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OPTICAL MAGNETOABSORPTION IN A MODEL OF A DISORDERED SEMICONDUCTOR

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Coefficient of optical magnetoabsorption of a disordered semiconductor in a strong magnetic field (quantum limit) has been calculated taking into account also quantum effects due to the random potential. The magnetic freeze-out of the spectra in the range of the absorption edge, owing to the Landau shift of energy as well as the quantum effects, has been found. In some cases, the "flare-up" of the spectra with an increasing magnetic field may occur.

ОПТИЧЕСКАЯ МАГНИТОАБСОРБЦИЯ В МОДЕЛИ НЕУПОРЯДОЧЕННОГО ПОЛУПРОВОДНИКА

В работе рассчитан коэффициент оптической магнитоабсорбции неупорядоченного полупроводника, находящегося в сильном магнитном поле (квантовый предел), с учетом также квантовых эффектов, обусловленных случайным потенциалом. Найдено магнитное вымораживание спектра в области границы поглощения, связанное со сдвигом Ландау, а также с квантовыми эффектами. В некоторых случаях может иметь место «разгорачивание» спектра при увеличении магнитного поля.

I. INTRODUCTION

In our previous work [1] the magnetic freeze-out of the states in the tails of the energy of disordered semiconductors in strong magnetic fields was theoretically investigated. Generally, the correlation of the density of states with the absorption coefficient suggests to expect this phenomenon in the optical magnetoabsorption spectra. Unfortunately, the optical magnetoabsorption in strongly doped and amorphous semiconductors was not intensively studied experimentally. In doped Ge there was observed the "flare-up" of magnetoabsorption depending on the magnetic field [2]. Dependence of the spectra on the concentration of impurities was studied by Hasegawa and Nakamura [3] related to the "magnetic freeze-out". Theoretical investigation of the interband magnetoabsorption in strongly doped semiconductors at low temperatures was performed by Djakonov, Efros and Mitchell [4]. They ascertained that with an increasing magnetic field the absorption edge will be shifted to lower frequencies, become sharper and approach that for the pure crystal.

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Starting from the model described in [1] we shall calculate the coefficient of the optical magnetoabsorption between the non-degenerated states of the randomly undulated Landau bands taking into account the quantum effects due to the random potential [5]. As we shall see, localization due to the essential one-dimensionality of the electron in a random potential and in a strong magnetic field is involved as well.

II. THE PROBABILITY OF TRANSITION IN A MAGNETIC FIELD

The theory of a linear response results in the known formula for the coefficient of optical magnetoabsorption

$$\varepsilon_2^{(H)}(\omega)=\left(\frac{e}{m\omega}\right)s^2\frac{\pi}{\hbar V}\,\mathrm{Re}\int_0^\infty \mathrm{d}t\; e^{i\omega t}\iint \mathrm{d}x\mathrm{d}y \lim_{\substack{x'\to x\\ y'\to y}}(P_x^*+P_{x'})(P_y+P_{y'}^*)\times \tag{1}$$
$$\times\langle R^{(2)}(\mathbf{x},\mathbf{y},\mathbf{y}',\mathbf{x}',it/\hbar;\mathbf{H})\rangle_{av},$$

where

$$\mathbf{P}=\mathbf{p}+\frac{e}{c}\mathbf{A}(\mathbf{r})$$

and

$$i\langle R^{(2)}\rangle=\langle G_r^{(2)}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{y}',\boldsymbol{x}',t;H)\rangle= \tag{2}$$
$$=\exp\left(-\frac{i}{\hbar}E_g t\right)\left\langle\int_x^{y'} D\big(\mathbf{r}_e(\tau)\big)\exp\left\{\frac{i}{\hbar}\int_0^t \mathrm{d}\tau\left[\frac{m_e}{2}\dot{\mathbf{r}}_e^2(\tau)+e\mathbf{r}_e(\tau)\,\mathbf{A}(\mathbf{r}_e(\tau))-\right.\right.\right.$$
$$\left.\left.-\eta_e U(\mathbf{r}_e(\tau))\right]\right\}\int_y^{x'} D(\mathbf{r}_h(\tau))\exp\left\{\frac{i}{\hbar}\int_0^t \mathrm{d}\tau\left[\frac{m_h}{2}\dot{\mathbf{r}}_h^2(\tau)-e\mathbf{r}_h(\tau)\times\right.\right.$$
$$\left.\left.\left.\times\mathbf{A}(\mathbf{r}_h(\tau))-\eta_h U(\mathbf{r}_h(\tau))\right]\right\}\right\rangle_{av}$$

is the Feynman path-integral representation of the averaged two-particle (electron-hole) retarded Green function in the magnetic field $A(\mathbf{r})=\frac{1}{2}\mathbf{H}\times\mathbf{r}$ and in the random potential $\eta U(\mathbf{r})$. Here we neglected the Coulomb correlation of the electron and the hole paths.

