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METHOD OF EXPERIMENTAL STUDY OF FLUCTUATIONS IN SEMICONDUCTORS

BOHUMIL KOKTAVÝ*, JOSEF ŠIKULA*, Brno

Methods of an experimental study of voltage or current fluctuations in semiconductors consisting in the measurement of the voltage or current noise spectral density are described. In the present paper we describe a measuring method for the experimental evaluation of the noise current, which is applicable whenever the real fluctuation process can be replaced by a stationary ergodic process. Some comments on the practical measurement performance are given.

ЭКСПЕРИМЕНТАЛЬНЫЕ МЕТОДЫ ИССЛЕДОВАНИЯ ФЛУКТУАЦИЙ В ПОЛУПРОВОДНИКАХ

В работе описаны экспериментальные методы исследования флуктуаций тока и напряжения в полупроводниках, основанные на измерении спектральной плотности шума тока и напряжения. Приведенный метод измерения, позволяющий экспериментально определить шум тока, применим в том случае, когда процесс действительной флуктуации можно заменить стационарным эргодическим процессом. Кратко описываются практические измерения.

I. INTRODUCTION

The spectral density and the correlation function of a stochastic process are — apart from the probability density — the most important characteristics of the process. It is known [1] that the information given by the spectral density or the correlation function is equivalent. From the theoretical point of view it is not substantial which of the mentioned characteristics is taken into account. From the point of view of the experiment, however, this is not the case. The study of the correlation function requires the amplification of the measured signal over a wide range of frequencies, while in the case of spectral density being measured only a narrow frequency band is amplified, the amplifier being tuned to the particular frequency.

For a stationary random process the spectral density is determined in the following way (see, for example [1]). If for an arbitrary realization $x^{(k)}(t)$ of the

* Katedra fyziky FAST VUT, Barvičova 85, CS-663 37 BRNO.

random process $\xi(t)$ the Fourier transform of the pertinent part of the realization in the time interval $(-T/2, +T/2)$ is

$$Z_T^{(k)}(j\omega) = \int_{-T/2}^{T/2} x^{(k)}(t) \exp(-j\omega t)\, dt, \tag{1}$$

then the spectral density $S(\omega)$ is defined as a statistical mean value

$$S(\omega) = \lim_{T\to\infty} m_1 \left\{ \frac{2}{T} |Z_T(j\omega)|^2 \right\}, \tag{2}$$

where m_1 is the first moment.

The correlation function $B(\tau)$ of the random process $\xi(t)$ with a zero mean value $[m_1\{\xi(t)\} = 0]$ is defined as follows

$$B(\tau) = m_1\{\xi(t)\xi(t+\tau)\} \tag{3}$$

and is related to the spectral density $S(\omega)$ through the Fourier transformation

$$S(\omega) = 2\int_{-\infty}^{\infty} B(\tau) \exp(-j\omega\tau)\, d\tau, \tag{4}$$

$$B(\tau) = \frac{1}{4\pi}\int_{-\infty}^{\infty} S(\omega) \exp(j\omega\tau)\, d\omega. \tag{5}$$

The spectral density is very important when the properties of stochastic processes, their sources and circuits with random sources are studied. The spectral density of the current noise in semiconductor materials or components as a function of macroscopic parameters gives valuable information which may be of high importance when the sources of the noise are studied and localized (see, e.g., [6—8]). Various conductivity mechanisms and other processes taking place in semiconductors exhibit diverse characteristic noise spectra. When the current, voltage, temperature, etc., are altered, the mechanisms or the relative weight of the individual mechanisms in the studied system may change.

The relation between the spectral density and the current, voltage, etc., often exhibits local extrema or other particular points, from which we can determine some microscopic parameters. The spectral density of the noise power of a given component is a fundamental characteristic of its noise properties. It plays, therefore, a very important role in applications of semiconductor components in electric circuits, particularly in the amplification and measurements of extremely low voltages.

Noise in semiconductor components is frequently caused by microscopic defects, which in turn lead to an excess current in the P—N junctions. From the shape of the noise spectra the technology of the material or component may be evaluated or modified with respect to a minimum noise.

