

## THE NUCLEAR EQUILIBRATION PROCESS — THE MASTER EQUATION AND CLOSED-FORM APPROXIMATIONS\*

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The general master equation is specified for the nuclear equilibration process and applied to the case where specific particle-hole configurations of exciton states are explicitly taken into account. In this way it is shown that  $\lambda_0$  transition rates do not appear in the usual master equation which contains transition and emission rates averaged over the configurations. Different closed-form approximations and their mutual connections are discussed within the pre-equilibrium exciton model. It is shown that under certain conditions such approximations yield fits to experimental data of about the same quality as more elaborate master-equation calculations.

### ЯДЕРНЫЕ РАВНОВЕСНЫЕ ПРОЦЕССЫ — ОПРЕДЕЛЯЮЩЕЕ УРАВНЕНИЕ И ДРУГИЕ БЛИЗКИЕ ПО ФОРМЕ ПОДХОДЫ

Для ядерных равновесных процессов найдено определяющее уравнение, которое применено для случая, когда в явном виде учтены специфические конфигурации частиц и дырок возбужденных состояний. На основе этого показано, что скорости  $\lambda_0$  переходов не входят в обычное определяющее уравнение, которое содержит скорости переходов и испускания, усредненное по конфигурациям. Осуждаются также разные близкие по форме приближения и их взаимосвязь в рамках предпринятой экситонной модели. Показано, что при определенных условиях такие приближения согласуются с экспериментальными данными с той же самой точностью, как и более тщательные вычисления на основе определяющего уравнения.

### 1. INTRODUCTION

The equilibration process after the initial projectile-target interaction in a nucleus is commonly envisaged as proceeding via a chain of intermediate states characterized by a number of excited particles and corresponding holes called

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together excitons. The master equation and different closed-from approaches used for the description of such a process follow from the general master-equation approach upon the introduction of a number of assumptions based on the properties of nuclear interaction.

The master equation for the nuclear equilibration process, widely used in literature [1—7], contains transition and emission rates averaged over specific particle-hole configurations. In this equation,  $\lambda_0$  transitions do not appear explicitly. However, the general master equation for a system of equilibrating nucleons in which each specific particle-hole configuration is treated separately, contains all transition rates,  $\lambda_{++}$ ,  $\lambda_-$  and  $\lambda_0$ . Starting from such a complete master equation, it is possible to derive a master equation valid for averaged transition and emission rates which do not contain terms with  $\lambda_0$ . This will be done in Section II.

Integrating the master equation over time, one obtains a system of algebraic equations for a time  $T(n, E)$  spent by the system (composite nucleus) in an  $n$ -exciton state at energy  $E$ . From these equations it is possible to obtain a closed-form solution for  $T(n, E)$  by the iterative method. This point will be discussed in Section III. Application of different approximations for transition and emission rates appearing at  $T(n, E)$  leads to different approximations for  $T(n, E)$ . The derivation of these closed-form approximate solutions and the conditions for their applicability will be discussed in Section IV.

## II. THE MASTER EQUATION FOR SPECIFIC PARTICLE-HOLE CONFIGURATIONS

A specific nuclear configuration  $\gamma$  of a given exciton state is determined by a fixed distribution of particles and holes. The probability of finding a system in such a specific exciton state  $(n, \gamma)$  at a time  $t$  is denoted by  $P(n, \gamma, t)$ . The transition rates (transition probability per unit time) from the state  $(n, \gamma)$  to a state  $(n', \gamma')$  are denoted by  $\lambda_{\pm,0}(n, \gamma, \gamma')$  ( $n' = n \pm 2$ , or  $n$ ). The total emission rate from the  $(n, \gamma)$  state is given by

$$W_c(n, \gamma) = \sum_{\beta} \int_0^{\epsilon_{\max}} W_{\beta}(n\epsilon_{\beta}, \gamma, \epsilon_{\beta}) d\epsilon_{\beta}, \quad (1)$$

where  $\beta$  denotes various emitted particles.