The averaging in formula (2) over the Gaussian distribution functional $P\{V(r)\}$ results in

$$\left\langle \exp\left\{\frac{i}{\hbar}\eta\int_0^t d\tau[U(\mathbf{r}_e(\tau))-U(\mathbf{r}_h(\tau)]\right\}\right\rangle = \tag{3}$$

$$\exp\left\{-\frac{\eta^2}{2\hbar^2}\int_0^t\int_0^t d\tau\, d\tau'[W(\mathbf{r}_e(\tau))-\mathbf{r}_e(\tau'))+W(\mathbf{r}_h(\tau)-\mathbf{r}_h(\tau'))-\right.$$

$$\left.-2W(\mathbf{r}_e(\tau)-\mathbf{r}_h(\tau)'))]\right\}.$$

In what follows we shall use the same model as in [1] for the calculation of the density of states. The consistency with the supposition of the negligible Coulomb interactions of the electron and the hole in the case $|\varrho|\sim L-D_e$ can be achieved when the mixed electron-hole correlations due to the random potential $W(\mathbf{r}_e(\tau)-\mathbf{r}_h(\tau'))$ in (3) are also neglected. Indeed, the harmonic approximation of this term would require a localization of the electron-hole pair in a small region within L. Then, expression (2) results in

$$\langle G^{(2)}\rangle=\exp\left\{-\frac{\eta^2 t^2}{\hbar^2}+\frac{i}{\hbar}E_g t\right\}(2\sqrt{\pi})^{-6}\int d\mathbf{q}_1\exp\left(-\frac{q_1^2}{4}\right)\times \tag{4}$$

$$\times\int_x^{y'}D(\mathbf{r}_e(\tau))\exp\left\{\frac{i}{\hbar}\int_0^t d\tau\left[\frac{m_e}{2}\dot{\mathbf{r}}_e^2(\tau)-e\dot{\mathbf{r}}_e(\tau)\mathbf{A}(\mathbf{r}_e(\tau)-\right.\right.$$

$$\left.\left.-\frac{1}{2}m_e\Omega_e^2 r_e^2(\tau)-\mathbf{E}(q_1)\mathbf{r}_e(\tau)\right]\right\}\int d\mathbf{q}_2\exp\left(-\frac{q_2^2}{4}\right)\int_{y}^{x'}D(\mathbf{r}_h(\tau))\exp\left\{\frac{i}{\hbar}\int_0^t d\tau\times\right.$$

$$\left.\times\left[\frac{m_h}{2}\dot{r}_h^2(\tau)+e\mathbf{r}_h(\tau)\mathbf{A}(\mathbf{r}_h(\tau))-\frac{1}{2}m_h\Omega_h^2 r_h^2+\mathrm{E}(q_2)\mathbf{r}_h(\tau)\right]\right\},$$

where

$$\Omega_{e(h)}^2=\frac{\eta^2}{L^2}\frac{2it}{\hbar m_{e(h)}},\ E(q_i)=\frac{\eta}{L}q_i,\ i=1,2.$$

Next, we shall use the following transformations in (4):

$$\dot{\mathbf{r}}_e(\tau)=\dot{\bar{\mathbf{r}}}_e(\tau)+\omega_e\times\bar{\mathbf{r}}_e(\tau),\ \omega_e=\frac{e\mathbf{H}}{2m_e c}$$

$$\dot{\mathbf{r}}_h(\tau)=\dot{\bar{\bar{\mathbf{r}}}}_h(\tau)+\omega_h\times\bar{\bar{\mathbf{r}}}_h(\tau),\ \omega_h=-\frac{e\mathbf{H}}{2m_h c}.$$

