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THE SELFCONSISTENT TREATMENT OF FLUCTUATIONS IN SUPERCONDUCTORS

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Fluctuations in superconductors within the framework of the Landau Ginzburg theory of superconductivity are discussed. Starting from the locally gauge invariant Lagrangian of this theory and considering selfconsistently the electromagnetic field fluctuations induced by the cooper pairs fluctuations it is shown that no divergent difficulties occur. Calculations give us a critical exponent different from that given in standard literature.

РАССМОТРЕНИЕ ФЛЮКТУАЦИЙ В СВЕРХПРОВОДНИКАХ В КАЧЕСТВЕ САМОСОГЛАСОВАННОГО ПОЛЯ

В статье обсуждается вопрос о флюктуациях в сверхпроводниках в рамках теории Гинзбурга–Ландау. Показано, что в теории не возникает расходимостей, если лагранжиан обладает локально градиентной инвариантностью и если флюктуации электромагнитного поля, вызванные флюктуациями куперовских пар, рассматриваются в качестве некоторого самосогласнованного поля. На основе вычислений получено значение критического показателя, отличное от значения, приводимого в соответствующей литературе.

I. INTRODUCTION

The calculations of fluctuations in superconductors, as it is well known in standard literature, presents various difficulties. That is why one has to take a cut-off in the momentum space to obtain the critical exponent equal to 1/2, as we show in the first part of this paper.

Hassing and Wilkins [1] have considered the mean-square fluctuations $\langle |\Psi_{OO}|^2 \rangle$ as a function of the temperature T, by using a renormalized temperature shift. By applying the functional transformation [2] and the cumulant expansion method [3] to the partition function, and a self-consistent mean-field approximation (similar to that of Marčelja [4]) within the framework of the time-dependent Ginsburg–Landau theory they have expressed the fluctuating free energy $F[\Psi]$ in a form suitable for the evaluation of functional integrals determinimg the quantity

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$\langle|\Psi_{00}|^2\rangle$. In their functional-integral approach to the critical fluctuations in superconductors they have considered a superconductor in contact with heat and particle reservoirs in the absence of external fields. However, in all the mentioned calculations the effect of fluctuations of the electromagnetic field induced by fluctuations of the Cooper pairs was ignored. That is why we are going to calculate fluctuations in superconductors by taking into account these fluctuations selfconsistently. The calculations, performed in 3-dimensional classical case analogously to that of the first section, give us the critical exponent equal to 3/4. The proposed method can be used also in the 2-dimensional case or in a 1-dimensional one (e.g. superconducting thin film).

It is important to emphasize that we need not make any cutoff because there are no divergent difficulties in the calculations of the mean-square fluctuations in superconductors.

In Marčelja's theory a supperssion of the divergence at T_c has been achieved by introducing a renormalized temperature shift, as it has been observed experimentally. Let us note that in the Marčelja theory the order parameter was considered as a quantized field. In our case the order parameter is considered as a classical field.

II. FLUCTUATIONS IN SUPERCONDUCTORS

The influence of fluctuating electron-pairing upon kinematical characteristics (conductivity, absorption coefficient, etc.) of „two dimensional" samples above the critical temperature T_c has been investigated by Aslamazov and Larkin [5] within the framework of the microscopic approach. In the transition region the BCS theory of superconductivity and the Ginsburg–Landau one give the same results.

The idea inerlying the Ginsburg–Landau theory of superconductivity is the expansion of the free energy density by the order parameter. When the electromagnetic field is absent, the free energy density in a superconductor has the following form [6]

$$f=f_n+\frac{\hbar^2}{2m^*}|\Psi|^2+a|\Psi|^2+\frac{b}{2}|\Psi|^4, \tag{1}$$

where f_n is the free energy density of the normal state, $m^*=2m_e$ is the mass of the Cooper pairs, m_e is the electron mass, $\hbar$ is the Planck constant, $a=\alpha(T-T_c)<0$ and $b/2$ is a positive phenomenological parameter (in such a case we have an eigenvalue spectrum of the Hamiltonian bounded from below). The field Ψ carries the charge $e^*=2e$, where e is the electron charge.

The free energy of system (1) plays the role of the Hamiltonian density of the solid and relates to the Lagrangian density as follows

$$f=\dot{\Psi}\frac{\partial L}{\partial\dot{\Psi}}+\dot{\Psi}^*\frac{\partial L}{\partial\dot{\Psi}^*}+\dot{\Psi}_i\frac{\partial L}{\partial\dot{\Psi}_i}+\dot{\Psi}_i^*\frac{\partial L}{\partial\dot{\Psi}_i^*}-L, \tag{2}$$

where the field Ψ_i describes ions carrying the charge $-e$, and the dots denote the time derivatives.

