acta phys. slov. 30. (1980), No. 3

STOCHASTIC MODEL FOR SEQUENTIAL PRODUCTION PROCESSES

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In the present note we formulate explicitly the probability to observe (arbitrarily) given numbers of several sorts of particles; it is assumed that in the intermediate state represented by a modified (i. e. including forces) urn the processes run stochastically. In the corresponding limits already that approach leads to several forms of asymptotic relations which can be interpreted as the Pomeranchuk theorem or as the alternative dimensional counting rules.

СТОХАСТИЧЕСКАЯ МОДЕЛЬ ДЛЯ ПОСЛЕДОВАТЕЛЬНЫХ ПРОЦЕССОВ РОЖДЕНИЯ

В статье приведена явная формула вероятности наблюдения (произвольных) заданных количеств разных сортов частиц. При этом предполагается, что в промежуточном состоянии, представленном модифицированным (т. е. с включением сил) ящиком, процессы протекают стохастически. Уже этот упрощенный подход приводит в соответствующих пределах к различным видам асимптотических соотношений, которые можно интерпретировать как теорему Померанчука или же как альтернативные правила размерного счета.

I. INTRODUCTION

Increasing number of different models trying to explain the data in the high energy region leads to the suggestions to create an approach based on a simple formulation of the fundamental properties of the production phenomena under consideration. Let us mention e. g. the recent calls for "a standard jet model" [1], [2], the renewed claims that "the physical phenomena must be explainable in a simple intuitive form" [3] and the suggestion "to critically examine what is the minimal amount of theory needed to interpret the data (or at least most of it), keep that much, throw away the rest, and rebuild" [4].

It is worthwhile to emphasize that especially the urn models [5] sufficiently generalized and modified might be considered as serious candidates to become eventually "the standard models" (e.g. in the sense of Ref. [1]). This observation

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arises mainly due to the fact that the urn models constructed by means of simple and transparent assumptions lead to the probability distributions which behave quite often similarly (or even identically) to the experimental ones, [6] (and see, e.g., Ref. [7] for comparison with quantum vs. classical distributions, Ref. [8] for the estimate of hyperon structure functions and especially §5 of Ref. [9] for photocount distributions as well as the part 3b of Ref. [10] for some results concerning the quantum theory of laser), thereby suggesting the views that the processes in the corresponding intermediate states obey similar (or identical) rules. Moreover, it is not unusual to recognize that different ingenious and considerably complicated models involve more or less clearly also the assumptions on which just the urn models are built up.

Even if in this area of research it is expected that more sets of what might be called the master equations will be formulated (and the connections with Markovian — cf. especially part V-9 of Ref. [11] — as well as with non Markovian processes might be studied), it should be emphasized that the urn models presented until our days allow to derive quite often explicit expressions for different kinds of probability distributions. It is the purpose of the present contribution to give in an explicit form the probability to observe (arbitrarily) given numbers of given sorts of particles; some details to that point can be found in the next Section. The limiting cases of that probability are discussed in the III. Section; especially the results are derived there which can be interpreted in terms of several well-known asymptotic theorems. The last part contains several conclusions concerning also the next possible development in that region of research.

II. FORMULATION AND SOLUTION OF THE PROBLEM

1. The most of the models trying to describe the reactions leading to particle productions introduce some kind (or kinds) of intermediate state with some (say s) sorts of particles (objects, constituents) therein and with some (say β_r) particles of the r-th sort ($r = 1, 2, \ldots, s$). In that intermediate state it is possible to introduce the (atractive) forces acting on the particles of the r-th sort: they might be characterized by the parameters b_r, with $0 \leq b_r \leq 1$, [6]; the case of free particles corresponds to $b_r = 1$ while the case $b_r \to 0$ gives evidence on strong increase of the attractive forces.

Considering the production processes, one of the most important facts concerning the corresponding intermediate state (or "urn") is that the initial numbers of particles change (say, with increasing energy or with time, etc.). Let us describe this fact at least in the first approximation in such a way that if some (conveniently interpreted) action is performed upon the particle of the f-th sort ($f = 1, 2, \ldots, s$) then c_{fr} new particles of the r-th sort are created (or annihilated if the corresponding c_{fr} are negative).