Then the Green functions in (4) turn out to be

$$G_e(\bar{\mathbf{x}}_e,\bar{\mathbf{y}}_e,t,\mathbf{q}_1)=\int_{\mathbf{x}_e}^{\mathbf{Y}_e'}D(\mathbf{r}_e(\tau))\exp\left\{\frac{i}{\hbar}\int_0^t dt\left[\frac{m_e}{2}\dot{\bar{\mathbf{r}}}_e^2(\tau)-\right.\right. \tag{6}$$

$$\left.\left.-\frac{1}{2}m_e\omega_e^2\bar{\mathbf{r}}_{e\perp}^2(\tau)-\frac{1}{2}m_e\Omega_e^2\bar{\mathbf{r}}_e^2(\tau)-\bar{\mathbf{E}}(\tau,\mathbf{q}_1)\bar{\mathbf{r}}_e(\tau)\right]\right\},$$

$$G_h(\bar{\bar{\mathbf{y}}}_h,\bar{\bar{\mathbf{x}}}_h',t,\mathbf{q}_2)=\int_{\bar{\bar{\mathbf{y}}}_h}^{\bar{\bar{\mathbf{x}}}_h'}D(\bar{\bar{\mathbf{r}}}_h(\tau)\exp\left\{\frac{i}{\hbar}\int_0^t dt\left[\frac{m_h}{2}\dot{\bar{\bar{\mathbf{r}}}}_h^2-\right.\right. \tag{7}$$

$$\left.\left.-\frac{1}{2}m_h\omega_h^2\bar{\bar{\mathbf{r}}}_{h\perp}^2-\frac{1}{2}m_h\Omega_h^2\bar{\bar{r}}_h^2+\bar{\mathbf{E}}(\tau,q_2)\bar{\bar{\mathbf{r}}}_h(\tau)\right]\right\}.$$

Here

$$\bar{\mathbf{E}}(\tau,\mathbf{q})=\frac{\eta}{L}[\mathbf{q}_{\parallel}+\mathbf{q}_{\perp}\cos(\omega\tau)-\mathbf{e}_3\times\mathbf{q}_{\perp}\sin(\omega\tau)]$$

is the vector of the local electric field in the coordinate system rotating with an electron (or hole) in the magnetic field with the frequency ω.

Owing to the symmetry of the Lagrangians in (6) and (7) it is convenient to separate the Green functions related to the time development of the system under consideration into two parts as follows:

$$G_{e\perp}(\bar{\mathbf{x}}_{e\perp},\bar{\mathbf{y}}_{e\perp}',t;\mathbf{q}_{1\perp})=\int_{\bar{\mathbf{x}}_{e\perp}}^{\mathbf{Y}_{e\perp}'}D(\bar{\mathbf{r}}_{e\perp}(\tau))\exp\left\{\frac{i}{\hbar}\int_0^t dt\left[\frac{m_e}{2}\dot{\bar{\mathbf{r}}}_{e\perp}^2(\tau)-\right.\right. \tag{8}$$

$$\left.\left.-\frac{m_e}{2}(\omega_e^2+\Omega_e^2)\bar{r}_{e\perp}^2(\tau)-\bar{\mathbf{E}}_{\perp}(\tau,q_{1\perp})\mathbf{r}_{e\perp}(\tau)\right]\right\},$$

$$G_{e''}(\mathbf{x}_{e''},\mathbf{y}_{e''}',t;\mathbf{q}_{1''})=\int_{\mathbf{x}_{e''}}^{\mathbf{Y}_{e''}'}D(\mathbf{r}_{e''}(\tau))\exp\left\{\frac{i}{\hbar}\int_0^t d\tau\left[\frac{m_e}{2}\dot{\mathbf{r}}_{e''}^2-\frac{m_e}{2}\Omega_e^2 r_{e''}^2-\right.\right.$$

$$\left.\left.-\mathbf{E}_{''}(q_{1''})\,\mathbf{r}_{e''}(\tau)\right]\right\}.$$

Similar expressions hold also for the respective hole propagators.

