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PION CHARGE RADIUS WITHIN THE FRAMEWORK OF THE VIRTON MODEL

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Electromagnetic pion and kaon radii were calculated within the nonlocal virton-quark model with the simplest interaction Lagrangians. A good agreement with experimental data has been achieved.

ЭЛЕКТРОМАГНИТНЫЙ РАДИУС ПИОНА В РАМКАХ ВИРТОННОЙ МОДЕЛИ

В рамках нелокальной виртон-кварковой модели с максимально простыми лагранжианами взаимодействия рассчитан электромагнитный радиус пиона и каона. Достигнуто хорошее согласие с экспериментальными данными.

I. INTRODUCTION

The progress of the quark theory compels us to assume that quarks exist objectively, perhaps, in some very unusual form. The fact that they have not yet been discovered experimentally indicates that possibly quarks are such objects of the microworld which we have not come across so far.

During the past few years a lot of models have appeared which make attempts to explain the quark confinement [1]. A common feature of these models is the assumption that quarks do exist as physical particles but they cannot arise on the basis of a certain dynamical mechanism. Such a mechanism could consist in an interaction with a gluon field, "bags", and so on.

In paper [2] the alternative hypothesis is proposed: "quarks do not exist at all as usual physical particles and exist in the virtual state only". This hyphothesis can be interpreted in the following way. Within the framework of the quantum field theory there have been found "particles" called "virtons", which possess the following properties. First, the field describing free virtons is identically zero, i.e. virtons do

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not exist in a free state, and secondly, the causal Green function of the virton field is nontrivial, i.e. virtons exist in a virtual state only. The virton field can be constructed by methods of the nonlocal quantum field theory [3].

The virton field is convenient to describe quarks and does not need any additional fields (like the gluon field) to ensure the quark confinement. The physical assumption here is that hadrons described by the standard quantum field equations do not interact with each other directly but through an intermediate virton-quark field.

A successful description of some strong, electromagnetic and weak decays of the octet of vector and pseudoscalar mesons, of octet and decuplet of baryons within the virton-quark nonlocal model [4] indicates that the model having only two free parameters ξ, L, is positively valid in the range of the quark confinement (≈ 2 Gev). It is a region in which the current algebra and its quantum field realization like the chiral field theories are also very successful and give results comparable with experimental data. However, the calculations within these standard methods, unlike the virton-quark nonlocal model, require some additional assumptions and different approximations.

The more stringent characteristics of any quantum field theoretical model are, e.g., the obtained values of pseudoscalar meson charge radii. They have been already calculated within the framework of the chiral quantum field theory of pions and other hadrons [5], in the standard relativistic quark model [6], in the SU(2)⊗SU(2) quark σ-model [7], etc. Thus in this paper we calculate the pion and kaon charge radii within the framework of the virton-quark nonlocal model by using the interaction Lagrangian invariant under the unbroken SU(3) group. In this case the pion charge radius and the kaon charge radius are equal and the radius of the neutral kaon is equal to zero.

II. THE INTERACTION LAGRANGIAN

In order to calculate the pion and kaon charge radii the interaction Lagrangian will be chosen in the following form

$$\mathcal{L}_I = \mathcal{L}_I^S + \mathcal{L}_I^{em}, \tag{2.1}$$

where the strong interaction part

$$\mathcal{L}_I^S(x) = g_p P^{KS}_{(x)}(\bar{q}^K_\alpha(x)\gamma_5 q^S_\alpha(x)) \tag{2.1a}$$

is invariant under the $SU(3)\times SU_c(3)$ symmetry. Here g_p is the coupling constant, $q(x)$ is the virton-quark field, $\alpha = 1, 2, 3$ are the colour indices and $k, s = 1, 2, 3$ are the indices of the SU(3) group, $P(x)$ is the the octet matrix of pseudoscalar mesons.

The electromagnetic interaction part

$$\mathcal{L}_I^{em}(x) = e[I_\mu^{em\ (\mathrm{hadrons})}(x) + I_\mu^{em\ (\mathrm{quarks})}(x)]A_\mu(x) \tag{2.1b}$$

is invariant under the $SU(3)\otimes SU_c(3)\otimes U(1)$ group. Here the hadron currents are taken in a standard way and the virton-quark currents in the following regularized form

$$[I_\mu^{em\ (\mathrm{quarks})}(x)]^\delta = \sum_{j=1}^{\infty}(-1)^j(q_{j\alpha}^{-\delta}(x)\gamma_\mu Q q_{j\alpha}^{\delta}(x)), \tag{2.2}$$

where

$$Q = \frac{1}{3}\begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is the charge matrix. We note that all questions about the regulirazation procedure can be found in ref. [2].