The master equation describing the equilibration of such a system can be written as follows (see Fig. 1):

$$\frac{dP(n, \gamma, t)}{dt} = \left\{ \frac{dP(n, \gamma, t)}{dt} \right\}_+ + \left\{ \frac{dP(n, \gamma, t)}{dt} \right\}_0 + \left\{ \frac{dP(n, \gamma, t)}{dt} \right\}_-, \quad (2)$$

where

$$\left\{ \frac{dP(n, \gamma, t)}{dt} \right\}_{\pm} = \sum_{\gamma'} \lambda_{\pm}(n \pm 2, \gamma', \gamma) P(n \pm 2, \gamma', t) - P(n, \gamma, t) \sum_{\gamma'} \lambda_{\pm}(n, \gamma, \gamma') \quad (2a)$$

$$\left\{ \frac{dP(n, \gamma, t)}{dt} \right\}_0 = \sum_{\gamma'} \lambda_0(n, \gamma', \gamma) P(n, \gamma', t) - P(n, \gamma, t) \sum_{\gamma'} \lambda_0(n, \gamma, \gamma') \quad (2b)$$

and

$$\left\{ \frac{dP(n, \gamma, t)}{dt} \right\}_c = -P(n, \gamma, t) W_c(n, \gamma). \quad (2c)$$

The four terms in eq. (2) represent transitions with exciton number change ( $+$ ,  $-$  and  $0$ ), no exciton-number change ( $0$ ) and the emission of particles ( $c$ ), respectively. It is understood that the probability  $P(n, \gamma, t)$ , the transition rates  $\lambda_{\pm,0}(n, \gamma, \gamma')$  and the emission rates  $W_c(n, \gamma)$  are functions of the excitation energy  $E$  of the nucleus; hence, for the time being, the energy is not written explicitly.

The summation of transition rates over final configurations leads to the quantities  $\lambda_{\pm,0}(n, \gamma)$ ,

$$\lambda_{\pm,0}(n, \gamma) = \sum_{\gamma'} \lambda_{\pm,0}(n, \gamma, \gamma'). \quad (3)$$

These relations are applied to eqs. (2a) and (2b).

The total probability of finding a system (at an excitation energy  $E$ ) in an  $n$ -exciton state at a time  $t$  can be defined as a sum of probabilities of finding it in any of its allowed particle-hole configurations, i. e.

$$P(n, t) = \sum_{\gamma} P(n, \gamma, t). \quad (4)$$

We may now proceed to the summation of eq. (2) over initial configurations. A general statistical assumption for the pre-equilibrium exciton model is that every partition of energy for a given exciton number occurs with equal a priori probability during the equilibration process. Having this in mind, we may write the specific probability  $P(n, \gamma, t)$  as independent of a given configuration  $\gamma$ , i. e.

$$P(n, \gamma, t) = P(n, t). \quad (5)$$

Furthermore,

$$\sum_{\gamma} P(n, \gamma, t) = N_{\gamma} P(n, t) = P(n, t), \quad (6)$$

where  $N_{\gamma}$  is the number of  $n$ -exciton configurations of a given type  $\gamma$ .

The average values of transition and emission rates are given by

$$\frac{\sum_{\gamma} \lambda_{\pm,0}(n, \gamma) P(n, \gamma, t)}{\sum_{\gamma} P(n, \gamma, t)} = \lambda_{\pm,0}(n, t) \quad (7a)$$

$$\frac{\sum_{\gamma} W_c(n, \gamma) P(n, \gamma, t)}{\sum_{\gamma} P(n, \gamma, t)} = W_c(n, t). \quad (7b)$$

These quantities appear as time-dependent. However, if we use the assumption of a priori equal probabilities for every energy partition (specific configuration) for a given excitation state (eq. 6), inherent to the excitation model, the relations (7a) and (7b) become

$$\frac{P(n, t) \sum_{\gamma} \lambda_{\pm,0}(n, \gamma)}{N_{\gamma} P(n, t)} = \frac{\sum_{\gamma} \lambda_{\pm,0}(n, \gamma)}{N_{\gamma}} = \lambda_{\pm,0}(n) \quad (8a)$$

$$\frac{P(n, t) \sum_{\gamma} W_c(n, \gamma)}{N_{\gamma} P(n, t)} = \frac{\sum_{\gamma} W_c(n, \gamma)}{N_{\gamma}} = W_c(n). \quad (8b)$$

It follows from eqs. (8a) and (8b) that the average values of transition and emission rates are time-independent.