The canonical density matrix related to the electron motion in the plane $\perp$ **H** can be expressed with the aid of the solution to the Schrödinger equation with the Hamiltonian of the forced harmonic oscilator

$$H_{e\perp}(u)=\frac{m_e}{2}\dot{\bar{\mathbf{r}}}_{e\perp}^2(u)+\frac{1}{2}m_e(\omega_e^2+\Omega_e^2)\bar{r}_{e\perp}^2(u)+\bar{\mathbf{E}}(u,q_1)_{\perp}\bar{\mathbf{r}}_e(u) \tag{9}$$

as follows

$$R_e(\bar{\mathbf{x}}_{e\perp},\bar{\mathbf{y}}_{e\perp}',\mathbf{q}_{1\perp},\beta)=\sum_{(n)}\Psi_{(n)}^*(\bar{\mathbf{x}}_{e\perp},\mathbf{q}_{1\perp})\Psi_{(n)}(\bar{\mathbf{y}}_{e\perp}',q_{1\perp})\exp(-\beta E_{(n)})= \tag{10}$$

$$=\frac{m\omega_{Ge}}{\hbar}\sum_{n_1n_2=0}^{\infty}\Phi(\bar{\xi}_{e\perp})_{n_1n_2}^*(\bar{\eta}_{e\perp}')\exp(-\beta E_{n_1n_2}).$$

Here

$$\hbar u=i\tau,\ \Omega_e^2=\frac{\eta^2}{L^2}\frac{2\beta}{m_e},\ \bar{E}(u,q_{1\perp})=\frac{\eta}{L}(q_{1\perp}\mathrm{ch}(\omega_e u)-i\mathbf{e}_3\times\mathbf{q}_{\perp}\mathrm{sh}(\omega_e u)).$$

$\Phi_{n_1n_2}$ are the eigenfunctions of the two-dimensional harmonic oscilator and

$$\mathbf{x}_{e\perp}=\left(\frac{\hbar}{m_e\omega_1}\right)^{1/2}\xi_{e\perp}-\frac{L}{2\beta\eta}\mathbf{q}_{1\perp},\mathbf{y}_{e\perp}=\left(\frac{\hbar}{m_e\omega_2}\right)^{1/2}\boldsymbol{\eta}_{e\perp}-$$

$$-\frac{L}{2\beta\eta}\mathbf{q}_{1\perp},\ \omega_{1,2}=(\omega_{Ge}^2+\omega_H^2)^{1/2}\mp\omega_H.$$

The normalization factor in Eq. (12) results from the normalization condition for $\beta=0$

$$R(\tilde{\mathbf{x}}_\perp,\mathbf{y}_\perp,0)=\delta(\tilde{\mathbf{x}}_1-\tilde{\mathbf{y}}_1)\delta(\tilde{\mathbf{x}}_2-\tilde{\mathbf{y}}_2)$$

as follows

$$R(\xi_\perp,\tilde{\boldsymbol{\eta}}_\perp,0)=\frac{m_e}{\hbar}(\omega_{1e}\omega_{2e})^{1/2}\delta(\xi_1-\tilde{\eta}_1)\,\delta(\xi_2-\tilde{\eta}_2)$$

The density matrix related to the hole can be expressed similarly in the coordinate system rotating with the hole

$$\tilde{\mathbf{x}}_{h_1}=x_{h_1}\cos(\omega_c t)+x_{h_2}\sin(\omega_c t)$$

$$\tilde{\mathbf{x}}_{h_2}=-x_{h_1}\sin(\omega_c t)+\tilde{x}_{h_2}\cos(\omega_c t),\ \omega_c=\omega_e-\omega_h=\frac{eH}{2\omega c}.$$

Then the probability of transition can be expressed as

$$P\sim\int\int d\mathbf{x}d\mathbf{y}\lim_{\substack{\mathbf{x}'\to\mathbf{x}\\ \mathbf{y}'\to\mathbf{y}}}P_x^*P_y\langle R_e(\tilde{\mathbf{x}}_e,\tilde{\mathbf{y}}_e,\beta,\mathbf{q}_1\rangle\ \langle R_h(\tilde{\bar{\mathbf{y}}}_h,\tilde{\bar{\mathbf{x}}}'_e,\beta,\mathbf{q}_2)\rangle= \tag{11}$$

$$=m\omega_G\hbar\sum_{\substack{n_1n_2\\ n'_1n'_2}}\langle|\langle n'_1,n'_2;\mathbf{q}_{2\perp}|\varepsilon\nabla_\xi|n_1n_2;\mathbf{q}_{1\perp}\rangle|^2\rangle_{\mathbf{q}_{1\perp},\mathbf{q}_{2\perp}}\exp(-\beta(E_{n_1n_2}-$$

