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STOCHASTIC CHARACTERISTICS OF THE BURST NOISE

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The burst noise (popcorn noise), which consists of a series of pulses whose amplitude is constant and width is random, is described by means of the generation-recombination process. Relations for the distribution function, spectral density and other statistical characteristics are derived. Furthermore, statistical characteristics of a combined process which consists of the burst noise and the other kinds of noise with normal probability density, are described. The probability density of the resulting process is then made up by a superposition of two normal distribution functions, separated from each other by the amplitude of the burst noise.

СТОХАСТИЧЕСКИЕ ХАРАКТЕРИСТИКИ ВЗРЫВНЫХ ШУМОВ

Взрывные шумы, содержащие последовательность импульсов с постоянной амплитудой и случайной шириной, описаны при помощи процесса генерации и рекомбинации. Выведены формулы для функции радпределения, спектральной плотности и других статистических характеристик. Описаны также статистические характеристики комбинированного процесса, содержащего взрывные шумы и другие виды шумов с плотностью вероятности, имеющей нормальное распределение. Плотность вероятности результирующего процесса представлена в виде суперпозиции двух функций нормального распределения, взаимно разделенных амплитудой взрывных шумов.

I. INTRODUCTION

In many semiconductor devices the burst noise makes up — beside the $1/f$ noise — a prominent component of the audiofrequency noise. The series of pulses of the burst noise is characterized by the random width of the pulses. The pulse width distribution is exponential and the average pulse width is within a very large interval. Experiments give values ranging from 10^{-6} s to 10^{3} s [1, 2]. The pulse current amplitude depends on the current, temperature and other quantities and can be as large as 10^{-10} A. The physical nature of this noise has been investigated by a number of authors [1 through 6]. W. H. Card [3] assumes that the burst noise

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is caused by a modulation of the current flowing through the potential barrier. In many cases a correlation between the burst noise and the excess current has been observed. S. T. Hsu and R. J. Whittier [4] propose that large defects in a single crystal localized in the P—N junction depleted layer builds up Schottky barriers that in turn produce the burst noise as well as the excess current.

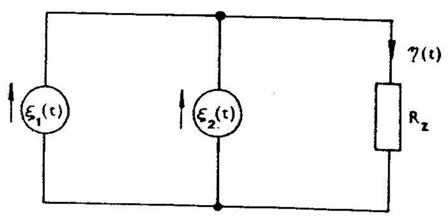

Fig. 1.

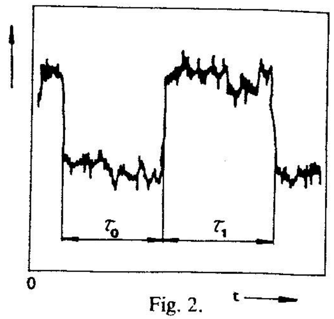

Fig. 2.

In the present paper we give a statistical model of the burst noise. In our model one component will be that of the burst noise while the other will comprise all the other noise sources, i.e. the $1/f$ noise, the white noise, etc.

In Fig. 1 the random process $\eta(t)$ represents the total noise current passing through the load resistor R_z. The process $\eta(t)$ itself is taken as a superposition of the burst noise current $\xi_2(t)$ and the current due to all the other noise sources, $\xi_1(t)$. A typical shape of the $\eta(t)$ realization is in Fig. 2.

Let the burst noise amplitude be A, the pulse width τ_1 and the pulse separation τ_0. The quantities τ_1 and τ_0 are random quantities, whose probability densities are $w(s_1)$ and $w(s_0)$, where s_1 and s_0 are the values of the stochastic quantities τ_1, τ_0, whose mean values are $\bar{\tau}_1$, $\bar{\tau}_0$. The total noise current $\eta(t)$ is

$$\eta(t)=\xi_1(t)+A\sum_i F\left(\frac{t-t_i}{\tau_{1i}}\right), \tag{1}$$

where $F(\Theta)=1$, for $\Theta \in (0, 1)$

$$F(\Theta)=0, \quad \text{if} \quad \Theta \notin (0, 1)$$

and the random quantity t_i corresponds to the instant of the i-th impulse birth. If the processes $\xi_1(t)$, $\xi_2(t)$ are independent, then the characteristics of the process $\xi_1(t)$ can be determined from the measurement od the noise of an equivalent element which does not exhibit the burst noise.

II. CHARACTERISTIC OF THE $\xi_1(t)$ PROCESS

The statistical characteristics of the $\xi_1(t)$ process depend on the kind of the investigated semiconductor device in question. Usually, this process is one of the following types: the $1/f$ noise, the shot noise and the thermal noise with normal probability density. So, for example, the element led to the result that the one-dimensional probability density of the $\xi_1(t)$ process has the normal form

$$w(x_1)=\frac{1}{\sqrt{2\pi}\sigma_1}\,\mathrm{e}^{-\frac{x_1^2}{2\sigma_1^2}} \tag{2}$$

where σ_1^2 is the dispersion of the $\xi_1(t)$ process.