II. THEORY OF DISPERSION AND SPECTRAL DENSITY MEASUREMENT

The choice of a suitable method for a given statistical characteristic measurement depends on the nature of the process to be studied. The measurement method is greatly simplified in the case of a stationary ergodic normal process. We confine ourselves to the types of noise processes that can be considered stationary and ergodic. The condition of stationarity of the noise process will be satisfied if the macroscopic parameters of the system are constant within the measurement time. It is necessary that the system be in a steady state, which means that we carry out the measurement after the transients die out. Of high importance for the experimental study are ergodic processes. A necessary and sufficient condition for a process to be ergodic is that the following equation holds

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T B(\tau)\, d\tau = 0, \tag{6}$$

where $B(\tau)$ is the correlation function of the process. Generally, condition (6) is fulfilled for all the known types of noise, as $\lim_{\tau\to\infty} B(\tau) = 0$ and $B(0) = \sigma^2$ is finite (σ^2 is a dispersion of the process).

In the case of transformation of stationary and ergodic processes by a physical system the stationarity and ergodicity of the process at the output of the system are to be checked. If the transformation is invariant in time, then the output process is stationary and ergodic, too [3].

II.1. Theory of the dispersion measurement

For the spectral density measurement the experimental determination of the random process dispersion is of primary importance. For a stationary random process $\xi(t)$ with a zero mean value $m_1\{\xi(t)\} = 0$ the dispersion σ^2 is defined as follows

$$\sigma^2 = m_1\{\xi(t)^2\} = \int_{-\infty}^{\infty} x^2 w(x) dx, \tag{7}$$

where $w(x)$ is a one-dimensional probability density of the process $\xi(t)$. The dispersion is related to the correlation function $B(\tau)$ and the spectral density in the following way:

$$\sigma^2 = B(0) = \frac{1}{2\pi}\int_0^{\infty} S(\omega)\, d\omega. \tag{8}$$

If the process is ergodic, then $\xi^2(t)$ is ergodic, too, and as an estimate of the parameter σ^2 we may choose

$$(\sigma^2)^* = \frac{1}{T}\int_0^T \xi^2(t)\,\mathrm{d}t. \tag{9}$$

The dispersion measurement is simplified in the case where $\xi(t)$ is a normal process, i.e., its one-dimensional probability density has the form

$$w(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{x^2}{2\sigma^2}\right]. \tag{10}$$

In this case we can make use of the mean absolute value of the process $\xi(t)$, i.e., $m_1\{|\xi(t)|\}$ [4]. To see this we write

$$m_1\{|\xi(t)|\} = \int_{-\infty}^{\infty} |x| \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{x^2}{2\sigma^2}\right] \mathrm{d}x = \sqrt{\frac{2}{\pi}}\,\sigma. \tag{11}$$

A non-linear, non-inertial transformation conserves the ergodicity of the process so that $|\xi(t)|$ is ergodic as well. As an estimate of the parameter $m_1\{|\xi(t)|\}$ we can therefore take the following quantity

$$m_1^* = \frac{1}{T}\int_0^T |\xi(t)|\,\mathrm{d}t. \tag{12}$$

For the estimate of σ^* we get

$$\sigma^* = \sqrt{\frac{\pi}{2}}\,\frac{1}{T}\int_0^T |\xi(t)|\,\mathrm{d}t. \tag{13}$$

IL2. Theory of the spectral density measurement

Now we turn to the measuring method of the spectral density of a stationary ergodic process. We feed this process into the input of a linear narrow-band system with an impulse transient function $h(t)$ and transfer function $K(\mathrm{j}\omega)$. The output process will be

$$\zeta(t) = \int_0^T \xi(t-\tau)h(\tau)\,\mathrm{d}\tau. \tag{14}$$

Generally, the process $\zeta(t)$ will not be stationary and ergodic, as this transformation is not invariant in time. As far as we do not study the transient phenomena and can limit the process $\xi(t)$ within a sufficiently large time interval $(-T, t)$, then for $\underset{T\to\infty}{h}\,(T+t)\to 0$ we can write