From eqs. (7) and (8) the following relations can be obtained:

$$\sum_{\gamma} \lambda_{\pm,0}(n, \gamma) P(n, \gamma, t) = \lambda_{\pm,0}(n) P(n, t) \quad (9a)$$

$$\sum_{\gamma} W_c(n, \gamma) P(n, \gamma, t) = W_c(n) P(n, t). \quad (9b)$$

When the summation over initial configurations in eq. (2) is performed using relations (3), eq. (2) takes the form

$$\begin{aligned} \frac{dP(n, t)}{dt} = & \sum_{\gamma} \lambda_{+}(n-2, \gamma) P(n-2, \gamma', t) + \sum_{\gamma} \lambda_{-}(n+2, \gamma') P(n+2, \gamma' t) + \\ & + \sum_{\gamma} \lambda_0(n, \gamma') P(n, \gamma', t) - \sum_{\gamma} \lambda_0(n, \gamma) P(n, \gamma, t) - \\ & - \sum_{\gamma} \lambda_{+}(n, \gamma) P(n, \gamma, t) - \sum_{\gamma} \lambda_{-}(n, \gamma) P(n, \gamma, t) - \sum_{\gamma} W_c(n, \gamma) P(n, \gamma, t). \end{aligned} \quad (10)$$

110

Using relations (9a) and (9b) in eq. (10) one obtains the wellknown expression for the master equation:

$$\begin{aligned} \frac{dP(n, E, t)}{dt} = & \lambda_{+}(n-2, E) P(n-2, E, t) + \lambda_{-}(n+2, E) P(n+2, E, t) - \\ & - [\lambda_{+}(n, E) + \lambda_{-}(n, E) + W_c(n, E)] P(n, E, t). \end{aligned} \quad (11)$$

Eqs. (10) and (11) differ in that in the latter the transition and emission rates are averaged over all allowed particle-hole configurations  $\gamma$ . Consequently, the terms with  $\lambda_0(n, E)$  have cancelled. The energy dependence  $E$  is written explicitly.

The procedure used in this paper differs somewhat from that used in ref. [6]. Our approach is based on the assumption that every partition of energy is equally probable, which leads directly to the independence of  $P(n, \gamma, t)$  on the configurations  $\gamma$ . In ref. [6] it is assumed that all transition and emission rates summed over final configurations are independent of initial configurations, i.e.

$$\begin{aligned} \lambda_{\pm}(n, \gamma) &= \lambda_{\pm}(n) \\ W_c(n, \gamma) &= W_c(n). \end{aligned} \quad (12)$$

In our approach the additional assumption (12) is superfluous.

### III. THE MASTER EQUATION AND CLOSED-FORM EXPRESSIONS FOR $T(n, E)$

The master equation (11) has been widely used to describe nuclear reactions, as it gives both pre-equilibrium and equilibrium components of the reaction cross section. A coupled system of master equations can be solved by numerical methods [2—5, 8], or transformed into a linear algebraic system for time-integrated occupation probabilities [7]. This time-integrated occupation probabilities  $T(n, E)$  determine the time spent by the composite system in an  $n$ -exciton state:

$$T(n, E) = \int_0^{\infty} P(n, E, t) dt. \quad (13)$$

The time  $T(n, E)$  should be distinguished from the mean lifetime  $\tau(n, E)$  of an  $n$ -exciton state defined as

$$\tau(n, E) = [\lambda_{+}(n, E) + \lambda_{-}(n, E) + W_c(n, E)]^{-1}. \quad (14)$$

Within certain approximations, closed-form expressions for the time  $T(n, E)$  can be obtained in two ways. The master equation (11) can be integrated over time. From this equation, a closed form expression for  $T(n, E)$  can be obtained using the iterative method. This method was suggested in ref. [7]. The same result for  $T(n, E)$  can also be obtained by calculating the depletion of states due to  $\lambda_{-}$  and

$W$ , transitions from each exciton state in a chain. This procedure was suggested in ref. [9]. We shall briefly outline both approaches.