III. CHARACTERISTICS OF THE $\xi_2(t)$ PROCESS

The $\xi_2(t)$ process is described by

$$\xi_2(t)=A\sum_i F\left(\frac{t-t_i}{\tau_{1i}}\right) \tag{3}$$

A typical realization of this process is in Fig. 3. The $\xi_2(t)$ process features two states. In the state 1 there holds $\xi_2(t) = A$; in the state 0 we have $\xi_2(t) = 0$. The durations of the states 1, 0 are τ_1, τ_0, respectively, and they are, evidently, random quantities. Now we find the probabilities P_1 and P_0 of the system to be in the states 1 and 0, respectively. Further, we find the probability densities $w(s_1)$ ans $w(s_0)$ assuming that the $\xi_2(t)$ random process is formulated as follows:

a) If at the time t system is in the state 0, then the probability of transition into the state 1 within the interval $(t, t+\Delta t)$ is $g\Delta t+0(\Delta t)$.

b) If at the time t system is in the state 1, then the probability of transition into the state 0 within the ingerval $(t, t+\Delta t)$ is $r\Delta t+0(\Delta t)$.

First we derive the probabilities $P_0(t)$ and $P_1(t)$ that the system is at the time t in the state 0 and 1, respectively. There holds

$$P_0(t+\Delta t)=P_1(t)\,r\Delta t+P_0(t)\,[1-g\Delta t]+0(\Delta t), \tag{4}$$

$$P_1(t+\Delta t)=P_0(t)\,g\Delta t+P_1(t)\,[1-r\Delta t]+0(\Delta t). \tag{5}$$

After rearranging and finding the limit for $\Delta t \to 0$ we obtain differential equations for the probabilities $P_0(t)$ and $P_1(t)$:

$$\frac{\mathrm{d}P_0(t)}{\mathrm{d}t}=rP_1(t)-gP_0(t), \tag{6}$$

$$\frac{\mathrm{d}P_1(t)}{\mathrm{d}t}=gP_0(t)-rP_1(t). \tag{7}$$

The Laplace transformation with the initial conditions $P_1(0)=1$, $P_0(0)=0$ leads provided that g and r are independent of time, to the following equations

$$pP_0(p)=rP_1(p)-gP_0(p), \tag{8}$$

$$pP_1(p)=gP_0(p)-rP_1(p)+1. \tag{9}$$

Solving this system and returning to the original we get

$$P_0(t)=\frac{r}{g+r}\,1(t)-\frac{r}{g+r}\,e^{-(g+r)t}, \tag{10}$$

$$P_1(t)=\frac{g}{g+r}\,1(t)+\frac{r}{g+r}\,e^{-(g+r)t}, \tag{11}$$

where the function $1(t)=0$ for $t<0$ and $1(t) = 1$ for $t\geqq 0$. For a stationary state we obtain

$$P_0=\lim_{t\to\infty} P_0(t)=\frac{r}{g+r}, \tag{12}$$

$$P_1=\lim_{t\to\infty} P_1(t)=\frac{g}{g+r}. \tag{13}$$

Now we determine the distribution function $F_2(x_2) = P\{\xi_2(t)\leqq x_2\}$. In the stationary state there holds

$$F_2(x_2)=0, \quad x_2<0 \tag{14}$$

$$F_2(x_2)=P_0, \quad 0\leqq x_2<A$$

$$F_2(x_2)=1, \quad x_2\geqq A.$$

The distribution function vs. the x_2 diagram is plotted in Fig. 4. There holds

$$F_2(x_2)=\frac{r}{g+r}\,1(x_2)+\frac{g}{g+r}\,1(x_2-A). \tag{15}$$

From (15) we can find the probability density of the $\xi_2(t)$ process:

$$w_2(x_2)=\frac{r}{g+r}\,\delta(x_2)+\frac{g}{g+r}\,\delta(x_2-A). \tag{16}$$

Other important characteristics of the burst noise are the probability densities $w(s_1)$ and $w(s_0)$ of the random quantities τ_1 and τ_0. We derive them for the $\xi_2(t)$ process. Let us look for the probability $P_1^-(t, t+s_1)$ of the event that within the interval $(t, t+s_1)$ the state of the system does not change provided that at the time t the system is in the state 1. There holds

$$P_1^-(t, t+s_1+\Delta s_1)=P_1^-(t, t+s_1)\,[1-r\Delta s_1]. \tag{17}$$

After rearranging and expressing the limit we obtain the following differential equation

$$dP_1^-/ds_1=-P_1^-r,$$

which has a solution

$$P_1^-(t, t+s_1)=e^{-rs_1}, \tag{18}$$

for $r(t)$ = const.

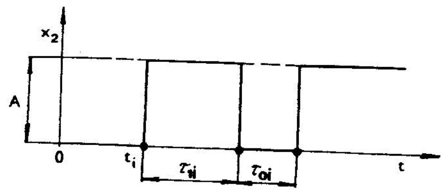

Fig. 3.

