector part of the electric proton form factor in the space-like region.

ПОЧЕМУ ПОЛОЖЕНИЕ ПОЛЮСА ϱ-МЕЗОНА НЕ МОЖЕТ БЫТЬ ОПРЕДЕЛЕНО ИЗ ДАННЫХ ОБ ЭЛЕКТРИЧЕСКОМ И МАГНИТНОМ ФОРМФАКТОРАХ ПРОТОНА В ПРОСТРАНСТВЕННОПОДОБНОЙ ОБЛАСТИ?

Анализ данных об изовекторной части электрического формфактора хуклона в переменной комформного отображения выполнен при помощи Паде-аппроксимации. Обнаружен один устойчивый полюс в месте короткого разреза на нефизическом листе, происхождение которого обусловлено проекцией борновского члена соответствующей парциальной волны в амплитуду πN-рассеяния. Результаты анализа показывают, что вклад от этого разреза на втором римановом листе совпадает с вкладом от неупругого $K\bar{K}$-канала и это не позволяет определить ϱ-мезон из данных об изовекторной части электрического формфактора протона в пространственноподобной области.

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I. INTRODUCTION

The nucleon form factors are believed to be analytic functions in the entire squared four-momentum transfer t-plane except for a cut from t_0 to infinity. There are no poles on the first (physical) sheet and all singularities are restricted only to a sequence of threshold branch points on the positive real axis.

It was 20 years ago, that Frazer and Fulco [1] while using these analyticity properties found that a resonance of suitable position and width in the $J=1$, $I=1$ state of the pion-pion system could bring the dispersion-theoretical calculation of the nucleon electromagnetic structure into a qualitative agreement with the experiment. At present this resonance, the so-called ϱ-meson, is well established and experiments on $e^+e^-\to\pi^+\pi^-$ have demonstrated how much it dominates the pion and hence the nucleon form factors. Since the time of Frazer and Fulco a large number of papers have been devoted to the investigation of the electromagnetic structure of the nucleon by using the analyticity and a complete review of references can be found in Höhler's paper [2].

As the experimental information on the nucleon form factors in the space-like region ($t<0$) has been improved and as in the near future one can also expect a considerable improvement of the present, poor, experimental information in the time-like region ($t>0$), more realistic models for the nucleon form factors, like in the case of the pion form factor [3], are desirable. In order to construct such a model we have to know, first of all, which of all the present singularities have to be taken into account. The only economical way to reveal the most important singularities seems to be to start with the description of the experimental data by means of the Padé-type approximation in a suitable chosen variable, which, as we know, are able to reproduce all the analytic structure of the function under consideration [4, 5]. Thus, if the ϱ-meson resonance is really dominant in the proton form factors as it follows from the Frazer and Fulco analysis [1], then one has to find it by means of the Padé approximants constructed from the coefficients of a polynomial fitting the experimental data as it was done in the case of the pion form factor [5].

It was really claimed by Dumbrajs [6] that the application of the Padé approximants to the electric proton form factor gives the stable ϱ-meson pole on the second Riemann sheet. This result was surprising with regard to the involved extrapolation through a cut, therefore Bowcock et al. [7] have decided to apply the above procedure, in order to test it, to a simple model based on a known mathematical function where singularities are easily identifiable. The comparison of the Dumbrajs procedure and the direct fit of the simulated data by means of a rational function (we call it the Padé-type approximation) clearly demonstrates the deficiencies of the former and the stability of the latter. Thus, the authors of paper [7] applied the direct fitting procedure to the electric proton form factor data to see what it predicts regarding poles and zeros. However, they found some poles

on the first sheet and no ϱ-meson poles. By means of this analysis they came to the conclusion that the very precise results obtained for the ϱ pole in [6] must be fortuitous. In the present paper we present some arguments why the authors in [7] could not find the ϱ-meson poles as they expected to find them according to the results of the Dumbrajs.

