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THEORY OF AN ELECTRIC SHEATH IN THE DIFFUSION REGIME DISCHARGE PLASMA*

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The electric field, potential and charged particle concentration are calculated in a plasma sheath at an infinite, electric non conducting plane. In the sheath, electrons are assumed to be distributed according to the Boltzmann law and the ions are accelerated in the sheath field and collide with the gas molecules.

ТЕОРИЯ ДВОЙНОГО ЭЛЕКТРИЧЕСКОГО СЛОЯ В ПЛАЗМЕ РАЗРЯДА В ДИФФУЗИОННОМ РЕЖИМЕ

В статье определено электрическое поле, потенциал и концентрация заряжённых частиц в двойном электрическом слое, образованном в плазме при непроводящей бесконечной плоскости. Предполагается, что электроны в двойном слое распределены в соответсвии с законом Больцмана и ионы в поле двойного слоя ускоряются и сталкиваются с молекулами газа.

It is known that the Schottky theory of a positive column is a very rough approximation of the real conditions in plasma bounded by a non-conducting wall. Some better results follow from a modified theory [1,2], in which also an inertia, heating and variation of the mobility of ions in the plasma sheath electric field at the wall are taken into consideration. That theory of a collisionless sheath was developed by Ott [3]. Forrest and Franklin [4] found a solution, valid in the diffusion regime for both the plasma region and the sheath, where they considered the particle production in the whole volume, but the heating, variation of the mobility and recombination of ions did not take place in their equations.

In the present paper, we deal with the profile of the electric field, potential, ion velocity and concentration in the plasma region, disturbed by the presence of the wall. The problem is solved in the planar geometry (the plasma is bounded by an

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infinite plane). We start from the Poisson and the momentum equation for ions, generally formulated in [5], but neglecting the term of stress tensor. Physically it represents the neglecting of the ion heating in the electric field, which, however, has not a large effect in the plasma region [6]. The variation of the ion mobility is not taken into consideration either.

In an one dimensional case, the system of equations has the following form:

$$\frac{d^2\varphi}{dz^2} = -\frac{e}{\varepsilon_0}(n_+ - n_-) \tag{1}$$

$$n_+ v_+ \frac{dv_+}{dz} = -\frac{n_+}{m_+} e \frac{d\varphi}{dz} - n_+ v_+ \nu_+ - v_+ n_-(\alpha - \beta n_+), \tag{2}$$

where n_+. v_+, m_+, ν_+, n_-, e, ε_0, α, β, φ, z correspond to concentration, drift velocity, mass and collision frequency of ions (for collisions with neutral atoms), electron concentration, elementary charge, permitivity of the vacuum, ionization and (binary) recombination coefficients, electric potential and the coordinate in a direction, vertical to the wall, respectively. The last term in equation (2) describes the production and volume destruction of charged particles and has its origin in the substitution of the term $v_+ \frac{dj_+}{dz}$ by the continuity equation:

$$\frac{dj_+}{dz} = \alpha n_- - \beta n_- n_+, \tag{3}$$

where j_+ is the ion flux density.

Further, we assume the electrons to be distributed in the electric field according to the Boltzmann law:

$$n_- = n_0 \exp\left(\frac{e\varphi}{kT_-}\right),$$

where $n_0 = \alpha/\beta$ is an equilibrium charged particle concentration, T_- is the electron temperature and k the Boltzmann constant.

When we introduce the Debye length $h = \sqrt{\frac{\varepsilon_0 kT_-}{e^2 n_0}}$ and put

$$u = \frac{v_+}{v_i} = v_+ \sqrt{\frac{m_+}{kT_-}},\ \eta = \frac{e\varphi}{kT_-},\ x_- = \frac{n_-}{n_0},\ x_+ = \frac{n_+}{n_0},\ \xi = \frac{z}{h},\ A = \frac{h\nu_+}{v_i},\ B = \frac{\alpha}{\nu_+}$$

the system of equations (1), (2) acquires the following form:

$$u\frac{du}{d\xi} = -\varepsilon - Au - ABue\eta \frac{1 - x_+}{x_+} \tag{4}$$

$$\frac{d\varepsilon}{d\xi} = e^{\eta} - x_+ \tag{5}$$

$$\frac{d\eta}{d\zeta} = \varepsilon\,. \tag{6}$$

In the following calculations we assume the total electric current to the wall to be zero, i. e. the equality of the electron and the ion flux densities: $j_+ = j_- = j$. In order to simplify the numerical calculations, we replace the continuity equation for ions by a solution of the equation en the ambipolar approximation, which can be written in normalizded variables as follows