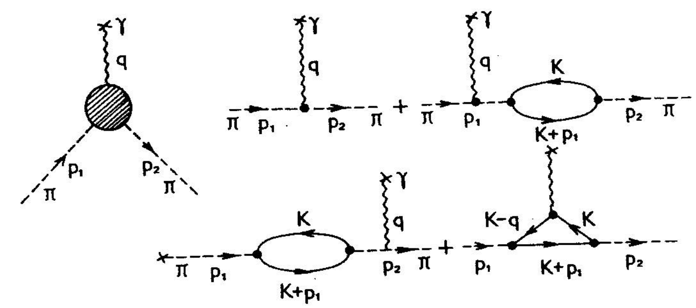

Fig. 1. Diagrams contributing to the pion electromagnetic form factor.

III. THE PION CHARGE RADIUS

The squared pion charge radius is defined through the pion form factor $F_\pi(q^2)$ by the following expression

$$\langle r_\pi^2 \rangle = -6\left.\frac{dF_\pi(q^2)}{dq^2}\right|_{q^2=0}. \tag{3.1}$$

The $F_\pi(q^2)$ is calculated by using the Lagrangian (2.1) within the framework of standard quantum field theory methods.

The relevant diagrams contributing to $F_\pi(q^2)$ are shown in Fig. 1. The corresponding T matrix element takes the form

$$T=-e\varepsilon_\mu(q^2)\left\{(p_1+p_2)_\mu+\left(\frac{i}{(2\pi)^4}3\left(\frac{2g_p}{ML}\right)^2\right)\frac{2}{l}\times\right. \tag{3.2}$$

$$\left.\times\left[\Lambda_\mu(p_1,p_2)+\frac{1}{2}(\Sigma'(p_1^2)+\Sigma'(p_2^2))(p_1+p_2)_\mu\right]\right\},$$

where

$$\Lambda_\mu(p_1,p_2)=\int d^4kSp\{d_\mu(-q)G(\hat{k})\gamma_5G(\hat{k}+\hat{p}_2)\gamma_5\}, \tag{3.2a}$$

$$\Sigma(p^2)=\int d^4kSp\{G(\hat{k})\gamma_5G(\hat{k}+\hat{p})\gamma_5\}, \tag{3.2b}$$

with

$$G(\hat{k})=\frac{1}{M}e^{-k^2}\left\{\cos\xi\sqrt{k^2}+\hat{k}\frac{\sin\xi\sqrt{k^2}}{\sqrt{k^2}}\right\}\quad\text{and}\quad\xi=\frac{2l}{L}$$

(ξ and L are free parameters defining all the dynamics of the virton-quark nonlocal model). Here all moments are taken in the Euclidean region.

The electromagnetic pion formfactor is obtained from eqs. (3.2)—(3.2a, b) and turns out to be

$$F_\pi(q^2)=1-\lambda(\mu_1^2-\mu_2^2)^2Y(\xi)+q^2\lambda\frac{L^2}{4}I(\xi), \tag{3.3}$$

where

$$Y(\xi)=\frac{3}{8}\int_0^\infty dt t^4\left\{([\cos\xi t\, e^{-t^2}]''')^2+\left(\left[\frac{\sin t\, e^{-t^2}}{t}\right]''''\right)^2\right\} \tag{3.3a}$$

and

$$I(\xi)=\frac{15}{4}\sum_{n=0}^{\infty}\frac{(-2)^n\xi^{2n+2}n!}{(2n+2)!}-\frac{1}{2}-\frac{\xi^2}{8}\left(1+\frac{\xi^2}{6}\right)-\xi^2\frac{5}{12}M\left(1,\frac{1}{2},-\frac{\xi^2}{2}\right)+ \tag{3.3b}$$

$$+\frac{\xi^2}{2}\left(3-\frac{\xi^2}{6}\right)M(2,3/2,-\xi^2/2)+\frac{\xi^2}{4}M(1,3/2,-\xi^2/2)+$$

$$+\frac{1}{3}M(3,1/2,-\xi^2/2)+\xi^2M(3,3/2,-\xi^2/2)+\left(\frac{2}{3}-\frac{\xi^2}{4}\right)M\left(2,\frac{1}{2},-\frac{\xi^2}{2}\right).$$

Here M are the degenerate hypergeometric functions and

$$\lambda=\frac{g_q^2}{(4\pi)^2}\frac{1}{(ML)^2}.$$

is the effective coupling constant <1 [2].

The expression (3.3) leads to the following form

$$\langle r_\pi^2\rangle^{1/2}=0.2718[I(\xi)]^{1/2}fm \tag{3.4}$$

of the square root of the mean squared pion radius, by taking the values of parameters of the model $L=\frac{1}{320\text{ MeV}}$, $\lambda=0.13$. The numerical values of these parameters were determined from the decay of mesons and baryons [2].

As one can see from (3.4) $\langle r_\pi^2\rangle^{1/2}$ is still the function of parameter ξ, for which the decay of mesons and baryons give $\xi=1.4$. For this value

$$\langle r_\pi^2\rangle^{1/2}=0.44\text{ fm}. \tag{3.5}$$

For the value $\xi=2.6$ the function (3.4) takes the maximum

$$\langle r_\pi^2\rangle^{1/2}=0.54\text{ fm}, \tag{3.6}$$

which is more realistic in comparison with the present knowledge on the pion and kaon charge radii [8].

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