II. THE ANALYTIC PROPERTIES OF ELECTRIC AND MAGNETIC PROTON FORM FACTORS

Aside from radiative corrections, the electron-nucleon elastic scattering, in which the electromagnetic structure of the nucleon is measured, can be described in the one-photon exchange approximation. As a consequence the corresponding differential cross section in the laboratory system takes the following form [8]

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{4E_0^2}\frac{\cos^2(\vartheta/2)}{\sin^4(\vartheta/2)}\frac{1}{1+\frac{2E_0}{m_N}\sin^2(\vartheta/2)}\times \tag{1}$$

$$\times\left\{F_1^2-\frac{t}{4m_N^2}\left[2(F_1+2m_NF_2)^2\,\mathrm{tg}^2(\vartheta/2)+(2m_NF_2)^2\right]\right\},$$

where $F_1(t)$ and $F_2(t)$ are the Dirac and the Pauli form factors, α the fine structure constant, E_0 the incident electron energy, ϑ the scattering angle, m_N the nucleon mass and the approximation $m_e/E_0\approx m_e/m_N\approx 0$ (m_e means the electron mass) was taken into account. $F_1(t)$ and $F_2(t)$ are real valued functions in the space-like region ($t<0$) and different for the proton and the neutron.

From the theoretical point of view it is convenient to decompose both form factors $F_1(t)$, $F_2(t)$ for the proton and the neutron into the isovector and isoscalar parts in the following way

$$F_1^p(t)=\frac{1}{2}(F_1^s+F_1^v);\quad F_2^p(t)=\frac{1}{2}(F_2^s+F_2^v) \tag{2}$$

$$F_2^n(t)=\frac{1}{2}(F_1^s-F_1^v);\quad F_2^n(t)=\frac{1}{2}F_2^s-F_2^v),$$

from which it is straightforward to find

$$F_1^s=F_1^p(t)+F_1^n(t);\quad F_1^v=F_1^p(t)-F_1^n(t) \tag{3}$$

$$F_2^s=F_2^p(t)+F_2^n(t);\quad F_2^v=F_2^p(t)-F_2^n(t).$$

The reason for the decomposition of $F_1(t)$, $F_2(t)$ into the isovector and isoscalar parts is that there is a general belief that the analytic properties of the isovector parts on the physical sheet of the t-variable are restricted only to the sequence of the threshold branch points at $t=4m_\pi^2$, $16m_\pi^2$, $4m_K^2$, $4m_N^2$, ... on the positive real axis and the analytic properties of isoscalar parts to the sequence of threshold branch points at $t=9m_\pi^2, 25m_\pi^2, 4m_K^2, 4m_N^2, \ldots$, where m_i $(i=\pi, K, N)$ is the mass of the pion, kaon and nucleon, respectively. Moreover, in the generalized vector meson dominance model the isovector parts are dominant only by pure isovector vector mesons like $\varrho, \varrho', \varrho''$ and the isoscalar parts only by pure isoscalar vector mesons like $\omega, \varphi, \omega', \varphi', \psi, \ldots$, which in a more realistic form factor model can be taken into account as poles on unphysical sheets of the corresponding Riemann surface.

Now it is not difficult to understand to what extent the analytic structure of the Dirac and the Pauli form factors is complicated. These complications are transformed also to the electric and magnetic proton form factors $G_E^p(t)$ and $G_M^p(t)$, respectively, as they are defined by the relations [9]

$$G_E^p(t)=F_1^p(t)+\frac{t}{4m_p^2}F_2^p(t) \qquad (4)$$

$$G_M^p(t)=F_1^p(t)+F_2^p(t),$$

in order to simplify the extraction of the experimental information on the electromagnetic structure of the nucleon from the differential cross section (1).