The simplest form of the Lagrangian density $\mathscr{L}$ satisfying the relations (1) and (2) is

$$\mathscr{L}=\frac{\hbar}{2i}(\Psi\dot{\Psi}^*-\dot{\Psi}\Psi^*)+\frac{\hbar}{2i}(\Psi_i\dot{\Psi}_i^*-\dot{\Psi}_i\Psi_i^*)-$$
$$-\frac{\hbar^2}{2m^*}|\nabla\Psi|^2-a|\Psi|^2-\frac{b}{2}|\Psi|^4-f_n. \tag{3}$$

The Langrangian density (3) is invariant under the global gauge transformation group. The equations of motion we obtain by using the minimal action principle with the Lagrangian density (3) are

$$i\hbar\dot{\Psi}=-\frac{\hbar^2}{2m^*}\nabla\Psi+a\Psi+b|\Psi|^2\Psi \tag{4}$$
$$i\hbar\dot{\Psi}_i=0$$

plus complex conjugated equations. The solutions of these equations must fulfil the following boundary condition

$$\boldsymbol{\tau}\left(\frac{\hbar}{i}\nabla\Psi\right)=0, \tag{5}$$

where $\boldsymbol{\tau}$ is the unit normal vector to the surface of the superconductor. For the ground state of the superconductor the solution of Eqs. (4) is the time and coordinates independent constant Ψ_0 and the following relation holds

$$\Psi_0\Psi_0^*=-\frac{a}{b}. \tag{6}$$

Let us express the parameter $\Psi(\mathbf{x})$ by the sum of the constant Ψ_0 and the fluctuating part as given by

$$\Psi(\mathbf{x})=\Psi_0+\frac{1}{\sqrt{V}}\sum_{\mathbf{k}}a_{\mathbf{k}}e^{i\mathbf{k}.\mathbf{x}}, \tag{7}$$

where $a_{\mathbf{k}}$ are dimensionless fluctuating parameters, and V is the total volume of the superconductor.

If we insert the expansion (7) in the expression (1) and integrate it over the total volume V, we get (in the lowest order of parameters $a_{\mathbf{k}}$) the free energy

$$F=F_n-\frac{a^2}{2b}V+\sum_{\mathbf{k}}a_{\mathbf{k}}^*a_{\mathbf{k}}\left(\frac{\hbar^2k^2}{2m^*}-a\right). \tag{8}$$

The mean-square fluctuation for a given wave vector **k** is [7]

$$\langle a_{\mathbf{k}}^* a_{\mathbf{k}} \rangle = K \int (a_{\mathbf{k}}^* a_{\mathbf{k}}) \exp\left[-\frac{F(a_{\mathbf{k}}^* a_{\mathbf{k}})}{k_B T}\right] \mathrm{d}a_{\mathbf{k}} = \frac{k_B T}{2\left(\frac{\hbar^2 \mathbf{k}^2}{2m^*} - a\right)}. \tag{9}$$

For the mean-square fluctuation at an arbitrary point of a superconductor we have

$$\langle a^* a \rangle = \frac{1}{2} k_B T \frac{V}{(2\pi)^3} \int \frac{\mathrm{d}^3\mathbf{k}}{\frac{\hbar \mathbf{k}^2}{2m^*} - a} = \frac{k_B T V}{(2\pi)^2} \int_0^\infty \frac{k^2 \mathrm{d}k}{\frac{\hbar^3 k^2}{m^*} - a}. \tag{10}$$

The last integral in (10) is divergent, that is why one is forced to introduce a cut-off in the momentum space $|\mathbf{k}| = \frac{2\pi}{\xi}$, where ξ is the coherent length

$$\xi = \sqrt{-\frac{\hbar^2 n}{2m^* a}} \tag{11}$$

(n denotes the number of Cooper pairs). Then the following relation holds

$$\langle a^* a \rangle \approx \frac{k_B T V}{(2\pi)^2} \int_0^{2\pi/\xi} \frac{k^2 \, \mathrm{d}k}{(-a)} = \frac{2\pi}{3} k_B T V \frac{1}{(-a)\,\xi^3} \sim (T_c - T)^{\frac{1}{2}}. \tag{12}$$

The critical exponent is equal to 1/2. We believe that in this simple model the difficulties encountered are due to an ignorance of the electromagnetic field fluctuations induced by fluctuations of the Cooper pairs.