Let us start with the method described in ref. [7]. Defining the initial exciton distribution as  $D(n)$ , the boundary condition for the time  $t = 0$  is

$$P(n, E, 0) = D(n), \quad (15)$$

and for the time  $t \rightarrow \infty$

$$\lim_{t \rightarrow \infty} P(n, E, t) = 0. \quad (16)$$

Integrating the master equation (11) over time and applying the conditions (15) and (16), we obtain the time-integrated master equation

$$-D(n) = T(n-2, E) \lambda_+(n-2, E) + T(n+2, E) \lambda_-(n+2, E) - T(n, E) \frac{1}{\tau(n, E)}. \quad (17)$$

The solution of eq. (17) in the zeroth approximation is

$$T^{(0)}(n, E) = \tau(n, E) \left[ \prod_{\substack{i=n_0 \\ \Delta i=2}}^{n-2} \lambda_+(i, E) \tau(i, E) \right] \quad n > n_0 \quad (18a)$$

and for  $n = n_0$ ,

$$T^{(0)}(n_0, E) = \tau(n_0, E). \quad (18b)$$

The above approximate solution is obtained assuming that  $D(n) = \delta_{nn_0}$  and  $\lambda_-(n+2, E) \ll \lambda_+(n-2, E)$ . The latter condition is satisfied for exciton states with a low exciton number ( $n$  not much different from  $n_0$ ).

The complete closed-form expression for the time  $T(n, E)$  spent by the nucleus in an  $n$ -exciton state is obtained using the iterative procedure from the time-integrated master equation (17):

$$T^{(l)}(n, E) = \tau(n, E) \left[ \prod_{i=n_0 \Delta i=2}^{n-2} \lambda_+(i, E) \tau(i, E) \right].$$

$$[1 + \lambda_+(n, E) \tau(n, E) \lambda_-(n+2, E) \tau(n+2, E)] + \tau(n, E) \delta_{nn_0}. \quad (19)$$

As already mentioned, the same result was obtained in ref. [9] using a different approach. The authors of ref. [9] start from the mean lifetime of an  $n$ -exciton state and calculated the depletion of states due to emissions and  $\lambda_-$  transitions from all states preceding a given exciton state. The correction factor to the mean lifetime due to such effects can be expressed as

$$\tau(n, E) \left[ \prod_{i=n_0 \Delta i=+2}^{n-2} \lambda_+(i, E) \tau(i, E) \right]. \quad (20a)$$

An additional correction factor is obtained when considering the  $\lambda_+(n, E)$  transition which forms the  $(n+2)$ -exciton state. From this state, the  $\lambda_-(n+2, E)$  transition leads back to the  $n$ -exciton state and acts as a feeding term. Therefore, the complete expression is

$$\begin{aligned} T(n, E) = & \tau(n, E) \delta_{nn_0} + \tau(n, E) \left[ \prod_{\substack{i=n_0 \\ \Delta i=+2}}^{n-2} \lambda_+(i, E) \tau(i, E) \right] + \\ & + \tau(n, E) \left[ \prod_{\substack{i=n_0+2 \\ \Delta i=+2}}^n \lambda_+(i, E) \tau(i, E) \right] \lambda_-(n+2, E) \tau(n+2, E) = \\ = & \tau(n, E) \left[ \prod_{\substack{i=n_0+2 \\ \Delta i=+2}}^{n-2} \Delta_+(i, E) \tau(i, E) \right] [1 + \lambda_+(n, E) \tau(n, E) \cdot \\ & \cdot \lambda_-(n+2, E) \tau(n+2, E)] + \tau(n, E) \delta_{nn_0}, \quad (20b) \end{aligned}$$

which is identical with (19). In expression (20b),  $\lambda_-$  transitions from the states with an exciton number higher than  $(n+2)$  are neglected.

If we use eq. (19), the closed-form expression for the differential pre-equilibrium cross section becomes

$$\frac{d\sigma(\alpha, \beta, E, \epsilon_g)}{d\epsilon_g} = \sigma_R(\alpha, E) \sum_{\substack{n=n_0 \\ \Delta n=+2}}^n W_g(n, E, \epsilon_g) T(n, E). \quad (21)$$

The simple physical meaning of the above expression that the emission from an  $n$ -exciton state is proportional to the emission rate multiplied by the time the system spends in this particular state up to the moment the equilibrium is reached.

#### IV. CLOSED-FORM APPROXIMATE EXPRESSIONS FOR $T(n, E)$

The above described closed-form expression for  $T(n, E)$  (eqs. 19 and 20b) obtained from the time-integrated master equation is also an approximation of the exact solution of the master equation. Besides this solution there are several other approximations for  $T(n, E)$  used in literature. These are based on specific assumptions on the absolute values of emission and transition rates and their respective relations. These approximations have been successfully used in analysing experimental data. We shall now (1) discuss these approximations in more detail and (2) show that they are in satisfactory agreement with experimental data under certain conditions.