Dumbrajs [6] and Bowcock et al. [7], by using these complicated analytic properties and a not very ingenious conformal mapping (the first sheet is mapped into the interior of the unit circle and the second sheet outside the unit circle), conjectured to determine the ϱ-meson pole position on the second Riemann sheet from the data on G_E^p and G_M^p in the space-like region. Of course, Bowcock et al. [7] have found some effective poles which in our opinion represent the contributions of important cuts on the unit circle in the conformally mapped plane, the ϱ-meson poles placed outside the unit circle and also the contribution of the isoscalar vector mesons like ω, φ, which are present in bot the G_E^p and the G_M^p form factors. And as to the results of Dumbrajs [6], they seem to be really fortuitous.

In order to shed more light on these problems we have carried out the analysis only with the isovector part of the electric proton form factor that is free of the cut contributions of the isoscalar part and also the contribution of isoscalar vector mesons like ω, φ, etc. The results of the analysis are presented in the next section.

III. THE ANALYSIS OF DATA ON THE ISOVECTOR PART OF THE ELECTRIC NUCLEON FORM FACTOR

If we substitute instead of F_1^p and F_2^p into (4) the relations (2) and define the isoscalar and isovector parts of the electric nucleon form factor by the relations

$$G_E^s(t)=F_1^s(t)+\frac{t}{4m_N^2}F_2^s(t)\,. \qquad (5)$$

$$G_E^v(t)=F_1^v(t)+\frac{t}{4m_N^2}F_2^v(t),$$

we can obtain the decomposition of the electric proton form factor into isoscalar and isovector parts as follows

$$G_E^p(t)=\frac{1}{2}\{G_E^s(t)+G_E^v(t)\}. \qquad (6)$$

In a like manner the expression for the electric neutron form factor

$$G_E^n(t)=\frac{1}{2}\{G_E^s(t)-G_E^v(t)\} \qquad (7)$$

can be found.

By solving (6) and (7) we get the isovector part of the electric nucleon form factor expressed through the electric proton and electric neutron form factor in the following way

$$G_E^v(t)=G_E^p(t)-G_E^n(t). \qquad (8)$$

By using the data on $G_E^p(t)$ and $G_E^n(t)$ one can obtain the data on $G_E^v(t)$.

For $G_E^p(t)$ we used the same data points [10, 11] for $-25.030\ \mathrm{GeV}^2\leq t\leq -0.0078\ \mathrm{GeV}^2$ as Dumbrajs [1] and Bowcock et al. [7], in order to make a comparison with their results. However, we have at our disposal the data on $G_E^n(t)$ only for $-1.530\ \mathrm{GeV}^2\leq t\leq 0.010\ \mathrm{GeV}^2$. Moreover, the data from this interval do not even coincide in t with the data on $G_E^p(t)$. For this reason, we have used for $G_E^n(t)$ the zero width approximation model of Zovko [12] that describes not only the data on $G_E^n(t)$ but also reproduces all the existing data on proton and neutron form factors in the space-like region only by four adjustable parameters. The latter is a sufficiently justified argument for the expression [12]

$$G_E^n(t)=\left\{\frac{1}{2}+\frac{\mu^s+2m_n^2b^s}{4m_n^2}t\right\}\left[\left(1-\frac{t}{m_\omega^2}\right)\left(1-\frac{t}{m_\varphi^2}\right)\left(1-\frac{t}{m_{\omega'}^2}\right)\right]^{-1}- \qquad (9)$$

$$-\left\{\frac{1}{2}-\frac{\mu^v+2m_n^2b^v}{4m_n^2}t\right\}\left[\left(1-\frac{t}{m_\varrho^2}\right)\left(1-\frac{t}{m_{\varrho'}^2}\right)\left(1-\frac{t}{m_{\varrho''}^2}\right)\right]^{-1},$$