III. FLUCTUATIONS OF THE COOPER PAIRS AND THE ELECTROMAGNETIC FIELD IN SUPERCONDUCTORS

Fluctuations of the Cooper pairs induce fluctuations of the electromagnetic field and the fluctuations of the electromagnetic field react on the fluctuations of the Cooper pairs. We are going to take these fluctuations selfconsistently. The electromagnetic potentials **A**, φ are fluctuating around the mean value $\mathbf{A} = 0$, $\varphi = 0$ (for the ground state) and selfconsistently influence the fluctuations of Cooper pairs in superconductors. We introduce the interaction with the electromagnetic field through the minimal coupling. The Lagrangian density of the free electromagnetic field

$$\mathscr{L}_{elmag} = \frac{1}{2}\left(\varepsilon_0 \mathbf{E}^2 - \frac{1}{\mu_0} \mathbf{B}^2\right) \tag{13}$$

is added to the Lagrangian density (3), where **E** is the fluctuating electric density and **B** is the fluctuating magnetic induction. If we take into account that the magnetic field in superconductor is highly suppressed (Meissner effect) we can put

$\mathbf{B} = 0$. The field derivatives in (3) are replaced by covariant derivatives, constructed by using the fluctuating potentials **A** and φ. The total Lagrangian density of the solid is

$$\mathscr{L} = \frac{1}{2} \varepsilon_0 \mathbf{E}^2 + \frac{\hbar}{2\mathrm{i}} (\Psi \dot{\Psi}^* - \dot{\Psi} \Psi^*) + \frac{\hbar}{2\mathrm{i}} (\Psi_i \dot{\Psi}_i^* - \dot{\Psi}_i \Psi_i^*) -$$
$$- \frac{\hbar^2}{2m^*} |\nabla \Psi|^2 + \frac{\mathrm{i}e^*\hbar}{2m^*} \mathbf{A} \; (\Psi \nabla \Psi^* - \Psi^* \nabla \Psi) - \frac{e^{*2}}{2m^*} \mathbf{A}^2 |\Psi|^2 -$$
$$- e^* \varphi |\Psi|^2 + e\varphi |\Psi_i|^2 - a|\Psi|^2 - \frac{b}{2} |\Psi|^4 \tag{14}$$

and it is locally gauge invariant. We introduce now the following parametrization for the order parameters

$$\Psi(\mathbf{x}, t) = \sqrt{n(\mathbf{x}, t)} \exp\left\{\frac{\mathrm{i}e^*}{\hbar} \zeta(\mathbf{x}, t)\right\}$$
$$\Psi_i(\mathbf{x}, t) = \sqrt{N(\mathbf{x}, t)} \exp\left\{\frac{-\mathrm{i}e}{\hbar} \vartheta(\mathbf{x}, t)\right\}, \tag{15}$$

where $n(\mathbf{x}, t)$, $\zeta(\mathbf{x}, t)$ $B(\mathbf{x}, t)$ and $\vartheta(\mathbf{x}, t)$ are real functions. If we use the minimal action principle with the Lagrangian density (14) the equations of motion take the following form

$$\frac{\partial n}{\partial t} + \mathrm{div}\,(n\mathbf{v}) = 0, \qquad \mathbf{v} = \frac{e^*}{m^*} (\nabla \zeta - \mathbf{A}), \tag{16}$$
$$\frac{\partial N}{\partial t} = 0$$
$$e^*(\dot{\zeta} + \varphi) = -\frac{1}{2} m^* v^2 + \frac{\hbar}{2m^* \sqrt{n}} \Delta \sqrt{n} - a - bn$$
$$+ e(\dot{\vartheta} + \varphi) = 0$$
$$-\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\varepsilon_0} e^* n \mathbf{v}$$
$$\mathrm{rot}\,\mathbf{E} = 0$$
$$\mathrm{div}\,\mathbf{E} = \frac{e^*}{\varepsilon_0} \left(n - \frac{1}{2} N\right).$$

The solutions of Eqa. (16) have to satisfy the following boundary conditions

$$\boldsymbol{\tau} \,.\, \mathbf{j} = 0, \qquad \boldsymbol{\tau} \,.\, \mathbf{E} = 0, \tag{17}$$

where the current **j** has the following form

$$\mathbf{j}=\frac{e^{*2}n}{m^*}(\nabla\zeta-\mathbf{A}). \tag{18}$$

Taking into account the gauge invariance of electromagnetic interactions we can choose the new electromagnetic potentials

$$\mathbf{A}-\nabla\zeta\to\mathbf{A},\qquad \varphi+\frac{\partial\zeta}{\partial t}\to\varphi \tag{19}$$

and introduce a new phase $\vartheta\to\Theta = \vartheta-\zeta$, in such a case we have $\varphi=-\dot{\Theta}$. We have fixed the gauge by choosing the potentials in the form (19) and the field $\zeta(\mathbf{x}, t)$ disappeared from the equations of motion (16).