$$\zeta(t) = \int_{-\infty}^{t} \xi(\tau)h(t-\tau)\,\mathrm{d}\tau. \tag{15}$$

This transformation is invariant in time and the process $\zeta(t)$ is therefore

a stationary and ergodic process. Its spectral density $S_\zeta(\omega)$ is related to the spectral density $S_\xi(\omega)$ of the input process as follows

$$S_\zeta(\omega) = |K(\mathrm{j}\omega)|^2 S_\xi(\omega). \tag{16}$$

The correlation function of the process $\zeta(t)$ is

$$B_\zeta(\tau) = \frac{1}{4\pi}\int_{-\infty}^{\infty} S_\zeta(\omega)\exp(\mathrm{j}\omega\tau)\,\mathrm{d}\omega = \frac{1}{4\pi}\int_{-\infty}^{\infty} S_\xi(\omega)|K(\mathrm{j}\omega)|^2\exp(\mathrm{j}\omega\tau)\,\mathrm{d}\omega. \tag{17}$$

The dispersion σ_ζ^2 is expressed by means of the spectral density by the relation

$$\sigma_\zeta^2 = B_\zeta(0) = \frac{1}{4\pi}\int_{-\infty}^{\infty} S_\xi(\omega)|K(\mathrm{j}\omega)|^2\,\mathrm{d}\omega = \frac{1}{2}\int_{-\infty}^{\infty} S_\xi(f)|K(\mathrm{j}f)|^2\,\mathrm{d}f. \tag{18}$$

For a linear narrow-band system and a centre frequency f_0 we denote the transfer function K_0 and the effective bandwidth Δf_e

$$\Delta f_e = \frac{1}{2K_0^2}\int_{-\infty}^{\infty} |K(\mathrm{j}f)|^2\,\mathrm{d}f. \tag{19}$$

If it is possible to consider the spectral density $S_\xi(f)$ constant within the frequency range $f_0 \pm \Delta f_e/2$, then for the estimate $(\sigma_\zeta^2)^*$ of the dispersion of the process $\zeta(t)$ there holds

$$(\sigma_\zeta^2)^* = S_\xi^*(f_0)\Delta f_e K_0^2 \tag{20}$$

and as an estimate of the spectral density $S_\xi(f_0)$ we can take the quantity

$$S_\xi^*(f_0) = \frac{\frac{1}{T}\int_0^T \zeta^2(t)\,\mathrm{d}t}{\Delta f_e K_0^2}. \tag{21}$$

For a normal process $\zeta(t)$ in the output of a narrow-band filter this estimate may be expressed using (11) as follows:

$$S_\xi^*(f_0) = \frac{\pi}{2}\,\frac{\left[\frac{1}{T}\int_0^T |\zeta(t)|\,\mathrm{d}t\right]^2}{\Delta f_e K_0^2}. \tag{22}$$

It is seen that the spectral density can be determined from the mean absolute value of the process $\xi(t)$.

III. METHOD OF THE FLUCTUATION SPECTRAL DENSITY MEASUREMENT

In the following we describe an apparatus for the measurement of the spectral density of the current noise of a given semiconductor diode. The current fluctuations make a random process $\eta(t)$. The diode current is

$$I(t) = I_0 + \eta(t), \tag{23}$$

where I_0 is the mean value of the current. To measure the characteristics of $\eta(t)$ we transform the current fluctuations into a voltage $\xi(t)$. For a real load impedance R_z in the diode circuit the noise voltage is proportional to the diode current

$$\xi(t) = R_z \eta(t). \tag{24}$$

The spectral density $S_\xi(\omega)$ of the noise voltage is related to the noise current spectral density by the equation

$$S_\xi(\omega) = R_z^2 S_\eta(\omega). \tag{25}$$

III.1. Analysis of the input circuit

The spectral density measurement is carried out according to Fig. 1., where R_z is the load resistance, R_v — the input resistance of the preamplifier, $\xi_z(t)$ — a source of the preamplifier random voltage reduced to the input, ξ_v — v random voltage in the input of an ideal amplifier.