with $m_\omega^2=0.614\ \mathrm{GeV}^2$, $m_\varphi^2=1.039\ \mathrm{GeV}^2$, $m_{\omega'}^2=1.4\ \mathrm{GeV}^2$, $m_\varrho^2=0.585\ \mathrm{GeV}^2$, $m_{\varrho'}^2=1.3\ \mathrm{GeV}^2$, $m_{\varrho''}^2=2.1\ \mathrm{GeV}^2$, $\mu^s=-0.12$, $b^s=-0.91\ \mathrm{GeV}^{-2}$, $\mu^v=0.925$, $b^v=-1.10\ \mathrm{GeV}^{-2}$ and m_n as the neutron mass to be used to give through (8) 68 experimental points on $G_E^v(t)$ in the range of moments $-25.030\ \mathrm{GeV}^2\leq t\leq -0.0078\ \mathrm{GeV}^2$, with the errors of $G_E^v(t)$ to be found only from the errors of $G_E^p(t)$.

We emphasize again that by means of the aforementioned procedure we have obtained data on the isovector part of the electric nucleon form factor, the analytic properties of which consist only of the threshold branch points at $t=4m_\pi^2, 16m_\pi^2,$

$4m_K^2$, $4m_N^2$, etc. According to the vector dominance model it should be dominated first of all by the ϱ-meson exchange contributions. Thus, if one has any chance to find the ϱ-meson pole position from the data on the nucleon form factor, then, first of all, it has to be found from the data on $G_E^v(t)$.

We have carried out two independent analyses of these data by means of the Padé-type approximstions for two different variables, on two different computers, and by two different minimization programs.

The first analysis was carried out at Bratislava on the computer SIEMENS by using the CERN minimization program MINUIT [13]. The results of the direct fit of the data on $G_E^v(t)$ by means of the following normalized to 1/2 (in that case the errors of G_E^v are taken to be one half of the errors of G_E^p) and respecting the real analyticity the Padé-type approximation

$$G_E^v(t)=\frac{1}{2}\frac{A_1+\sum_{n=1}^{M}A_{2n+1}(iq)^n}{1+\sum_{n=1}^{M}A_{2n}(iq)^n}\;\frac{1+\sum_{n=1}^{M}A_{2n}(-1)^n}{A_1+\sum_{n=1}^{M}A_{2n+1}(-1)^n} \tag{10}$$

on the pion c.m. momentum q-plane are presented in Table 1. The q-plane is obtained from the t-plane by the conformal mapping

$$q=\sqrt{\frac{t-4m_\pi^2}{4}} \tag{11}$$

with $m_\pi=1$.

The second analysis was carried out at Dubna on the computer CDC-6500 by using the minimization program FUMILI [14]. In this latter case a more ingenious conformal mapping was used. The q-plane was first turned at 90° in the anti-clockwise direction and then shifted so that all the data on $G_E^v(t)$ from the range of momenta — $25.0300\ \mathrm{GeV}^2\leq t\leq -0.0078\ \mathrm{GeV}^2$ be placed symmetrically on the real axis around the origin of this new k-plane. All this could be achieved by the following conformal mapping

$$k=\sqrt{\frac{4-t}{4}}+k_0, \tag{12}$$

where $k_0\approx 9.5$. The results of the direct fit of the data on $G_E^v(t)$ by the Padé-type approximation normalized to one and respecting the real analyticity

$$G_E^v(t)=\frac{1+\sum_{n=1}^{M}A_{2n-1}(k_0-1)^n+\sum_{n=1}^{M}A_{2n}[k^n-(k_0-1)^n]}{1+\sum_{n=1}^{M}A_{2n-1}k^n} \tag{13}$$

on the k-plane are presented in Table 2.

