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SPIN GLASSES WITH SHORT-RANGE INTERACTION IN THE BPW APPROXIMATION[1]

СПИНОВЫЕ СТЕКЛА С БЛИЗКОДЕЙСТВУЮЩИМ ВЗАИМОДЕЙСТВИЕМ В ПРИБЛИЖЕНИИ БЕТЕ–ПАЙЕРЛСА–ВЕЙСА

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The static properties of the system of spins interacting via a special short-range interaction within the Bethe–Peierls–Weiss (BPW) approximation are discussed. The positive sign is discussed of the slope of the temperature dependence of the susceptibility below freezing temperature T_f.

Following the results of the papers [1, 2] the equation for the determination of the molecular magnetic field λ_1 has the form

$$\left.\frac{\partial^2 F}{\partial \lambda_0^2}\right|_{\lambda_0=0} = \frac{1}{z^2}\left.\frac{\partial^2 F}{\partial \lambda_1^2}\right|_{\lambda_0=0}, \qquad (1)$$

where F is the average value of the free energy,

$$F = -kT\,\langle \ln Z_{BPW}\rangle_{\bar{J}}$$

$\langle\ldots\rangle_{\bar{J}}$ denotes an averaging over the distribution $P(\bar{J})$ of the effective coupling

$$\bar{J} = \frac{1}{z}\sum_{i=1}^{z} J_{0i},$$

λ_0 is the external magnetic field, Z_{BPW} is the partition function determined by Brown and Luttinger [1] for the classical-spin approximation and for a large z. By using the explicit form of F from (1) we obtain the spin glass freezing temperature T_f in the form

$$kT_f = \frac{1}{3}\left(\frac{\langle \bar{J}^2\rangle_{\bar{J}}}{1-\frac{1}{z}}\right)^{1/2}. \qquad (2)$$

The suspectibility λ is derived from the free energy F via

$$\chi = \frac{(g\mu_B)^2}{3kT}(1-q), \qquad (3)$$

where

$$q = \frac{1}{3}\left(\frac{z\lambda_1}{3kT}\right)^2 \langle \bar{J}^2\rangle_{\bar{J}}$$

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is the Edwards–Anderson spin glass order parameter, which we can write by using the equation for λ_1 and Eq. (2) in the form

$$g=b\left(1-\frac{T}{T_f}\right),\quad b=\frac{2\left(1-\frac{1}{z}\right)}{\left(1-\frac{1}{z}\right)\frac{\langle \bar{J}^4\rangle_J}{\langle \bar{J}\rangle_J^2}-1}. \tag{4}$$

The slope of the susceptibility $\chi(T)$ is determined by the variation of the spin-glass parameter q with temperature. For that we obtain

$$\frac{3kT_f^2}{(g\mu_B)^2}\frac{d\chi}{dT}=\begin{cases}-1 & \text{for } T\to T_f^+\\ -1+b & \text{for } T\to T_f^-\end{cases} \tag{5}$$

One obtains a cups in the susceptibility if $b>1$. For the limit $z\to\infty$

$$b_\infty=\frac{2}{\langle \bar{J}^4\rangle_J/\langle \bar{J}^2\rangle_J^2-1}. \tag{6}$$

All specific information about spin glasses is contained in the distribution $P(\bar{J})$. We assume the special short-range interaction J_{0i}

$$J_{0i}=\begin{cases}\pm J & \text{if } i \text{ is in a volume } v \text{ around the site } 0\\ 0 & \text{otherwise}\end{cases}$$

and the distribution of the effective coupling $\bar{J}$ with randomly distributed sites in the volume V is a Poisson distribution [3]

$$P\left(\frac{\bar{J}}{J/z}=k\right)=\frac{(cv)^k}{k!}\exp(-cv),\qquad c=z/V. \tag{7}$$

For the distribution (7) we have

$$\langle \bar{J}^2\rangle_J=\left(\frac{J}{z}\right)^2((cv)^2+cv) \tag{8}$$

$$\langle \bar{J}^4\rangle_J=\left(\frac{J}{z}\right)^4((cv)^4+6(cv)^3+7(cv)^2+cv). \tag{8a}$$

The condition for the positive slope in the BPW ($z\to\infty$) approximation below T_f is $1<\langle \bar{J}^4\rangle_J/\langle \bar{J}^2\rangle_J^2<3$ and in the molecular field approximation (MFA) $0<\langle \bar{J}^4\rangle_J/\langle \bar{J}^2\rangle_J^2<3$.

The expression for a slope of the susceptibility in the BPW approximation does not reduce to one in the MFA. For small cv the results in the two approximations differ little. By increasing the cv value the results in the MFA and the BPW approximation differ more. When $(cv)_M$ is a solution of the equation $\langle \bar{J}^4\rangle_J=5/3\langle \bar{J}^2\rangle_J^2$, the positive slope in BPW approximation has the maximum value $+2$. This means that up to $(cv)_M$ the system with a short-range interaction does not obey the Poisson distribution but the Gaussian distribution. The Poisson distribution changes into a Gaussian one if the range of the interaction is much larger than the mean distance between the spins. In the MFA we do not obtain this limit value of cv. In principle cv in MFA can proceed to infinity, where $\langle \bar{J}^4\rangle_J=\langle \bar{J}^2\rangle_J^2$ and the slope of $\chi(T)$ is $+2$. In the BPW approximation we have a limit value up to $(cv)_M$ that the Poisson distribution does not satisfy but the Gaussian distribution does. We expect the BPW approximation for a short-range interaction to be a more realistic approximation of spin glasses than the MFA.

REFERENCES

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