The problem is to find the probability $\mathcal{R}$ that in N attempts (draws or steps in which e.g. the energy is increased) the sequence of particles of the following sort will be observed,

$$f_1, f_2, \ldots, f_H \equiv \{f\}_H, \tag{1}$$

$0 \leq H \leq N$, while in the remaining $N - H$ attempts no particle at all will be observed (the last possibility exists due to the introduction of the parameters b_r).

(All f_i's in rel. (1) are equal to any natural number from 1 to s and some (or all) of them are allowed to be equal: the last case leads to multiple production.)

The last fundamental assumption involved in the present note says that in the intermediate state the processes run stochastically.

2. For a given sequence (1) we get the probability

$$\mathcal{R} = \mathcal{R}(\{f\}_H). \tag{2}$$

Let us denote the sum of the probabilities of type (2) obtained for all permutations of the given sequence (1) as well as of all other admissible sequences together with all possible repetitions as

$$(\text{sum perm. } \mathcal{R})_H \tag{3}$$

(for a given H). The probability $\mathcal{R}$ is normalized in such a way that

$$\sum_{H=0}^{N} (\text{sum perm. } \mathcal{R})_H = 1. \tag{4}$$

For instance, let us consider two sorts of constituents ($s = 2$) and let $N = 2$; then rel. (4) has the following meaning,

$$\mathcal{R}(H=0) + \overbrace{[\mathcal{R}(1) + \mathcal{R}(2)]}^{H=1} + \overbrace{[\mathcal{R}(1,1) + \mathcal{R}(1,2) + \mathcal{R}(2,1) + \mathcal{R}(2,2)]}^{H=2} = 1.$$

3. If an attempt is performed (e.g. the energy increased in a given step) while no particle is observed at all then, in the present approach, let c_{or} particles of the r-th sort be created in the intermediate state. Therefore we take all parameters $c_{\varphi r}$ (with $\varphi = 0, 1, 2, \ldots, s$) as known from outer considerations. Moreover, let us introduce the parameters $c_{\varphi 0}$ and b_0 as follows,

$$c_{\varphi 0} = \beta_0 [\Sigma' c_{\varphi r}(1 - b_r)] / [\Sigma' \beta_r (1 - b_r)], \tag{5a}$$

$$b_0 = [\Sigma' \beta_r (1 - b_r)] / \beta_0 \tag{5b}$$

where β_0 is a real, positive (nonvanishing) and finite (otherwise arbitrary) number and

$$\Sigma' \equiv \sum_{r=1}^{s}.$$

Conditions (5a, b) guarantee the fulfillment of the normalization (4) (cf. Ref. [6]).

We assume that $b_0 \neq 0$, i.e. that at least one of the parameters b_r is not equal to unity.

4. With the procedure outlined essentially in Ref. [6] we obtain the probability $\mathcal{R}$ in the following form

a) if $H=0$,

$$\mathcal{R}=\left(\frac{b_0 c_{00}}{\Sigma' c_{0r}}\right)^N \frac{\Gamma\left(\frac{\beta_0}{c_{00}}+N\right)\Big/\Gamma\left(\frac{\beta_0}{c_{00}}\right)}{\Gamma\left(\frac{\Sigma'\beta_r}{\Sigma' c_{0r}}+N\right)\Big/\Gamma\left(\frac{\Sigma'\beta_r}{\Sigma' c_{0r}}\right)} \tag{6}$$