The q-plane and the k-plane were preferred to be used in the analysis of the data on $G_E^v(t)$ by the following motivations: First, $G_E^v(t)$ by the mapping (11) and (12) is regularized at the first threshold branch point $t=4m_\pi^2$ and the contribution of the corresponding two-pion cut is thus automatically eliminated. Secondly, as the $e^+e^-\to 4\pi$ experiments indicate that the cross section remains small below 1 GeV [15], we neglect the four-pion cut and as a consequence, the ϱ-meson pole is undoubtedly placed on the second Riemann sheet reaching through the two-pion cut from $t=4m_\pi^2$ to $t=4m_K^2$. Thirdly, both Riemann sheets are in the q-plane as well as in the k-plane placed equivalently. And we avoid the difficulty of an analytic continuation from the interior of the unit circle through branch cuts to the outside of the unit circle, which is characteristic for the Dumbrajs [6] as well as the Bowcock et al. [7] procedures.

As one can see from Table 1 and Table 2 immediately, the almost identical results are obtained independently of the variable used and the minimization programs. One stable pole is found in the q-plane around the value $q\approx -i0.65$ that corresponds to $k\approx 10.15$ in the k-plane just below the threshold $t=4m_\pi^2$ in the second Riemann sheet. There are other two symmetrically placed stable poles (see

Table 1

The results of analysis of the data on G_E^v by the Padé-type approximations (10) in the q-plane

[N/M]	X	X/ndf	Position of zeros	Position of poles
[2/2]	818.607	12.99	i 14.321	1.640 + i 0.624
			i 12.856	−1.640 + i 0.624
[2/3]	129.242	2.08	i 21.044	2.160 + i 1.632
			i 20.143	−2.160 + i 1.632
				−i 0.513
[3/3]	112.973	1.85	−i 25.912	2.226 + i 1.489
			3.678 + i 15.569	−2.226 + i 1.489
			−3.678 + i 15.659	−i 0.651
[3/4]	112.351	1.87	−i 31.218	−i757.837
			3.830 + i 15.743	2.236 + i 1.497
			−3.830 + i 15.743	−2.236 + i 1.497
				−i 0.661
[4/4]	112.532	1.91	+ i197.676	−i289.494
			−i 24.613	2.230 + i 1.504
			3.917 + i 15.848	−2.230 + i 1.504
			−3.917 + i 15.848	−i 0.644
[5/5]	109.365	1.92	−i 7.977	3.478 − i 0.470
			−i 7.977	−3.478 − i 0.470
			−5.419 + i 17.791	2.833 + i 1.191
			5.419 + i 17.791	−2.833 + i 1.191
			−i 7.977	−i 1.305

Table 2

The results of analysis of the data on G_E^v by the Padé-type approximation (13) in the k-plane

$[N/M]$	X	X/ndf	Position of zeros	Position of poles
[1/2]	3932.0	60.491	−3.213	8.7152 + i1.2459
				8.7152 − i1.2459
[2/2]	735.5	11.492	−3.6422 + i 0.9213	8.8908 + i1.6666
			−3.6422 − i 0.9213	8.8908 − i1.6666
[2/3]	104.8	1.664	−7.0315 + i 4.1079	10.2895
			−7.0315 − i 4.1079	8.0323 + i2.3115
				8.0323 − i2.3115
[3/2]	225.9	3.586	−7.0690	9.0289 + i2.0591
			−1.8304 − i 3.5258	9.0289 − i2.0591
			−1.8304 + i 3.5258	
[3/3]	104.0	1.677	−12.6486	10.4563
			−9.2957 − i 7.4849	8.0721 + i2.3888
			−9.2957 + i 7.4849	8.0721 − i2.3888
[4/3]	102.3	1.677	2.8032 + i10.0472	10.8927
			2.8032 − i10.0472	8.0803 + i2.5540
			−6.6987 + i 2.4955	8.0803 − i2.5540
			−6.6987 − i 2.4955	

Table 1 for $q \approx \pm 2.5 + i1.5$ and Table 2 for $k \approx 8 \pm i2.5$) but no ϱ-meson has been found on the second Riemann sheet, which is placed exactly at $q_\varrho = \pm 2.59 - i0.30$ and $k_\varrho = 9.8 \pm i2.59$ in the q- and the k-plane, respectively.

