acta phys. slov. 29. (1979), No. 1

THE MULTIPLICITY FUNCTIONS OF COSMIC RAYS PROTONS IN THE REGION OF NEUTRON SUPERMONITOR SENSITIVITY

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In this paper the multiplicity functions of cosmic rays protons are determined in the rigidity interval 1—15 GV on the basic of the latitude effect of the cosmic ray neutron component during the time of minimal solar activity. The differential solar protons spectrum in the interval 1—5 GV for the solar flare of January 28th, 1967 is also specified. The obtained values are approximated by the power and the exponential function. The results are compared with the publications by other authors regarding the wide solar proton rigidity interval.

КРАТНЫЕ ФУНКЦИИ ПРОТОНОВ КОСМИЧЕСКОГО ИЗЛУЧЕНИЯ В ОБЛАСТИ ЧУВСТВИТЕЛЬНОСТИ НЕЙТРОННЫХ СУПЕРМОНИТОРОВ

В предлагаемой работе на основе широтного эффекта потока нейтронной составляющей космического излучения в период минимума солнечной активности определены кратные функции протонов космического излучения в интервале жесткостей 1—15 Гв. Определён дифференциальный спектр солнечных протонов в интервале 1—5 Гв для случая услучения солнечных частиц 28-го января 1967 г. Апроксимация полученных значений спектра проведена с помощью степенной и экспоненциальной функций. Проведено сравнение полученных результатов с работами других авторов в широком интервале жесткостей солнечных протонов.

I. INTRODUCTION

For the theories of accelerating mechanisms of the particles of solar cosmic rays, for the models describing the spread of these particles in the interplanetary magnetic fields as well as for the solution of many geophysical problems it is very important to know the absolute fluxes of solar particles in the vicinity of the Earth in various rigidity intervals $R(R = pc/Ze)$ at various times after their ejections into the interplanetary space.

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For the precise description of the primary spectrum in the wide solar cosmic ray rigidity interval it is desirable to harmonize the measured results obtained by help of rockets, satellites, riometers and ionospheric probes in the small rigidity region (0.045—0.45) GV with data from the balloon measurements at the boundary of the Earth's atmosphere in the registration interval (0.45—1.0) GV as well as with the results of ground level registrations in the rigidity region $R \geqslant 1$ GV. To achieve this it is necessary to know the absolute sensitivity of the corresponding detectors of the various parts of the primary spectrum. Studying primary particles with the rigidity $R \geqslant 1$ GV, the sensitivity of the ground level detectors, registering the cosmic ray neutron component — the neutron supermonitors — acquires extraordinary importance.

The world network of neutron supermonitors localized at places where the vertical particle cut-off rigidity is $\leqslant 5$ GV are of foremost importance in the study of the supplementary flux of cosmic rays generated during some of the proton flares in the Sun.

II. THE LATITUDE EFFECT OF THE COSMIC RAY NEUTRON COMPONENT IN ABSOLUTE UNITS

The absolute integral flux F_g of the galactic cosmic rays in the time t registered by the apparatus localized at the atmospheric level h with the cut-off rigidity R_c is:

$$F_g(>R, H, t) = \int_{R_c}^{\infty} m_z(R, H)\, D_z(R, t)\, \mathrm{d}R \tag{1}$$

where: $D_Z(R, t)$ is the differential spectrum of the primary particles with the change Z; $m_Z(R, h)$ is the total multiplicity function for the primary component with the charge Z, i.e. the ratio of the particles registered by the apparatus of particles with the charge Z from the small spherical angle in the neighbourhood of the vertical with the rigidities in the interval $(R, R+\mathrm{d}R)$.

Differentiating Eq. (1) according to R we get:

$$\left|\frac{\mathrm{d}F_g}{\mathrm{d}R}\right| = m_Z(R, h) D_Z(R, t), \tag{2}$$

where the left-hand side of the equation represents the absolute particle flux in the apparatus which registers one of the components of the galactic cosmic rays.