$$-E_{n'_1n'_2}))Z_{e'}Z_{h'}-\int d\mathbf{x}_\cdot d\mathbf{y}_\cdot\exp\left[\frac{(m_{Ge}+m_{Gh}}{2\hbar^2\beta}(\mathbf{x}_\cdot-\mathbf{y}_\cdot)^2\right]+$$

$$+Z_{e\perp}^{(H)}Z_{h\perp}^{(H)}\int d\mathbf{x}_\perp d\mathbf{y}_\perp\exp\left[-\frac{(m_{GHe}+m_{GHh}}{2\hbar^2\beta}(\tilde{\mathbf{y}}_\perp-\tilde{\mathbf{x}}_\perp)^2\right]$$

$$\sum_{n_3n'_3}\langle|\langle n'_3,\mathbf{q}_{2'}|\varepsilon\mathbf{P}_\cdot|\,n_3,\mathbf{q}_{1'}\rangle|^2\rangle_{\mathbf{q}_{1'},\mathbf{q}_{2'}}\exp(\beta(E_{n_3}-E_{n'_3})).$$

Here Z_e and Z_h are partition sums of the electron and the hole in a one-dimensional harmonic potential $\frac{1}{2}m_e\omega_{Ge}^2x_\cdot^2$,

$$Z_{e'}=\left(\frac{m\beta}{8\pi}\right)^{1/2}\frac{\omega_{Ge}}{\mathrm{sh}\left(\frac{\hbar\beta\omega_{Ge}}{2}\right)}=\frac{\eta\beta}{L\sqrt{\pi}}\sum_{n_3=0}^{\infty}\exp\left(-it\omega_{Ge}\left(n_3+\frac{1}{2}\right)\right).$$

$Z_{e\perp}$, $Z_{h\perp}$ are two-dimensional partition sums of the electron and the hole in a magnetic field and in a random potential, which have been calculated in [1]. The values of m_{GHe} and m_{GHh} are given in [6],

$$m_{GH}=\frac{m\hbar\beta}{\omega_1+\omega_2}\left(\frac{\omega_1^2}{1-e^{-\omega_1\hbar\beta}}+\frac{\omega_2^2}{e^{\omega_2\hbar\beta}-1}\right).$$

The first term in Eq. (11) represents "intraband" transitions between the Landau oscillatory states $|n_1n_2\rangle$ broadened by the random fields $\mathbf{q}_{1\perp}$, $\mathbf{q}_{2\perp}$. The second represents "interband" transitions between the Stark levels in a one-dimensional random potential.

III. THE COEFFICIENT OF MAGNETOABSORPTION

The coefficient of optical magnetoabsorption can be calculated approximately for strong magnetic fields, $\omega_H\gg\omega_G$ and providing $m_e\approx m_h$. For the "intraband transitions", represented by the first term of Eq. (11), one obtains the result

$$\varepsilon_2^{intra}(\hbar\omega)\approx\left(\frac{e}{m\omega}\right)^2\frac{\hbar^3 2^5}{\sqrt{\pi m}}\left(\frac{\eta}{L}\right)^6\sum_{\substack{n_1n_2n_3\\ n'_1n'_2n'_3}}\eta'^{-1}(\eta'\sqrt{2})^{-\frac{13}{2}-\frac{3}{2}l}\times \tag{12}$$

$$\times\frac{(-1)^l}{l!}\left(\frac{\hbar\eta}{L}\sqrt{\frac{2}{m}}(n_3+n'_3+1)\right)^l\Gamma\left(\frac{15}{2}+\frac{3}{2}l\right)\exp\left(-\frac{E^2}{8\eta^2}\right)\times$$

$$\mathrm{Re}\left\{-\left(\frac{i}{\eta'\sqrt{2}}\right)^{\frac{13}{2}+\frac{3}{2}l}\ D_{-\frac{15}{2}-\frac{3}{2}l}\left(-\frac{iE}{\eta'\sqrt{2}}\right)\right\}.$$