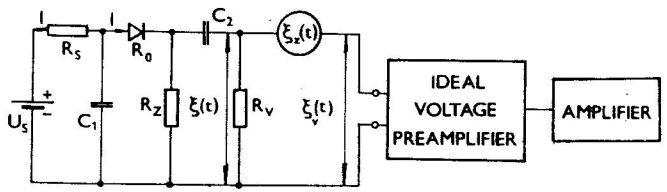

Fig. 1.

First we determine the voltage ξ_v in the input of an ideal amplifier. We assume that the resistance R_z and R_v can be replaced by ideal noiseless resistors in parallel to a thermal noise current source $\eta_{tz}(t)$ and current generator $\eta_{tv}(t)$. We assume that the diode to be measured exhibits a thermal noise characterized by a current generator $\eta_t(t)$ and excess current noise — $\eta_d(t)$. The thermal noise of the diode is generated in the differential resistance of the diode R_0.

The circuit equivalent to that in Fig. 1 is drawn in Fig. 2. The random process then is

$$\xi_v(t) = \xi_z(t) + R_p[\eta_d(t) + \eta_t(t) + \eta_{tz}(t) + \eta_{tv}(t)], \tag{26}$$

where we denote

$$R_p = (R_0 R_z R_v)/(R_0 R_z + R_v R_0 + R_z R_v). \tag{27}$$

Let us assume that the partial random processes are statistically independent. Then for the dispersion of the process $\xi_v(t)$ there holds

$$D\{\xi_v(t)\} = D\{\xi_z(t)\} + R_p^2[D\{\eta_d(t)\} + D\{\eta_t(t)\} + \tag{28}$$
$$+ D\{\eta_{tz}(t)\} + D\{\eta_{tv}(t)\}].$$

If we replace the diode with a resistance R_0, then $\eta_d(t) = 0$. The input process of the ideal amplifier is

$$\xi'_v(t) = \xi_z(t) + R_p[\eta_t(t) + \eta_{tz}(t) + \eta_{tv}(t)], \tag{29}$$

its dispersion is

$$D\{\xi'_v(t)\} = D\{\xi_z(t)\} + R_p^2[D\{\eta_t(t)\} + D\{\eta_{tz}(t)\} + D\{\eta_{tv}(t)\}]. \tag{30}$$

The dispersion of the excess noise of the diode is found from (28) and (30) to be

$$D\{\eta_d(t)\} = [D\{\xi_v(t)\} - D\{\xi'_v(t)\}]/R_p^2. \tag{31}$$

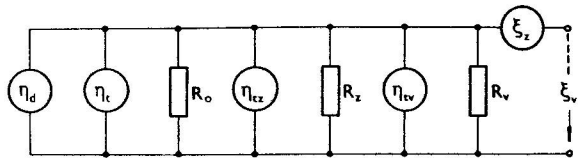

Fig. 2.

Similarly, for the spectral density of the random process $\eta_d(t)$ in the case of a non-coherent stationary process we get

$$S\{\eta_d(t)\} = [S\{\xi_v(t)\} - S\{\xi'_v(t)\}]/R_p^2. \tag{32}$$

This formula makes it possible to study the spectral density of the excess noise.

III.2. Stochastic characteristics analyser

An apparatus used for the spectral density measurement — the analyser — has a block diagram represented in Figs. 3 and 4. If we measure the mean absolute

value, we use the apparatus according to Fig. 3; if we wish to measure the mean square value, we make use of the apparatus drawn in Fig. 4.

The arrangement of Fig. 3 is used whenever the one-dimensional probability density of the process $\xi(t)$ is normal, for then the process $\zeta(t)$ is normal, too. Even when the process $\xi(t)$ is not normal, we can also use this arrangement, as it is known [5] that a narrow-band system normalizes any process. The higher the ratio of the time constant of a linear inertial system τ_s to the correlation time τ_k, the better the approximation to normality. The correlation time τ_k is

$$\tau_k = \frac{1}{\sigma^2} \int_0^\infty |B(\tau)| \, d\tau, \tag{33}$$

where $B(\tau)$ and σ^2 are the correlation function and the dispersion of the input process, respectively.