$$\frac{d^2x}{d\zeta^2} = -\frac{\alpha h^2}{D_a}(x - x^2)\,. \tag{7}$$

Here the particle flux density in the ambipolar diffusion case was substituted for $j: j = -D_a \frac{dn}{dz}$; D_a is an ambipolar diffusion coefficient and $x = n/n_0$, where n is the concentration of the charged particles. The equation (7) can be slightly once integrated to obtain the following formula for the normalized flux density $j/n_0 v_i$:

$$x_+ u = -\sqrt{B}\sqrt{\frac{2x^3 - 3x^2 + 1}{3}}\,. \tag{8}$$

This relation will be used to establish x_+ and it is assumed to be applicable also in the sheath, when putting $x = x_-$. Then the system (4—6) reduces to

$$\frac{d\xi}{du} = -\frac{u}{\varepsilon + Au\left[1 + Be^{\eta}\left(\frac{1}{x_+} - 1\right)\right]} \tag{9}$$

$$\frac{d\varepsilon}{du} = (e^{\eta} - x_+)\frac{d\xi}{du} \tag{10}$$

$$\frac{d\eta}{du} = \varepsilon \frac{d\xi}{du} \tag{11}$$

$$x_+ = \frac{\sqrt{B}(1 - e^{\eta})}{u}\sqrt{\frac{2e^{\eta} + 1}{3}}, \tag{12}$$

where we have used u as an independent variable.

In order to solve the system consisting of the differential equations (9) — (11) and algebraic relation (12), the suitable initial conditions (i.e. values of variables u, η, ε, x_+ and ζ) must be chosen. For this purpose we have searched for solutions η and ε in the form of power expansion with regard to the zero values of u, η and ε, in the plasma. In search of coefficients of the power expansion, it has been found that also the quadratic terms must be included:

$$\varepsilon \doteq (a_0 + a_1 u) u \qquad (13)$$
$$\eta \doteq (b_0 + b_1 u) u \, .$$

The term $e^{\eta} - x_+$ in equation (10) is especially sensitive; here the first three terms of the power series of the function $e^{\eta} \doteq 1 + \eta + \frac{1}{2}\eta^2$ must be used. Then we obtain (neglecting the powers of η and u higher than 2):

$$x_+ \approx -\frac{\eta}{u}\sqrt{B} - \frac{5}{6}\frac{\eta^2}{u}\sqrt{B}$$

$$e^{\eta} - x_+ \approx 1 + \eta\left(1 + \frac{\sqrt{B}}{u}\right) + \eta^2\left(\frac{1}{2} + \frac{5}{6}\frac{\sqrt{B}}{u}\right) \approx \qquad (14)$$

$$\approx b_0\sqrt{B} + u(b_0 + b_1\sqrt{B} + \frac{5}{6}b_0^2\sqrt{B}) +$$

$$+ u^2\left(b_1 + \frac{b_0^2}{2} + \frac{5}{3}\sqrt{B}b_0 b_1\right) .$$

Eliminating $d\xi/du$ from equations (10) and (11), substituting the expansions (13) and neglecting the powers of u higher than 2, we obtain by comparison of the coefficients with the same powers of u:

$$b_0(1 + b_0\sqrt{B}) = 0 \qquad (15a)$$

$$b_0\left(b_0 + b_1\sqrt{B} + \frac{5}{6}b_0^2\sqrt{B}\right) + 2b_1(1 + b_0\sqrt{B}) = a_0^2 . \qquad (15b)$$

Substituting (9) into (11) (the term e^{η}/x_+ was replaced by unity) and using (13) and (14), we obtain in the same way:

$$b_0[a_0 + A + AB(1 + b_0\sqrt{B})] = 0 \qquad (15c)$$

$$a_0 + b_0\left[a_1 + AB\sqrt{B}\left(b_1 + \frac{5}{6}b_0^2\right)\right] + 2b_1[a_0 + A + AB(1 + b_0\sqrt{B})] = 0 . \qquad (15d)$$

Formally, equation (15a) admits two solutions; one of them ($b_0 = 0$) yields the condition

$$a_0[1 + a_0(a_0 + A + AB)] = 0 .$$

which offers either identically zero solutions for ε and η, or complex roots when $A(1+B) < 2$, thus they are not applicable.