Here

$$E=\hbar\omega-E_g-2\omega_H(n_1+n'_1+1) \tag{13}$$

$$\eta'^2=\eta^2\left(1-\frac{\lambda_H^2}{L^2}(n_1+n'_1+n_2+n'_2+2)\right),\ \lambda_H=\left(\frac{\hbar c}{eH}\right)^{1/2}.$$

The asymptotic behaviour of expression (12) for $|E|\gg\eta'$, $E<0$, determining the shape of the absorption edges, results in

$$\varepsilon_2^{intra}(\hbar\omega)\approx\left(\frac{e}{m\omega}\right)^2\left(\frac{\eta^2\hbar}{2L^2}\right)^3\frac{1}{\sqrt{m}}\sum_{\substack{n_1n_2\\ n_1n_2}}\frac{|E|^{\frac{13}{2}}}{\eta'^{14}}\frac{\exp\left(-\frac{E^2}{4\eta'^2}\right)}{\mathrm{sh}^2\left(\frac{\eta\hbar}{4L\sqrt{m}}\frac{|E|^{3/2}}{\eta'^3}\right)}. \tag{14}$$

The evaluation of the second term in Eq. (11) representing the "interband transitions" yields

$$\varepsilon_2^{inter}(\hbar\omega)\approx\left(\frac{\eta}{L}\right)^6\left(\frac{2e\hbar}{m\omega}\right)^2\frac{1}{\omega_H\sqrt{\pi m}}\sum_{\substack{n_1n_2n_3\\n_1n_2n_3}}(\eta'\sqrt{2})^{-\frac{13}{2}-\frac{3}{2}l}\frac{(-1)^l}{l!}\times \qquad (15)$$

$$\times\left[\frac{\eta\hbar}{L}\sqrt{\frac{2}{m}}(n_3+n_3'+1)\left(\eta'\sqrt{2}\right)\right]^{-\frac{3}{2}l}\Gamma\left(\frac{13}{2}+\frac{3}{2}l\right)D_{-\frac{13}{2}-\frac{3}{2}l}\left(-\frac{iE}{\eta'\sqrt{2}}\right)$$

$$\exp\left(-\frac{E^2}{8\eta'^2}\right).$$

Here E and η' are given by Eq. (13).

The absorption edge, given by the asymptotic behaviour of Eq. (17) for $|E|\gg\eta'$, yields

$$\varepsilon_2^{inter}(\hbar\omega)\approx\left(\frac{e\hbar}{m\omega}\right)^2\left(\frac{\eta}{L}\right)^6(2\omega_H\sqrt{2m})^{-1}\sum_{\substack{n_1n_1'\\n_2n_2'}}\eta'^{-12}\frac{|E|^{\frac{11}{4}}\exp\frac{E^2}{4\eta'^2}}{\mathrm{sh}^2\left(\frac{|E|^{3/2}\hbar\eta}{2\eta^3L\sqrt{m}}\right)} \qquad (16)$$

It is obvious that, similarly as for the optical absorption with $\mathbf{H}=0$, the neglection of the electron-hole correlation $W(\mathbf{r}_e(\tau)-\mathbf{r}_h(\tau'))$ in (3) results in the reproduction of the density of states in the absorption coefficients (15) and (16), having the same characteristic features as described in [1]:

1. The Landau shift of the absorption due to the magnetic oscillations of electrons and holes in the plane $\perp\mathbf{H}$.

2. The magnetic field dependence of the effective damping constant η'^{-2} in (14) and (16).

3. The quantum shift of the density of states decreasing with increasing $\mathbf{H}$ and being nonzero also for $\mathbf{H}=0$ [5].

This effect corresponds to the localization due to the effective one--dimensionality of the problem of the electron in a random potential owing to the magnetic field.

Generally speaking, for strong magnetic fields there occurs the shift of the density of states to higher energies caused by the three above mentioned effects, as well as the contingent "flare-up" of the oscillatory behaviour of the magnetoabsorption at the increasing magnetic field if the condition for the dominancy of the magnetic effects $\omega_H\gg\omega_G$ has been fulfilled. This effect has been actually observed in doped Ge [2] but, unforunately, the investigation of strongly doped semiconductors has not yet been performed. The dependence of the width of the impurity band on the magnetic field and on the concentration of impurities has been investigated by Hasegawa and Nakamura [3]. These investigations imply the existence of the magnetic "freeze-out" in doped InSb.