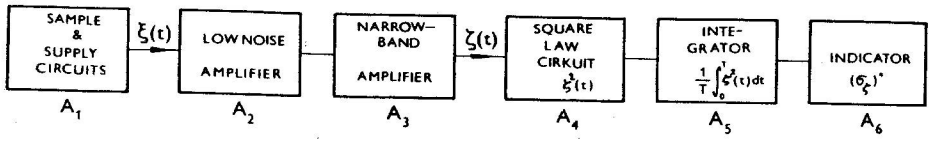

Fig. 3.

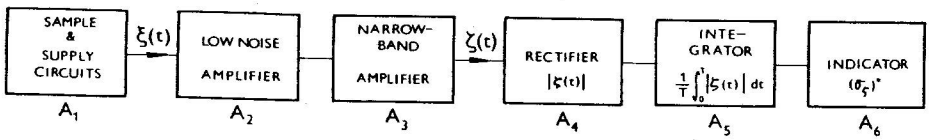

Fig. 4.

Let us describe briefly the details of the apparatus drawn in Fig. 3. The A_1 system consists of the measured sample and the power supplies. It must be carefully protected from external electromagnetic fields.

As the A_2 through A_6 systems we may use a Unipan selective nanovoltmeter, the preamplifier of which has good noise properties and a high input impedance absolute value. For example, the 233.7 preamplifier has a gain of 20 dB extending over a frequency range from 1 Hz to 150 kHz, an input resistance of 100 MΩ and its noise factor is less than 3 dB at $f > 300$ Hz and the source resistance within 1 kΩ to 10 MΩ. The 237 nanovoltmeter has a frequency range from 1 Hz to 100 kHz, a sensitivity from 0.1 μV to 100 mV and a selectivity adjustable in three steps to 0, 25, and 40 dB per octave ($\Delta f_e = kf_0$).

The operation

$$\frac{1}{T} \int_0^T |\zeta(t)| \, dt \tag{34}$$

is realized in a two way linear rectifier with an operational amplifier and the mean value measuring instrument. The indicator is calibrated in rms values of a sine input voltage. Then the mean value is

$$U_s = \frac{2\sqrt{2}}{\pi} U \tag{35}$$

and the ratio

$$\frac{\sigma_\xi^*}{K_0} = \sqrt{\frac{\pi}{2}} U_s = 1.13U. \tag{36}$$

For the spectral density of the noise voltage we get

$$S_\xi^*(f) = \frac{1.13^2(U^2 - U_0^2)}{kf} \tag{37}$$

U_0 is the reading corresponding to the noise background, the factor $k = \Delta f_e / f$ is determined from a standard signal measurement.

The A_5 integrator circuit must have a sufficiently high time constant τ, especially at low frequencies, where the amplifier bandwidth is low (about 1 Hz). The probable error of one measurement due to the fluctuations of the indicator pointer is $\delta = 1/\sqrt{2\Delta f_e \tau}$.

If we require that the probable error δ does not exceed 2 %, we find the necessary time constant in the order of 1000 seconds.

Furthermore it is necessary to test the dynamic properties of the apparatus. For the mentioned selective nanovoltmeter it has been found that its output current follows the input voltage according to a linear law up to the full scale deflection voltage multiplied by a factor of 3.5. The error due to the finite linear range in the case of a normal process with a dispersion σ^2 will be less than ϑ, where

$$\vartheta = \frac{\int_0^\infty x \exp\left[-\frac{x^2}{2\sigma^2}\right] dx}{\int_0^{m\sigma} x \exp\left[-\frac{x^2}{2\sigma^2}\right] dx} - 1 = 0.7 \text{ \% for } m = 3.4. \tag{39}$$

IV. CONCLUSIONS

Unlike the substitution method, which has been frequently used in practice, the method we describe in the present paper is a direct method of the spectral density

measurement. This method is based upon the noise voltage measurement by means of either a linear or a square detector. From the theory of measurement we have derived the requirements for the amplifying system properties. A method of the spectral density measurement by means of a real amplifier is discussed in detail. The formulae for the spectral density calculation from the narrow-band-amplified noise voltage are given.

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