(rel. (6) gives the probability that in all N attempts no particle at all will be observed);

b) if $1 \leqslant H \leqslant N-1$,

$$\mathcal{R}=\left(\frac{b_0 c_{00}}{\Sigma' c_{0r}}\right)^{N-H} \frac{\Gamma\left(\frac{\beta_0+\sum_{i=1}^{H} c_{f_i,0}}{c_{00}}+N-H\right)\Big/\Gamma\left(\frac{\beta_0}{c_{00}}\right)}{\Gamma\left(\frac{\Sigma'(\beta_r+\sum_{i=1}^{H} c_{f_i,r})}{\Sigma' c_{0r}}+N-H\right)\Big/\Gamma\left(\frac{\Sigma'\beta_r}{\Sigma' c_{0r}}\right)} \times$$

$$\times \prod_{\sigma=1}^{H} \sum_{\substack{\tau_\sigma=\tau_{\sigma-1} \\ (\tau_0=0)}}^{N-H} b_{f_\sigma} \frac{\beta_{f_\sigma}+\sum_{i=1}^{\sigma} c_{f_i,f_\sigma}-c_{f_\sigma,f_\sigma}+\tau_\sigma c_{0,f_\sigma}}{\Sigma'\left(\beta_r+\sum_{i=1}^{\sigma} c_{f_i,r}-c_{f_\sigma,r}+\tau_\sigma c_{0r}\right)} \times$$

$$\times \frac{\Gamma\left(\frac{\beta_0+\sum_{i=1}^{\sigma} c_{f_i,0}-c_{f_\sigma,0}}{c_{00}}+\tau_\sigma\right)\Big/\Gamma\left(\frac{\beta_0+\sum_{i=1}^{\sigma} c_{f_i,0}}{c_{00}}+\tau_\sigma\right)}{\Gamma\left(\frac{\Sigma'(\beta_r+\sum_{i=1}^{\sigma} c_{f_i,r}-c_{f_\sigma,r})}{\Sigma' c_{0r}}+\tau_\sigma\right)\Big/\Gamma\left(\frac{\Sigma'(\beta_r+\sum_{i=1}^{\sigma} c_{f_i,r})}{\Sigma' c_{0r}}+\tau_\sigma\right)}; \tag{7}$$

c) if $H=N$

$$\mathcal{R}=\prod_{\sigma=1}^{H}\left[b_{f_\sigma}\frac{\beta_{f_\sigma}+\sum_{i=1}^{\sigma} c_{f_i,f_\sigma}-c_{f_\sigma,f_\sigma}}{\Sigma'(\beta_r+\sum_{i=1}^{\sigma} c_{f_i,r}-c_{f_\sigma,r})}\right] \tag{8}$$

(rel. (8) gives the probability that in all attempts a particle will be observed and namely in the sequence (1)).

III. ASYMPTOTICS

1. In the limit when all $\beta_r \to \infty$ while all $|c_{fr}| \ll \beta_r$ (i.e. we have the sea of every individual sort of constituents, say quarks or other objects, distinguishable at those energies) we obtain from rel. (7)

$$\mathcal{R} \to \binom{N}{H} \frac{(b_0\beta_0)^{N-H}}{(\Sigma'\beta_r)^N} \prod_{i=1}^{H} (b_i\beta_i). \tag{9}$$

The r.h.s. of rel. (9) can be rewritten in the form of the ("modified") binomial distribution,

$$\eta^N \binom{N}{H} P^H Q^{N-H}$$

where $P+Q=1$,

$$P=1/(1+\varepsilon), \quad Q=1/(1+\varepsilon^{-1}) \tag{10}$$

and

$$\varepsilon=b_0\beta_0\Big/\sqrt[H]{\prod_{i=1}^{H}(b_i\beta_i)}, \quad \eta=\left(1+\frac{1}{\varepsilon}\right)b_0\beta_0/\Sigma'\beta_r. \tag{11}$$

If all β_r approach infinity with the same speed, rel. (9) gives

$$\mathcal{R} \to \binom{N}{H}\left[\left(\prod_{i=1}^{H} b_i\right)/(s-\Sigma' b_r)^H\right][1-(\Sigma' b_r)/s]^N. \tag{12}$$

Let the probability $\mathcal{R}$, rel. (7) be interpreted as a (convenient) cross section. The asymptotic expression (9) for the particle-particle scattering (say, $p+p \to p+p$) will be equal to that expression for the antiparticle-particle scattering (say, $\bar{p}+p \to \bar{p}+p$) if we assume that (i) all β_r approach infinity with the same speed for particles as well as for antiparticles (in other words, that the sea of different sorts of constituents is equally dense), (ii) in the limit under consideration the forces acting on constituents which form the particles, are the same as the forces acting on constituents which form the antiparticles, (iii) the particles contain the same number of (valence, i.e. left after the limit $\beta_r \to \infty$) constituents as do the antiparticles. All three assumptions are physically plausible. As far as they are fulfilled the result can be interpreted as the Pomeranchuk theorem.