In the following we shall deal with the cosmic ray neutron component registered by the world neutron monitor network at sea level ($h = 1030$ g cm^{-2}). The function $\mathrm{d}F_g/\mathrm{d}R$, which appears on the left-hand side of Eq. (2) can be determined in the rigidity interval (1—15) GV on the basis of the latitude effect of the cosmic ray neutron component counting rate.

In the analysis we have used measurements of the latitude effect of the cosmic ray neutron component counting rate obtained during voyages of Japanese expeditions of the Antarctic and from spectral experiments performed between 1954—1962. The results have been summarized in [1]. Thus we have obtained results for the period between the solar minimum and the solar maximum activities. Taking into account the deviations in the geometry of the testing monitors the results have been normalized by data from the Chicago neutron monitor (R_c = 1.72 GV) operating during the whole period of the latitude measurements. For a long-time comparability the results of the neutron monitor counting rates are time normalized by the agreement that the relative intensity at the time of the minimal solar activity in 1954 for $R_c = 15$ GV is equal to 100. As shown in [2] the latitude dependence of the neutron counting rates is best approximated by the following analytical form:

$$N_g^n(>R) = N_g^n(0)\,\{1 - \exp(-\alpha R^{-k})\}. \tag{3}$$

Using the least square method and daily neutron counting rate average values with the deviation of 0.2% per day we have obtained for the constants in Eq. (3) the values: $N_g^n(0) = 188.5$, $\alpha = 6.58$ and $k = 0.71$.

Most valuable results, concerning the experimental determination of the absolute neutron flux registered by a neutron monitor at the period of minimal solar activity have been obtained by a group of workers at the Leeds observatory [3]. By calibration measurements it has been shown that the absolute neutron flux registered by the neutron monitor at Leeds ($R_c = 2.2$ GV) is $(10 \pm 0.7) \times 10^{-4}$ neutron cm^{-2}s^{-1}, which represents approximately 82% of the total counting rate registered by the neutron monitor. By help of Eq. (3) and calibration measurements we have obtained the latitude effect for the cosmic ray neutron component in absolute units:

$$F_g^n(>R) = 1.024 \times 10^{-3}\{1 - \exp(-6.58\, R^{-0.71})\}\ \text{neutr. cm}^{-2}\text{s}^{-1}. \tag{4}$$

Then for the neutron differential flux in the interval (1—15) GV at sea level we have

$$\frac{\mathrm{d}F_g^n}{\mathrm{d}R}(R) = 4.78 \times 10^{-3} R^{-1.71} \exp(-6.58 R^{-0.71})\ \text{neutron cm}^{-2}\text{s}^{-1}\text{GV}^{-1}. \tag{5}$$

III. COSMIC RAY PROTONS MULTIPLICITY FUNCTIONS

In the sensitivity region of neutron supermonitors direct measurements of the integral spectrum of the primary cosmic ray protons carried out by the Čerenkov counters for the period close to that of the minimal solar activity are summarized in publication [4]. The experimental values of the proton integral spectrum in the interval (1—15) GV have been approximated by the relation:

$$J_g^p(>R) = 3.18\{1 - \exp(-3.19\, R^{-1.51})\}\ \text{proton cm}^{-2}\text{s}^{-1}. \tag{6}$$

Hence for the primary protons differential spectrum

$$D_g^p(R) = 15.28\ R^{-2.51} \exp(-3.19\ R^{-1.61})\ \text{prot. cm}^{-2}\text{s}^{-1}\text{GV}^{-1} \qquad (7)$$

In Fig. 1 differential spectra of primary protons and secondary neutrons are compared.

If we know the latitude dependence of the cosmic ray neutron component at sea level and the differential spectrum of the primary protons at the boundary of the Earth's atmosphere in absolute units, then substituting Eqs. (5) and (7) into Eq. (2) we get the absolute values for the multiplicity functions of the cosmic ray protons $m_p(R)$. To be able to compare the absolute values with the normalized ones we shall use the normalization condition as the authors did in [5]:

$$m_p(R) = m_p'(R) \int_{15}^{\infty} (dF_g^n/dR)\, dR \Big/ \int_{15}^{\infty} D_g^p(R)\, dR.$$

The integral in the nominator represents the flux of neutrons originating from primary protons and registered by the neutron supermonitor in the position with the vertical cut-off rigidity 15 GV; the integral in the denominator represents the integral spectrum of protons with the rigidity above 15 GV. After substitution of appropriate values we get the relation between absolute and normalized multiplicity functions of the primary cosmic radiations.

