

# THE MAGNETIC PROPERTIES OF A RANDOM DILUTE FERROMAGNET<sup>1</sup>

МАГНИТНЫЕ СВОЙСТВА СЛУЧАЙНО РАЗБАВЛЕННЫХ  
ФЕРРОМАГНЕТИКОВ

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The paramagnetic susceptibility and relative magnetization of a random ferromagnet for bcc, fcc and some plane lattices are studied in the molecular field approximation. The temperature dependence of susceptibility is numerically calculated for different concentrations.

A simple theoretical model for a randomly diluted magnetic system, which has been widely studied, is the randomly diluted Ising model. We can write the Hamiltonian of such a system as follows

$$\mathcal{H} = -\mathcal{J} \sum_{\langle ij \rangle} p_i p_j \mu_i \mu_j, \quad (1)$$

where  $\langle ij \rangle$  indicates that the sum is over pairs of the nearest neighbours ( $m$ -sites),  $\mu_i$  is an Ising spin variable, and  $p_i$  is a random variable equal to one if the site is occupied and is zero otherwise,  $\mathcal{J}$  is the exchange constant.

The mean field theory (MFT) equation for this random system may written as follows

$$\sigma_i = B_s(\beta \mathcal{J} \sum_{\langle ij \rangle} p_j + \beta H), \quad (2)$$

in which  $\sigma_i$  is the relative magnetization at an occupied site  $i$ ,  $\beta = 1/kT$ ,  $k$  — the Boltzmann constant,  $T$  — the absolute temperature,  $H$  is proportional to the applied field, and  $B_s$  is the Brillouin function for the spin  $S$ . All occupied sites are assumed to have the same value of the local moment, independent of the environment and all unoccupied sites are taken to have the zero moment.

The average relative magnetization  $\bar{\sigma}$  is determined by averaging over all possible configurations of occupied and unoccupied sites:

$$\bar{\sigma} = \sum_{N=0}^z P_N(z) \sigma_N(N), \quad (3)$$

where  $z$  is the coordination number of the considered lattice,  $P_N(z) = c^N (1-c)^{z-N} Z_1(N) / (z-N)!$  is the probability of finding  $N$  occupied  $m$  sites for the concentration  $c$  of the magnetic element, and

$$\sigma_N(N) = B_s[\beta \mathcal{J} N \sigma_N(N) + \beta H] \quad (4)$$

$$\sigma_N(N) = B_s[\beta \mathcal{J} \sum_i p_i \sigma_i(N) + \beta H] \quad (5)$$

$$\sum_i p_i \sigma_i(N) = \sigma_N(N) + \sigma_1(N) \sum_i p_i + \sum_i p_i \sigma_i(N), \quad (6)$$

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where  $\sum_l$  indicates a summation over those sites which are  $mn$  of 1;  $\sum_l'$  indicates the summation over

those sites which are  $mn$ 's both of site 0 and 1, and  $\sum_l''$  is a sum over the remaining sites exclusive of 0, which are  $mn$ 's of 1.

This treatment accounts for the correlation in a lattice between the numbers of magnetic neighbours of two atoms which are themselves the nearest neighbours. This feature has also been used to explain the clustering of magnetic moments in an alloy.

The equations (3)–(6) give the closed system by assuming that  $\sigma_l(N)$  in (6), for which site  $l$  is two  $mn$  steps removed from site 0, can be expressed in terms of  $a_0(N)$ ,  $\sigma_1(N)$  and  $\bar{\sigma}$ :

$$\sigma_l(N) = a_0(N) + b\sigma_1(N) + d\bar{\sigma}, \quad (7)$$

where  $a_l + b_l + d_l = 1$ , and the relative values of  $a_l$ ,  $b_l$  and  $d_l$  are proportional to the number of  $mn$  sites in common with 0, in common with 1, and common to neither 0 or 1, respectively.

The results for the sc and fcc lattices were calculated in papers [1, 2]. It is possible to show that the result for the bcc lattice is the same as that for the sc and for the hcp lattice, corresponding to the fcc lattice.

The result of the triangular lattice was obtained in the following form:

$$\sigma_1^2 = B_3 \left\{ \beta\delta \left[ \alpha_0 + \frac{2}{5}(N-1)\sigma_1 + \frac{3}{5}(N-1)\bar{\sigma} \right] + \beta H \right\}. \quad (8)$$

This closed system of equations can be solved numerically and we can obtain the temperature dependence of magnetization, which can be experimentally verified. For example, in paper [1] a very good agreement is obtained with the measuring of magnetization on Ni–Cu alloys.

In the paramagnetic region, we can linearize the equations and obtain the results for the paramagnetic susceptibility. For example, for a triangular lattice:

$$\chi^A = \frac{\beta \sum_{N=0}^6 P_N \frac{1 + \beta\delta \frac{S+1}{3S} N - \frac{2}{5} \frac{S+1}{3S} (N-1)}{1 - \frac{2}{5} \beta\delta \frac{S+1}{3S} (N-1) - (\beta\delta)^2 \left( \frac{S+1}{3S} \right)^2 N}}{\frac{3}{5} N(N-1)} \quad (9)$$

$$\frac{3S}{S+1} - (\beta\delta)^2 \frac{S+1}{3S} \sum_{N=0}^6 P_N \frac{1 - \frac{2}{5} \beta\delta \frac{S+1}{3S} (N-1) - (\beta\delta)^2 \left( \frac{S+1}{3S} \right)^2 N}{\frac{3}{5} N(N-1)}$$

The temperature dependence of the susceptibility numerically calculated for different lattices and concentrations is shown in Fig. 1, where we used relative quantities.

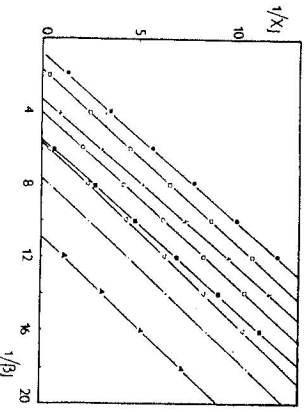


Fig. 1. The paramagnetic susceptibility as a function of temperature for a triangular lattice (●)  $c = 0.1$ , □  $c = 0.4$ , ×  $c = 0.6$ , ○  $c = 0.9$  and fcc (hcp) lattice ■  $c = 0.1$ , △  $c = 0.4$ , △  $c = 0.6$ , △  $c = 0.9$ .

## REFERENCES

- [1] Richards, P. M.: Phys. Lett. 44 A (1973), 389.
- [2] Corrias, M., Cullen, J. R., Wohlfarth, E. P.: Phys. Lett. 46 A (1974), 341.

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