2. Using the normalization condition (5b), the limit $N \to \infty$ brings the r.h.s. of rel. (9) into the form

$$\mathcal{R} \to c \exp[-N(\Sigma' b_r \beta_r)/\Sigma' \beta_r] \tag{13}$$

where

$$c = (N^H/H!)\left\{\left[\prod_{i=1}^{H}(b_i\beta_i)\right]\Big/(\Sigma'\beta_r)^H\right\}[1-(\Sigma' b_r\beta_r)/(\Sigma'\beta_r)]^{-H}. \tag{14}$$

If our interest is not concentrated on the multiplicative factor in rel. (13), we put $c \approx 1$. Moreover, if

a) the sea is reached for all respective constituents with the same speed, rel. (13) gives

$$\mathcal{R} \to \exp[-N(\Sigma' b_r)/s] \tag{15}$$

where s is the number of sorts of constituents;

b) we do not introduce the normalization of the probability at the very beginning, the asymptotic form (13) will not contain the expression $\Sigma'\beta_r$; in this case rel. (13) gives

$$\mathcal{R} \to \exp[-N\Sigma' b_r \beta_r]. \tag{16}$$

3. The fact that in the first place the limit $\beta_r \to \infty$ was performed and then $N \to \infty$ (i.e. N tends to infinity more slowly than β_r), might be taken into account by the relation

$$N \sim \ln t$$

where t is a new variable (say, with the standard physical meaning). In this case rel. (15) gives

$$\mathcal{R} \to t^{-(\Sigma' b_r)/s} \tag{17}$$

while rel. (16) gives

$$\mathcal{R} \to t^{-\Sigma b_r \beta_r} \tag{18}$$

or

$$\mathcal{R} \to t^x \tag{19}$$

$$x = -\Sigma' \sum_{p=1}^{s} b_r^{(p)} \beta_r^{(p)}$$

where the constituents are written down separately for every subgroup of particles involved in the urn (i. e. a process is considered which involves s-sorts of particles, each sort containing s_r subgroups; a subgroup contains $\beta_r^{(p)}$ constituents). Rel. (17) allows to determine (at least in principle) the number of sorts of constituents from the asymptotic form (say, of the cross section) while rels. (16), (18), (19) represent alternative forms of the dimensional counting rule (which was discussed in some more detail in Ref. [12]).

IV. CONCLUSIONS

It is natural to expect that the results of the present note will be valid also in more refined models as far as the corresponding assumptions will be somehow contained (and not cancelled) there.

Several considerations involved in the present contribution allow extensions in different ways. For instance the region of independent variable N can be divided by thresholds into several sub-regions in which different sets of parameters s, β_r, $c_{\varrho\varphi}$ can be introduced. Moreover, it would be very convenient to include into those considerations also the appropriate conservation laws.

The procedure leading to the probability $\mathcal{R}$ of the present paper can be extended also to the case when the objects are observed which have not been contained in the intermediate state under consideration at its early stages or which are glued from several combinations of constituents.

Even if the parameters $c_{\varphi\varrho}$ (with $\varrho = 0, 1, 2, \ldots, s$) depend on the variable N or even on the starting numbers β_r (say, in such a way that the numbers of created particles depend on the numbers of existing particles), the asymptotic relations of the present form are obtained again as far as the condition $|c_{f\varrho}| \ll \beta_\varrho$ is fulfilled (for all indices) when the seas are arrived at (i.e. when $\beta_r \to \infty$).

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Received March 6th, 1980