(1a) An approximate solution could be obtained assuming that both  $\lambda_-(n, E)$

and  $W_c(n, E)$  could be neglected with respect to the dominant  $\lambda_+(n, E)$  transitions. Then expression (19) takes the form

$$T(n, E) = [\lambda_+(n, E)]^{-1}. \quad (22)$$

Such a form was used at an early stage of development of the exciton model [1, 10]. This approximation is valid for the very first excitation states ( $n$  close to  $n_0$ ).

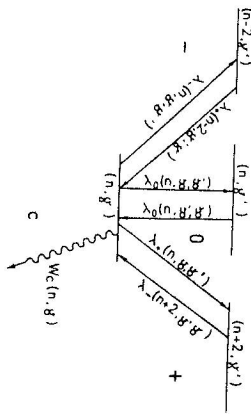


Fig. 1. A schematic presentation of allowed transitions within excitation states if specific particle-hole configurations are explicitly considered.

(1b) Another approximate expression for the time  $T(n, E)$  is obtained if only the  $\lambda_-(n, E)$  transition is neglected. Then, eq. (19) takes the form

$$T(n, E) = \frac{1}{\lambda_-(n, E) + W_c(n, E)} \prod_{i=n_0}^{n-2} \left[ 1 - \frac{W_c(i, E)}{\lambda_+(i, E) + W_c(i, E)} \right]. \quad (23)$$

This expression has been widely used in the exciton model as formulated in ref. [11]. An expression of the same form has also been used in the hybrid model [12], but with  $\lambda_+$  being defined in another way. In ref. [12],  $\lambda_+$  is obtained from the mean free path of nucleons in nuclear matter, is independent of the excitation number and depends on the energy of the emitted particles. In ref. [11],  $\lambda_+$  is the decay rate of an  $n$ -exciton configuration obtained as average values over the allowed energies and configurations of particle and hole collision probabilities, and depends on the excitation energy of the system.

The approximation (23) is valid until  $\lambda_-$  and  $\lambda_+$  transitions become comparable, i.e. until  $n$  reaches  $\bar{n}$ . However, since the emission probability decreases rapidly with increasing  $n$ , this approximation still describes pre-equilibrium emission with sufficient accuracy even for  $n$  close to  $\bar{n}$ .

(1c) The third possibility is to take for  $T(n, E)$  the zero-order approximation (18a and 18b) obtained from the time integrated master equation (17). In this approximation,  $\tau(n, E)$  is treated exactly, but the feeding term due to  $\lambda_-(n+2, E)$  transitions is neglected. We use this approximate closed-form expressions to analyse  $(n, 2n)$  excitation functions in the incoming energy  $E_n = E_{threshold}(n, 2n)$  up to 24 MeV for a number of nuclei from  $A=45$  to  $A=209$  [13].

In order to verify the conditions for the applicability of the above three

approximate closed-form expressions for  $T(n, E)$ , we compare the following quantities for a chosen nucleus at a given excitation energy: (i) absolute values of  $\lambda_+$ ,  $\lambda_-$  and  $W_c$  transition and emission rates; (ii) ratios between these transition and emission rates; (iii) values for  $T(n, E)$  obtained from various approximate expressions, and (iv) spectra of primary neutrons emitted due to the pre-equilibrium mechanism, calculated using various expressions for  $T(n, E)$ .

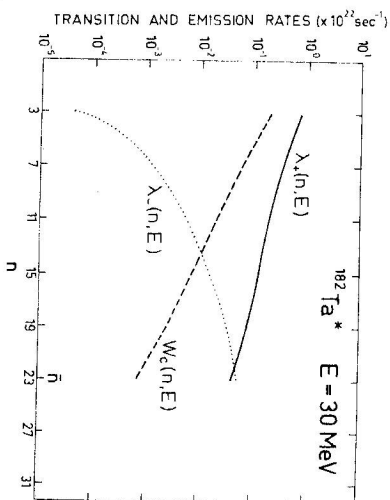


Fig. 2. Transition and emission rates as functions of the excitation number  $n$  for a system  $n + {}^{161}\text{Ta} \rightarrow {}^{182}\text{Ta}^*$  at the excitation energy  $E = 30$  MeV.