The first stable pole at $q \approx -i0.6$ or $k \approx 10$ visibly simulates the contribution of the short branch cut just below the threshold $t = 4m_\pi^2$ in the second Riemann sheet which comes from the partial wave projection of the nucleon Born term in the πN scattering amplitude [2]. The two symmetrically placed poles at $q \approx \pm 2.5 + i1.5$ or $k \approx 8 \pm i2.5$ in our opinion take effectively into account both the ϱ-meson contribution as well as the $K\bar{K}$ branch cut contribution which, in comparison with the ϱ-meson seems to be considerable in the isovector part of the electric nucleon form factor.

Thus, the results of our analysis support the conclusion of Höhler et al. [16] that unlike the pion form factor $G_E^v(t)$ apart from the ϱ-meson and the $K\bar{K}$ branch cut contributions is dominated by the nucleon exchange partial wave projection branch cut in the second Rieman sheet which is responsible, for instance, for the difference between the nucleon and the pion charge radii.

IV. CONCLUSIONS

By means of a direct least-square fit of the normalized and respecting the real analyticity Padé-type approximstions in a conformally mapped variable to the data on the isovector part of the electric nucleon form factor we have found most important singularities dominating $G_E^v(t)$. Unlike the pion form factor, the isovector part of the electric nucleon form factor is not given mainly by the ϱ-exchange contribution, but the short branch cut just below the threshold $t = 4m_\pi^2$ on the second Riemann sheet that comes from the partial wave projection of the nucleon Born term in the πN scattering amplitude [2] plays an important role. The latter explains the fact why many scientists [10, 17, 18] fitting the data on the nucleon form factor in the framework of the extended vector dominance model, fixed the ϱ-meson mass and did not find it in the fitting procedure.

REFERENCES

[1] Frazer, W. R., Fulco, J. R.: Phys. Rev. *117* (1960), 1609.
[2] Höhler, G.: Lecture given at the Int. Summer institute Current Induced Reaction. DESY-Hamburg, Sept. 15—25, 1975.
[3] Dubnička, S., Dubničková, A. Z.; Meshcheryakov, V. A.: Czech. J. Phys. *B 29* (1979), 142.
[4] Basdevant, J. L.: Fortschritte der Physik *20* (1972), 283.
[5] Dubnička, S., Krupa, D., Martinovič, L.: Acta Phys. Slov. *29* (1979), 201.
[6] Dumbrajs, O.: Rev. Roum. Phys. *21* (1976), 273.
[7] Bowcock, J. E., Dacunha, N. M. C., Qeen, N. M.: Rev. Roum. Phys. *23* (1978), 549.
[8] Rosenbluth, M. N.: Phys. Rev. *79* (1950), 615.
[9] Sachs, R. G.: Phys. Rev. *126* (1962), 2256.
[10] Blatnik, S., Zovko, N.: Acta Phys. Austriaca *39* (1974), 62.
[11] Akimov, Yu. K. at al.: Zh. Eksp. Teor. Fiz. *62* (1972), 1231; Sov. Phys. JETP *35* (1972), 651.
[12] Zovko, N.: Fortschr. Phys. *23* (1975), 185.
[13] James, F., Roos, M.: Computer Phys, Comm. *10* (1975), 343.
[14] The program library in FORTRAN, JINR, D-520 Vol. 2 Dubna 1970 (in Russian).
[15] Conversi, M. et al.: Phys. Lett. *52 B* (1974), 493.
[16] Höhler, G., Kiehlmann, H. D., Schmidt, W.: Phys. Rev. *D 11* (1975), 2667.
[17] Mehrotra, S., Roos, M.: *University of Helsinki preprint No 97*, Helsinki 1975.
[18] Skachkov, N. B., Solovtsev, I. L.: JINR, E2-9504, Dubna 1976.

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