In the Fig. 2 there are graphically represented the absolute $m_p(R)$ and normalized $m_p'(R)$ multiplicity functions for the cosmic ray protons in dependence on the rigidity of the primary protons in the rigidity intervals (1—15) GV.

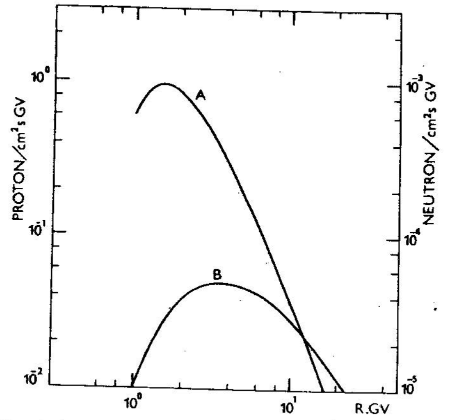

Fig. 1. The comparison of differential spectra in dependence on the particle rigidity during minimal solar activity (1954, 1965); A — primary protons at the boundary of the Earth's atmosphere, B — secondary neutrons at sea level.

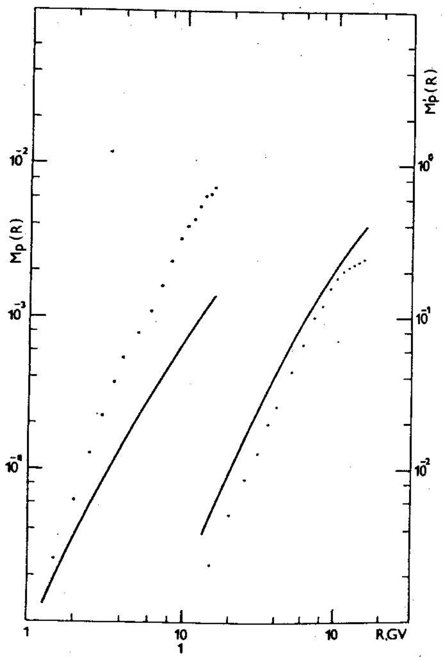

Fig. 2. The absolute $m_p(R)$ and normalized $m_p'(R)$ multiplicity cosmic ray proton functions; ○ — values $m_p(R)$ from [12], ● — values $m_p'(R)$ from [5]. The continuous curves represent results from the present paper.

IV. SOLAR PROTONS SPECTRUM AT THE BOUNDARY OF THE EARTH'S ATMOSPHERE

In some solar proton flares in the Sun the solar cosmic ray protons are generated and accelerated with rigidities sufficient to penetrate to the Earth's surface. In such cases neutron monitors register a supplementary flux which is greater than the usual level of the galactic radiation. The above determined proton multiplicity functions allow us to determine on the basis of the latitude dependence of solar cosmic radiation its rigidity spectrum on the boundary of the Earth's atmosphere.

If the integral neutron flux registered by the neutron supermonitor at sea level at the place with cut-off rigidity R_c is denoted by $F_s^n(>R)$, then, according to Eq. (2), we obtain for the rigidity spectrum of the solar protons at the boundary of the Earth's atmosphere

$$D_s^p(R) = (\partial F_s^n/\partial R)/m_p(R), \qquad (8)$$

where $\partial F_s^n/\partial R$ represents the differential flux of the solar particles and $m_p(R)$ are the above determined absolute multiplicity functions.

In the following part we shall apply the mentioned methods for the determination of the rigidity spectrum at the boundary of the Earth's atmosphere to the case of the proton flare of January 28th, 1967. The supplementary flux of the solar particles was registered by neutron monitors at medium and high geographic latitudes. The time profiles of the neutron counting rates show that the initial and decreasing phases have an isotropic character, see [6]. The values of percentual increases with regard to the galactic phon were corrected at the individual stations by the methods of the effective absorption length [7] and recomputed to the absolute integral flux by help of Eq. (4). The percentual increases and $F_s^n(>R)$ given in Tab. 1 are related to 1200UT of January 28th, 1967. The values of the vertical cut-off rigidity were taken from [8].