As a test nucleus we have chosen  ${}^{182}\text{Ta}$  excited to an energy of 30 MeV. Fig. 2 shows the values obtained for  $\lambda_+(n, E)$ ,  $\lambda_-(n, E)$  and  $W_c(n, E)$  for this nucleus. The transition rates  $\lambda_+$  and  $\lambda_-$  and the emission rate  $W_c(n, E)$  have been calculated as described in ref. [13], with the value of the parameter  $K = 700$  MeV<sup>2</sup>. It can be seen from Fig. 2 that the  $\lambda_+$  transition is dominant along all excitation states from  $n = n_0$  to  $n$ . The  $\lambda_-$  transition can be neglected, especially at the beginning of the excitation chain, while  $W_c$ , although a decreasing function of  $n$ , cannot be neglected.

Fig. 3 represents the ratios  $\lambda_+/\lambda_-$ ,  $W_c(n)/W_c(n_0)$  and  $\lambda_+/W_c$ . The ratio  $\lambda_+/\lambda_-$  is a very rapidly decreasing function, approaching unity at  $n = \bar{n}$ ; it is thus possible to neglect  $\lambda_-$  transitions and use the appropriate closed-form expression for  $T(n, E)$  (eqs. 23 and 18). Another condition for the applicability of the same approximate closed-form expression is that  $W_c(n)/W_c(n_0)$  should also be a decreasing function of  $n$ . This means that the bulk of the emitted particles should be emitted during the first stages of the equilibration process. Since in this case the emission from the states around  $n \sim \bar{n}$  can be neglected, the approximation which consists in neglecting  $\lambda_-$  can be used even when this transition rate is comparable with  $\lambda_+$ .

Fig. 4 shows a comparison between  $\tau(n, E)$  and  $T(n, E)$  obtained with different

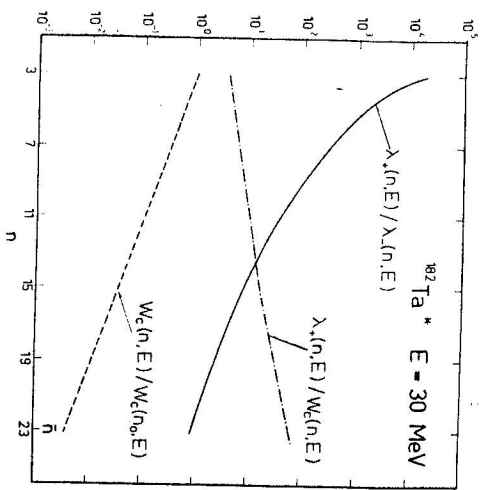


Fig. 3. Ratios  $\lambda_+(n, E)/\lambda_-(n, E)$ ,  $\lambda_+(n, E)/W_c(n, E)$  and  $W_c(n, E)/W_c(n_0, E)$  for a system  $n + {}^{181}\text{Ta} \rightarrow {}^{182}\text{Ta}^*$  at the excitation energy  $E = 30$  MeV.

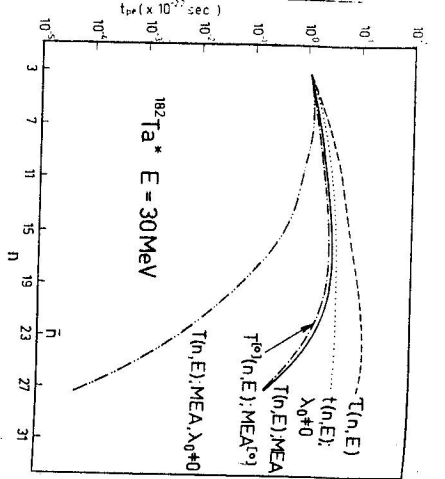


Fig. 4. Mean lifetimes and the times spent by the composite system  $n + {}^{182}\text{Ta} \rightarrow {}^{182}\text{Ta}^*$  at the excitation energy  $E = 30$  MeV. Symbols:  $\tau(n, E)$ : eq. (14);  $t(n, E)$ ;  $\lambda_0 \neq 0$ : eq. (24);  $T(n, E)$ , MEA: eq. (19);  $T(n, E)$ , MEA<sup>(0)</sup>: eq. (18);  $T(n, E)$ , MEA,  $\lambda_0 \neq 0$ : ref. [9].