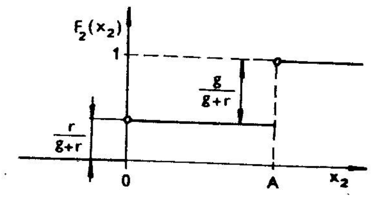

Fig. 4.

The probability that within the interval $(t, t+s_1)$ the state 1 of the system does change is then $1-e^{-rs_1}$. The distribution function of the random quantity τ_1 is

$$F(s_1)=P\{\tau_1\leqq s_1\}=1-e^{-rs}. \tag{19}$$

The probability density of τ_1 is

$$w(s_1)=\frac{dF}{ds_1}=re^{-rs_1}. \tag{20}$$

The mean value of τ_1 is

$$\bar{\tau}_1=r\int_0^\infty s_1e^{-rs_1}ds_1=\frac{1}{r}. \tag{21}$$

Similarly, for the random quantity τ_0 we may express the probability density:

$$w(s_0)=ge^{-gs_0} \tag{22}$$

and the mean value

$$\bar{\tau}_0=\frac{1}{g}. \tag{23}$$

To determine the spectral density of $\xi_2(t)$ we may start from the correlation function $B(t, u) = \overline{\xi_2(t)\xi_2(t+u)}-\overline{\xi_2(t)}\,\overline{\xi_2(t+u)}$. If the $\xi_2(t)$ process has the form (3), then, assuming that the individual impulses are statistically independent, we get (see, e.g. [7]):

$$B(u)=A^2\frac{rg}{(r+g)^2}e^{-(g+r)|u|}. \qquad (24)$$

By the Fourier transformation of (24) we obtain the spectral density of the $\xi_2(t)$ process:

$$S(\omega)=\frac{grA^2}{\pi(g+r)}\frac{1}{(g+r)^2+\omega^2}. \qquad (25)$$

IV. STATISTICAL CHARACTERISTICS OF THE $\eta(t)$ PROCESS

On the assumption that the $\xi_1(t)$ and $\xi_2(t)$ processes are statistically independent the mean value is

$$\overline{\eta(t)}=\overline{\xi_1(t)}-\overline{\xi_2(t)}, \qquad (26)$$

the dispersion

$$\sigma_\eta^2=\sigma_{\xi_1}^2+\sigma_{\xi_2}^2 \qquad (27)$$

and the spectral density

$$S_\eta(\omega)=S_{\xi_1}(\omega)+S_{\xi_2}(\omega). \qquad (28)$$

Let us find the probability density $w(y)$ of the $\eta(t)$ process, where $\eta(t)=\xi_1(t)+\xi_2(t)$. For independent processes the mutual two-dimensional probability density of $\xi_1(t)$ and $\xi_2(t)$ is

$$w(x_1,x_2)=w_1(x_1)\,w_2(x_2). \qquad (29)$$

The probability density $w(y)$ of the $\eta(t)$ process is [7]

$$w(y)=\int_{-\infty}^{\infty} w_1(u)\,w_2(y-u)\,du. \qquad (30)$$

If we put for $w_2(y-u)$ from (16), we obtain

$$w(y)=\int_{-\infty}^{\infty} w_1(u)\left[\frac{r}{g+r}\delta(y-u)+\frac{g}{g+r}\delta(y-u-A)\right]du=$$

$$=\frac{r}{g+r}w_1(y)+\frac{g}{g+r}w_1(y-A). \qquad (31)$$

If, for example, the $\xi_1(t)$ process has a normal distribution function (2), then the probability density of the resulting process is

$$w(y)=\frac{1}{\sigma_1\sqrt{2\pi}}\left\{\frac{r}{g+r}\exp\left[-\frac{y^2}{2\sigma_1^2}\right]+\frac{g}{g+r}\exp\left[-\frac{(y-A)^2}{2\sigma_1^2}\right]\right\}. \qquad (32)$$

The probability density of the resulting process, $\eta(t)$, is given as a superposition of two normal distributions separated by the amplitude A of the burst noise. In Fig. 5 the probability density $w(y)$ for $A=4$, $\sigma_1=1$, $r/g=0.428$ is plotted. The shape of the $w(y)$ curve depends on the ratio $A/\sigma_1\gg 1$, the amplitude of the burst noise is equal to the separation of the maxima of the individual probability density curves. From the relative heights of the maxima we then determine the value of the r/g ratio.

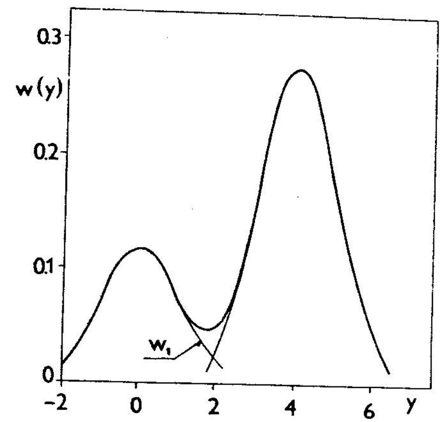

Fig. 5.

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