The relation $F_s^n = F_s^n(R)$ is best approximated by the equation:

$$F_s^n(>R) = 1.8 \times 10^{-4}\ R^{-1.52},$$

hence for the differential flux of the solar particles we have:

$$\frac{dF_s^n}{dR}(R) = 2.7 \times 10^{-4}\ R^{-2.52}.$$

Table 1

The parameters of neutron supermonitor stations used in the analysis

Station	Vertical cut-off rigidity	Altitude m	Latitude	Longitude °E	Increase %	$F_s^p(>R)$ $\mathrm{cm^{-2}s^{-1}}$
Deep River	1.02	145	46.10	282.50	14.83	1.516×10^{-4}
Durham	1.41	0	43.10	289.17	11.27	1.147×10^{-4}
Uppsalla	1.43	0	59.85	17.58	10.76	1.095×10^{-4}
Leeds	2.20	70	53.83	358.42	6.04	6.411×10^{-5}
Dallas	4.35	208	32.98	263.27	1.91	1.763×10^{-5}

Table 2

The comparison of the differential proton rigidity spectra from January 28th, 1967 at 1200 UT

Spectrum shape				
a		*b*		References
$D_s^p(0)$	γ	$(\mathrm{d}F/\mathrm{d}R)_0$	R_0	
31.0 ± 3.3	4.47 ± 0.14	32.4 ± 2.3	0.69 ± 0.07	the present work
6.3 ± 3.1	$4.9\ \pm0.2$	12.6 ± 6.3	0.55	[9]
81.7	$5.0\ \pm1.0$	49.0	0.60	[10]

Substituting into Eq. (8) and dividing by the appropriate value of $m_p(R)$ we get values of the proton rigidity spectrum at the boundary of the Earth's atmosphere. In Tab. 2 the differential spectrum is approximated a) by the power and b) by the exponential dependence in the following way:

$$\text{(a)}\quad D_s^p(R)=D_s^p(0)R^{-\gamma}\qquad \text{(b)}\quad D_s^p(R)=\left(\frac{\mathrm{d}F}{\mathrm{d}R}\right)_0\exp\left(-R/R_0\right),$$

where $D_s^p(0)$, γ, $(\mathrm{d}F/\mathrm{d}R)_0$, R_0 are characteristic parameters for the differential spectrum of a proton flare.

The decisive criterium for the comparison of the achieved results is the computation of the primary spectrum of the solar proton cosmic radiation by using various values of the multiplicity functions and its comparison while it is passing from ground level measurements to the direct ones in the stratosphere.

Assuming that the differential proton spectrum at the boundary of the atmosphere is described by the power dependence both in the low and the high rigidity regions, we want to compare results obtained by various authors (Tab. 2) for the region (1—5) GV with experimental data in the rigidity region (0.45—1.0) GV.

The balloon measurements [11] performed on January 28th, 1967 at the antarctic station Byrd showed that the proton rigidity spectrum in the interval (0.45—0.96) GV is approximated by the relation $D_s^pR=41.5\,R^{-4.5}$ proton $\mathrm{cm^{-2}s^{-1}GV^{-1}}$. Now we shall compare the values of spectra for $R=1$ GV and $R=0.45$ GV obtained by direct stratospheric measurements with the results in Table 2 and extrapolated to the mentioned rigidity spectrum region. The results are summarized in Tab. 3, where $|\Delta|$ indicates the deviation of values of differential spectra obtained from experimental values of direct stratospheric measurements from the values obtained in the present publication [9, 10] for various values of R.