Approximate closed-form expressions. It can be seen that the zeroth approximation for  $T(n, E)$ (eq. 18) agrees very well with the value  $T(n, E)$  obtained by the iterative procedure from the time-integrated master equation. In ref. [9], the same approximate expression for  $T(n, E)$  (eq. 19) was used, but the mean lifetime of an  $n$ -exciton state was defined as

$$t(n, E) = [\lambda_+(n, E) + \lambda_-(n, E) + \lambda_0(n, E) + W_c(n, E)]^{-1}. \quad (24)$$

We have shown in Section II that the terms with  $\lambda_0$  transitions should not appear in the master equation (11). In this respect Fig. 4 also shows the result for  $\tau(n, E)$  and  $T(n, E)$  obtained in ref. [9]. The value of  $T(n, E)$  from this reference differs markedly from the value of  $T(n, E)$  calculated from eq. (19). As the emission cross section is given by eq. (21), it is clear that the results of ref. [9] should be different from those obtained by using eq. (19). Let us note again that the extension of master equations to include  $\lambda_0$  transitions as was done by Wu and Chang [9] is not correct. This point was already criticized by Běťák and Dobeš [14].

Figs. 5 and 6 show the calculated spectra of primary neutrons due to the pre-equilibrium mechanism for  ${}^{182}\text{Ta}$  (excitation  $E = 30$  MeV) and  ${}^{56}\text{Mn}$  (excitation  $E = 25$  MeV). The spectra were calculated using the closed-form expression (21) with different approximate expressions for  $T(n, E)$ . In  ${}^{182}\text{Ta}$  (Fig. 5), the closed-form approximate expression for  $T(n, E)$  (eq. 19) obtained from the time-integrated master equation (MEA) and its zeroth approximation (eq. 18) give

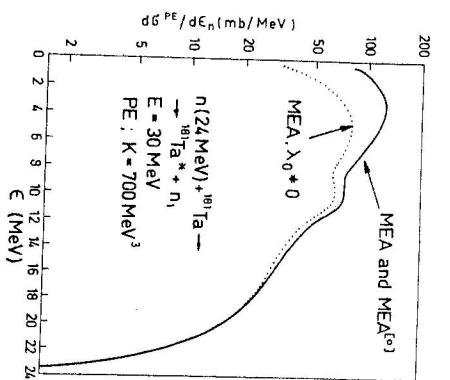


Fig. 5. Pre-equilibrium spectrum of primary neutrons (eq. 21) from  ${}^{182}\text{Ta}^*$  excited to 30 MeV. The following approximate expressions are used for  $T(n, E)$ : MEA: eq. (19), MEA<sup>(0)</sup>: eq. (18) and MEA,  $\lambda_0 \neq 0$ : from ref. [9].

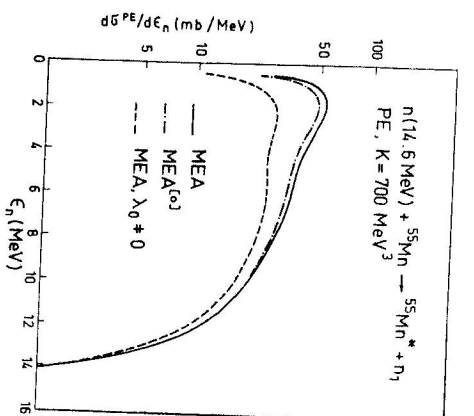


Fig. 6. Pre-equilibrium spectrum of primary neutrons (eq. 21) from  ${}^{56}\text{Mn}^*$  excited to 25 MeV. The following approximate expressions are used for  $T(n, E)$ : MEA: eq. (19), MEA<sup>(0)</sup>: eq. (18) and MEA,  $\lambda_0 \neq 0$ : from ref. [9].

the same results for the pre-equilibrium component of primary neutrons. For  ${}^{56}\text{Mn}$  (Fig. 6), the results obtained using these two equations differ slightly at low energies of primary neutrons. For both nuclei, the result obtained using the expression for  $T(n, E)$  with the  $\lambda_0$  transition (ref. [9]) differs from those obtained using eq. (18) and (19).

From the analysis performed we may conclude that for excitation energies in the region studied [13] ( $E = 15$ – $30$  MeV) and for a wide range of nuclei  $A = 45$ – $209$  the zero-order approximation to the time  $T(n, E)$  (the time which the system spends in an  $n$ -exciton state) (eq. 18) is a satisfactory approximate solution of the time-integrated master equation. In this energy region it is possible to neglect  $\lambda$ -transitions completely (eq. 23) or partly (eqs. 18 and 19).

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