The only physically usefull root of equation (15a) is then

$$b_0 = -\frac{1}{\sqrt{B}} ,$$

which gives (after determining the roots a_0, a_1, b_1 from the other 3 equations) the following formulae for ε and η:

$$\varepsilon \approx -Au - A\sqrt{B}(2 - A^2 B)u^2 \qquad (16)$$

$$\eta \approx -\frac{1}{\sqrt{B}}u + \left(\frac{1}{6B} - A^2\right)u^2 .$$

The relations (16) enable to start computations by equations (9) — (12) for given A and B. The value of u can be picked arbitrary, with the limitation that the approximate relations (13) and (14) are to be valid, i.e. u must be sufficiently small.

Equations (9) — (12) have been solved for $A = 1$, $B = 0.01$ and the initial values of ε and η determined from (16) for $u = 0.1$; then $\varepsilon = -0.101990$, $\eta = -0.843$. The results are shown in Figs. 1 and 2.

As it can be seen from equations (9—12), the variable ξ is not present in equations (10) and (11) and so the initial value of ξ is arbitrary. It is convenient to

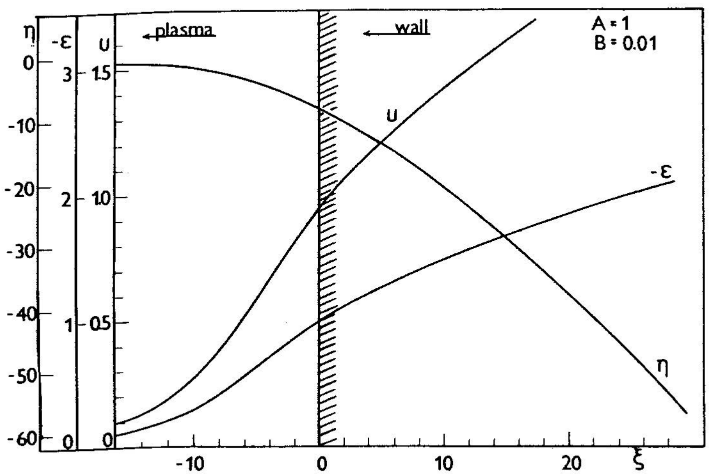

Fig. 1. The normalized ion velocity u, the electric field $-\varepsilon$ and the potential η as functions of normalized distance ξ for $A = 1$ *and* $B = 0.01$.

put $\xi = 0$ at the wall. The wall is physically defined as the place where the electron drift velocity reaches the half-value of the thermal electron velocity: $v_- = 1/2\langle c_- \rangle$. By using the normalized electron velocity $u_- = v_-/v_i$, we obtain a condition for the wall location

$$\frac{x_+ u}{x_-} = u_- = \sqrt{\frac{2m_+}{\pi m_-}},$$

where m_- is the mass of the electron.

The curves in Figs. 1 and 2 are universal ones (they are determined only by the values of the parameters A and B), but the wall location corresponds to the case of the argon plasma.

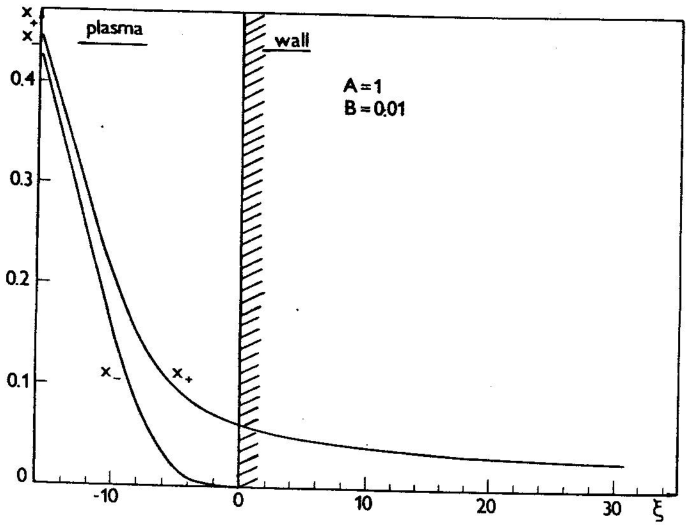

Fig. 2. The normalized electron x_- and the ion x_+ concentrations versus the normalized distance ξ for $A = 1$ and $B = 0.01$.

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