Table 3

$R_c=0.45$ GV		$R_c=1.0$ GV		References
$D_s^p(R)$	$\lvert\Delta\rvert$%	$D_s^p(R)$	$\lvert\Delta\rvert$%	
1.51×10^3				[11]
1.13×10^3	25	31.0	25	the present work
3.15×10^2	79	6.3	85	[9]
4.43×10^3	194	81.7	97	[10]

V. CONCLUSION

1. From the latitude dependence of the neutron component of the cosmic radiation at the time of minimum solar activity in 1954 we have determined by using calibration measurements of the neutron absolute flux in a locality with the threshold rigidity $R_c=2.2$ GV the latitude dependence of the neutron flux in units of neutron $\mathrm{cm^{-2}s^{-1}}$.

2. On the basis of results mentioned in point 1 we have determined the absolute multiplicity functions for cosmic ray protons in the rigidity (1—15) GV. The statistical accuracy of the measurement of the latitude dependence of the neutron component is given by the precision of the calibration measurements of the absolute neutron flux in Leeds. The values of the integral primary proton spectrum [4] entails a systematic error in the interval $\pm(3—6)$%. Thus, from Eq. (7) it follows that the mean square deviation of the absolute multiplicity functions increases proportionally with R and is equal to $\pm(5—7)$% in the rigidity interval (1—15) GV. Comparing our results with results from [5, 12] it can be seen from Fig. 2 that we have obtained a good agreement with the normalized values $m_p'(R)$, however, the comparison of the absolute values $m_p(R)$ considerably exceeds the r.m.s. error, especially at higher values of R.

3. By help of the absolute multiplicity functions $m_p(R)$ we have determined the rigidity spectrum of the solar protons at the boundary of the atmosphere for the case of January 28th, 1967. The differential spectrum in the interval of (1—15) GV was approximated by the power and exponential dependence. The results compared with the results of other authors are given in Tab. 2. The computation serves

at the same time as a test for verifying the correctness of the multiplicity functions $m_p(R)$ values. The differential spectra of solar protons in the form a) are graphically represented in Fig. 3.

4. Extrapolating the power spectrum of protons to the region of $R \leq 1$ GV and comparing its values for $R = 0.45$ GV and $R = 1.0$ GV with the values of direct stratospheric measurements from January 28th, 1967 (Tab. 3) we have shown that the values of devitions are not greater than 25% and are the smallest compared with results in others publications.

5. The detectors for registering cosmic rays in the rigidity interval of uninterrupted measurements of neutron monitors are rarely fitted on boards of satellites. The above metioned methods and the obtained results allow to complement the section of solar cosmic ray primary proton spectrum in absolute units on the basis of the world neutron supermonitor network measurements and to compare it with its low rigidity part.

The authors wish to thank Dr. L. I. Miroshnichenko from the Institute of Earth Magnetism and Propagation of Radiowaves in Moscow for valuable advice given during his stay at our Institute and to the heads of the stations of cosmic radiation at Deep River, Durham, Upsala, Leeds and Dallas for providing the neutron monitor data used in the analysis.

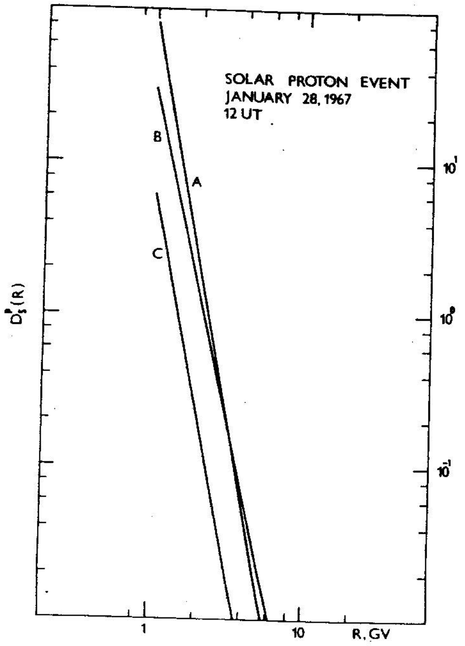

Fig. 3. Differential spectra of solar protons at the boundary of the Earth's atmosphere determined from neutron supermonitor data. A — is from [10], B — from the present paper and C — from [9].

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Received June 6th, 1977