IV. EVALUATION OF THE ELECTRON DENSITY OF STATES WHEN $\omega_G\ll\omega_H$

The condition for the dominancy of the magnetic effects over the effects of the random fields [1]

$$\frac{\omega_G}{\omega_H}\ll\frac{\lambda_H}{\lambda}(\sqrt{2}(n_1+n_2+1)^{-\frac{1}{2}}\ll 1,\ \lambda=\frac{\hbar}{(2m\beta)^{1/2}2\eta}$$

can be fulfilled only for weak random potentials $\eta<10^{-1}$ eV and for extremly strong magnetic fields (10^4—10^5 Γ). For realistic values $\eta\sim10^{-1}$ eV, $L\sim10^{-6}$ cm, $H\sim10^4$ Γ the inverse condition $\omega_G\gg\omega_H$ is more likely fulfilled.

The partition function $\langle Z(\beta,H)\rangle$ can be expressed as

$$\langle Z(\beta,\omega_H)\rangle=\left(\frac{m\beta}{8\pi}\right)^{3/2}(2\omega_G)^3\exp\left(\frac{\eta^2\beta^2}{2}\right)\sum_{n_1,n_2,n_3=0}^{\infty}\exp(-\beta E_{n_1n_2n_3}),$$

where

$$E_{n_1n_2n_3}=\left(n_1+\frac{1}{2}\right)\hbar(\Omega+\omega_H)+\left(n_2+\frac{1}{2}\right)\hbar(\Omega-\omega_H)+\left(n_3+\frac{1}{2}\right)\hbar\omega_G,$$

$$\Omega=(\omega_G^2+\omega_H^2)^{1/2},\ \omega_H=\frac{eH}{mc},\ \omega_G^2=\frac{\eta^2}{L^2}\frac{2\beta}{m}.$$

For $\omega_G\gg\omega_H$ we have $\Omega'=(\omega_G^2+\omega_H^2)^{1/2}\pm\omega_H\approx\omega_G\pm\omega_H+\frac{\omega_H^2}{2\omega_G}$,

so that

$$E_{n_1n_2n_3}=\left(n_1+n_2+n_3+\frac{3}{2}\right)\hbar\omega_G+\left(n_1-n_2\right)\hbar\omega_H+\frac{\hbar\omega_H^2}{2\omega_G}(n_1+n_2+1).$$

The respective density of states results in

$$n(E,H)=\left(\frac{\eta}{L}\right)^3\pi^{-5/2}\sum_{\substack{n_1n_2n_3\\n_1n_2n_3}}\eta^{-4-\frac{3}{2}l-\frac{1}{2}k}\frac{(-1)^{l+k}}{l!k!}A^lB^k\Gamma\left(\frac{3}{2}l+\frac{k}{2}+4\right) \qquad (17)$$

$$\exp\left(-\frac{E'^2}{4\eta^2}\right)\mathrm{Re}\left\{i^{\frac{3}{2}l+\frac{k}{2}}D_{-\frac{3}{2}l-\frac{k}{2}-4}\left(-\frac{iE'}{\eta}\right)\right\},$$

where

$$E'=E-\hbar\omega(n_1-n_2),\ A=\frac{\eta\hbar}{L}\sqrt{\frac{2}{m}}(n_1+n_2+n_3+2n_1'+2n_2'+n_3'+4),$$

$B=\frac{\omega_H^2\hbar L}{2\eta}\sqrt{\frac{m}{2}}(n_1+n_2+2n_1'+2n_2'+3)$. In the asymptotic range $|E'|\gg\eta',\ E'<0$ Eq. (17) yields

$$n(E,H) \approx (\sqrt{2} L^3 \pi^2 \eta^4)^{-1} \sum_{n_1 n_2 = 0}^{\infty} |E'|^3 \exp\left\{ -\frac{E'^2}{2\eta^2} - \frac{|E'|^{3/2}\hbar}{\eta^2 L} \sqrt{\frac{2}{m}} (n_1 + n_2 + 1) - |E'|^{1/2} \frac{\hbar\omega_H^2 \sqrt{m}}{2\sqrt{2}\,\eta^2}. \right. \tag{18}$$

As expected, the Landau shift does not appear in this case. Only the magnetic field dependent quantum shift of the spectra occurs, represented by the last two terms in the exponential of Eq. (18).

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