Letters to the Editor

A NOTE ON THE FERROMAGNETIC FARADAY ROTATION OF X-RAYS*

ЗАМЕЧАНИЯ ПО ПОВОДУ ФАРАДАЕВСКОГО ФЕРРОМАГНИТНОГО ВРАЩЕНИЯ РЕНТГЕНОВСКИХ ЛУЧЕЙ

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The question of the ferromagnetic Faraday rotation (FFR) arose in connection with its possible role in the contrast of x-ray topographs of ferromagnetic domains [1], and superficial answers seemed to be at hand. At optical frequencies, FFR attains values of the order of 10^3 rad cm^{-1}, due to electron spinorbit coupling and atomic orbital resonances; however, far above the resonances this effect should be quite negligible since it is proportional to the off-diagonal condutivity, i.e., in the high frequency limit, inversely proportional to the square of frequency [2]. On the other hand, FFR originating in spin precession is proportional to the off-diagonal susceptibility multiplied by frequency, and has a frequency-independent tail far above the Zeeman resonance. Using the Landau–Lifshitz equation of motion for the magnetization in an $\boldsymbol{H}$ field

$$\dot{\boldsymbol{M}} = -\gamma \boldsymbol{M} \times \boldsymbol{H} \tag{1}$$

to get the macroscopic response to a light wave, we get the specific FFR

$$\varphi / l = 2\pi\gamma M / c \tag{2}$$

(c is the speed of light, $\gamma = e/mc$; cgs units are used for historical reasons). This phenomenon was often discussed in optical literature [3, 4] and should be measurable with modern x-ray techniques: for nickel the value obtained from (2) is 1.7 rad cm^{-1}. In 1932 Froman [5] reported x-ray experiments apparently confirming eq. (2) (derived earlier by Drude [6][1)]).

However, Hrdý, Krouský and Renner [7] proved that FFR of CuK_α radiation passing through a magnetized nickel foil is about fifty times smaller than the value predicted by eq. (2). They also notice that eq. (2) disagrees with field-theoretical calculations [8] which give zero FFR for the first-order forward scattering of photons on polarized electrons. Since the macroscopic argument concerning spin precession is not expected to fail so completely at wavelengths large compared to $\lambda_c = \hbar/mc$, we have to look for some compensating mechanism.

In fact, considering spin-orbit interaction in connection with atomic $\boldsymbol{E}$ fields, we did not take into

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[1)] The intuitive derivation given here is interesting: a magnetization field $\boldsymbol{M}_0$, rigidly connected to a medium deformed by the light wave, will also deform, at a rate $\boldsymbol{M} = -\boldsymbol{M}_0 \times \text{rot}\, \boldsymbol{v}$ if $\boldsymbol{v}$ is the local displacement rate in the medium and $\boldsymbol{v} . \boldsymbol{M}_0 = 0$. The Landau–Lifshitz equation (1) is obtained if $\boldsymbol{v}$ is identified with the free-electron dielectric response velocity, $\boldsymbol{v} = \gamma \boldsymbol{A}$.

account the $\boldsymbol{E}$ field of the light wave itself. Classically, a magnetic dipole $\boldsymbol{\mu}$ moving with velocity $\boldsymbol{v}$ develops an electric dipole moment $-\boldsymbol{\mu}\times\boldsymbol{v}/c$ in the rest frame; in addition to the purely magnetic effect the wave is also influenced by the induced electric polarization for which we get, with the average $\dot{\boldsymbol{v}} = -e\boldsymbol{E}/m$,

$$\dot{\boldsymbol{P}} = \gamma \boldsymbol{M} \times \boldsymbol{E}. \tag{3}$$

The Faraday rotation produced by (3) is of the same magnitude as (2) but of the opposite sign; thus the total FFR vanishes.

The compensation in the response of the Dirac field to electromagnetic radiation [8] can be analysed as follows. Ordinary first-order perturbations of a wave function containing only a uniform φ_2 in the four-component Dirac basis (belonging to an electron at rest, $s_z = -1/2$), in the field of two circularly polarized waves with real vector potential amplitudes $A^{\pm}$, are

$$\varphi_3^{\pm} = \frac{e}{\hbar} \frac{A^{\pm} e^{\pm i(\omega t - qz)}}{\omega_D \pm c^2 q^2/\omega \mp \omega} \varphi_2 , \tag{4}$$

where $\omega_D = 2mc^2/\hbar$, and $\varphi_1^{\pm} = -\varphi_3^{\pm}$. The response is gyrotropic if $\varphi_3^{+} \neq \varphi_3^{+}$; here, the two terms with different signs in the denominators of φ_3^{+} and φ_3^{-} mutually cancel ($cq = \omega$). However, their origin is diferent (the first comes from $\varphi_1^{\pm}$, i.e., from spin motion), and a separate calculation of the two gyrotropic current density perturbations to the order ω/ω_D exactly reproduces the response described by eqs. (1) and (3), respectively. For the spin-orbital effect alone, eq. (4) indicates a resonance at ω_D; the frequency-independent compensating (negative) FFR is thus a low-frequency limit with respect to ω_D. More precise calculations [8] show indeed a broad resonance of negative FFR around $\omega = 0, 6\omega_D$; however, only in the next-higher order in the fine-structure constant.

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