Similarly as in [8] at the stable point

$$n=\frac{1}{2}n_0\neq 0 \tag{20}$$

the linearized equation

$$\frac{\partial^2\mathbf{E}}{\partial t^2}+\frac{\hbar^2}{4m^{*2}}\Delta\Delta\mathbf{E}-\frac{n_0 b}{m^*}\Delta\mathbf{E}+\omega_0^{*2}\mathbf{E}=0, \tag{21}$$

where

$$\omega_0^{*2}=\frac{e^{*2}n_0}{m^*\varepsilon_0} \tag{21a}$$

is obtained in the first order of the perturbation theory. The solution of Eq. (21) has the form of superposition of the standing waves of the following form

$$\mathbf{E}(\mathbf{x}, t)=\frac{4e^*\hbar}{\varepsilon_0\sqrt{2Vbm^*}}\sum_{\mathbf{k}}\Lambda(\mathbf{k})\sin k_1x\sin k_2y\sin k_3z\cos\Omega(\mathbf{k})t, \tag{22}$$

where

$$\Omega^2(\mathbf{k})=\frac{\hbar^2}{4m^{*2}}(k^2)^2+\frac{n_0 b}{m^*}\mathbf{k}^2+\omega_0^{*2} \tag{23}$$

and $\Lambda(\mathbf{k})$ are dimensionless fluctuating parameters.

If we use the parametrization (15) for the order parameters and take the last three relations from equations (16), we can rewrite the free energy density, corresponding to (14), as follows

$$f=\frac{\hbar^2\varepsilon_0^2}{8m^*e^{*2}n_0}(\Delta\mathbf{E})^2+\frac{m^*\varepsilon_0^2}{2e^{*2}n_0}\left(\frac{\partial\mathbf{E}}{\partial t}\right)^2+\frac{1}{2}\varepsilon_0\mathbf{E}^2-$$
$$-\frac{b\varepsilon_0^2}{2e^{*2}}\mathbf{E}\Delta\mathbf{E}+\operatorname{div}\left\{\mathbf{E}\left[\frac{\varepsilon_0}{e^*}\left(\frac{5a}{2}+\frac{bn}{2}\right)\right]\right\}-\frac{a^2}{2b}+f_n. \tag{24}$$

By putting solution (22) into (24) and integrating over the total volume V we obtain

$$F=\int_V f\,\mathrm{d}V=F_n-\frac{a^2}{2b}V+\sum_{\mathbf{k}}\Lambda^2(\mathbf{k})\left(\frac{\hbar^2k^2}{2m^*}+\frac{e^{*2}\hbar^2}{2m^*b\varepsilon_0}+\right.$$
$$\left.+\frac{\hbar^4k^4}{8m^{*2}bn_0}\right). \tag{25}$$

The dimensionless parameters $\Lambda^2(\mathbf{k})$ can be expressed as $\Lambda^2(\mathbf{k}) = a_{\mathbf{k}}^* a_{\mathbf{k}}$ so that they may correspond to the relation (8). The mean-square fluctuation for a given wave vector $\mathbf{k}$ has the following form

$$\langle a_{\mathbf{k}}^* a_{\mathbf{k}}\rangle=\frac{1}{2}\left(\frac{k_BT}{\frac{\hbar^2\mathbf{k}^2}{2m^*}+\frac{e^{*2}\hbar^2}{2m^*b\varepsilon_0}+\frac{\hbar^4\mathbf{k}^4}{8m^{*2}bn_0}}\right). \tag{26}$$

After integration over $\mathrm{d}^3\mathbf{k}$ we obtain for the mean-square fluctuation the following expression

$$\langle a^*a\rangle=\frac{V}{(2\pi)^3}\int\langle a_{\mathbf{k}}^* a_{\mathbf{k}}\rangle\,\mathrm{d}^3\mathbf{k} \tag{27}$$

$$=\frac{1}{2}k_BT\frac{4\pi V}{(2\pi\hbar)^3}\frac{8m^{*2}bn_0}{\hbar}\int_0^\infty\frac{k^2\,\mathrm{d}k}{k^4+2\beta k^2+\alpha}$$

$$=\frac{2^{1/2}k_BTbV}{4\pi^{5/4}\hbar^{7/2}e^{*1/2}}n_0^{3/4}\sim(T_c-T)^{3/4}, \tag{27}$$

where

$$\alpha=\frac{4m^{*2}\omega_0^{*2}}{\hbar^2},\qquad 2\beta=\frac{4m^*bn_0}{\hbar^2} \tag{28}$$

and we have taken $\alpha\gg\beta V^{-2/3}$.

We see that the critical exponent is equal to 3/4 in a case when the mutual influence of fluctuations of the electromagnetic field and those of the pairs have been selfconsistently taken into account.

IV. CONCLUSION

By using the local gauge invariant Lagrangian instead of the global gauge invariant we have shown within the framework of the Landau–Ginzburg theory of superconductivity that there are no divergent difficulties in calculations of the mean-square fluctuations below the critical point T_c.

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