acta phys. slov. 28. (1978), No. 4

CAUSAL CORRELATION BETWEEN HELICON DISPERSION AND ABSORPTION IN PLASMA

JAROSLAV FRANEK*, ŠTEFAN VEIS*, Bratislava

In the present paper, using the generalized Kramers-Kronig relations between the real and the imaginary part of the function of the dielectric response, the absorption of longitudinal waves in plasma is calculated. The basic assumption for the calculation of this absorption is the knowledge of dispersion of longitudinal waves. In our case the dispersion in long-wave approximation was used. Energy losses of longitudinal waves calculated here, which are causally connected with the chosen dispersion, are compared with published data.

ПРИЧИННАЯ СВЯЗЬ МЕЖДУ ГЕЛИКОНОВОЙ ДИСПЕРСИЕЙ И ПОГЛОЩЕНИЕМ В ПЛАЗМЕ

В работе проведён расчёт поглощения продольных волн в плазме, исходя из обобщённых дисперсионных соотношений Кронига-Крамерса, связывающих вещественную и мнимую части функции диэлектрического отклика. Расчёт такого поглощения возможен при условии, что известна дисперся продольных волн. В данном случае для дисперсии используется длинноволновое приближение. Рассчитанные энергетические потери продольных волн, которые причинно связанны с выбранной дисперсией, сопоставляются с существующими экспериментальными данными.

I. INTRODUCTION

The aim of the present paper is to show that even on the basis of simplified concepts it is possible to create a model connecting dispersion with absorption in the plasma in a causal way. We shall consider as a causal one such a transfer (Green) function $G(t)$, the Fourier transformation of which $G(\omega) = G_R(\omega) + iG_I(\omega)$ has its real (dispersion) and imaginary (absorption) part interrelated by the Kramers-Kronig dispersion relations [1]. It means that from the dispersion properties of the plasma its absorption properties can be determined and vice versa. In the present paper we shall determine in this way the absorption causally corresponding to the helicon dispersion in long-wave approximation and calculate from the dispersion relations the attenuation of plasma oscillations in non-magnetic plasma.

* Dept. of experimental physics PFUK, Mlynská dolina, CS-816 31 BRATISLAVA.

II. METHOD AND CALCULATIONS

In the kinetic theory of plasma there are often determined so-called transfer functions described in linear relation between two physical magnitudes, where one of them is in causal relation with the other. As an example of a transfer function we shall introduce the function of the dielectric response in plasma. This function correlates the external charge, affecting the plasma with the induced charge

$$\varrho_{IND}(\omega) = G(\omega)\varrho_{EXT}(\omega)\,. \tag{1}$$

As it can be seen, all the magnitudes in equation (1) are expressed with the aid of the Fourier transformation and so they depend on frequency. In Eq. (1) the dependence of physical magnitudes on spatial coordinates is not considered. This relation can be neglected in the case of the long-wave approximation ($k \to 0$). Eq. (1) describes in plasma the waves known as longitudinal waves of the space charge induced by the external charge $\varrho_{EXT}(\omega)$. A particular case of Eq. (1) is when the induced charge in plasma arises in the absence of the external charge. Such waves of the space charge are characteristic for the plasma and they are called plasma oscilations or plasmons.

The real part of the function $G(\omega)$ describes the dispersion of waves of the space charge in the plasma and the imaginary one their attenuation. The function of dielectric response of $G(\omega)$ is connected with plasma pemittivity by the relation [2]

$$G(\omega) = \frac{1}{\varepsilon(\omega)} - 1\,. \tag{2}$$

Further we shall transcribe this expression for the case of the magnetically active plasma. It is necessary, however, to know the plasma permittivity in the magnetic field. In literature there is the following expression in the permittivity of the electron collisionless plasma for a right-handed circularly polarized wave, progressing in the direction of the magnetic field [3]

$$\varepsilon(k,\omega) = 1 - \frac{\omega_p^2}{\omega(\omega - |\omega_c|)}[1 - W(Z)]\,, \tag{3}$$

$$Z = (\omega - |\omega_c|)/|k|\sqrt{\varkappa T/m}$$

ω_p — plasma frequency, ω_c — cyclotron frequency, k — wave vector, $\varkappa$ — Boltzmann's constant, T — temperature, m — electron mass. In long-wave approximation there is $Z \gg 1$ and the function $W(Z)$ can be expressed by the following series

$$W(Z) = \mathrm{i}\sqrt{\frac{\pi}{2}}\exp\{-z^2/2\}\, Z - \frac{1}{Z^2} - \frac{3}{Z^4}\cdots \tag{4}$$

It can be seen that $W(Z)$ converges to zero with increasing Z and the Eq. (3) obtains the simplified form

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega - |\omega_c|)}\,. \tag{5}$$

This expression of $\varepsilon(\omega)$ is frequently encountered in the literature in description of the propagation of circularly polarized low-frequency waves in magnetically active plasma [3—5]. These waves are called helicons and so we shall further deal with helicon dispersion. Our final task is to determine absorption, i.e. the imaginary part of $G(\omega)$ from the helicon dispersion $G_R(\omega)$. On substituting (5) into (2) we obtain for the real part of the transfer function

$$G_R(\omega) = \frac{\omega_p^2}{\omega^2 - \omega|\omega_c| - \omega_p^2} = \frac{\omega_p^2}{(\omega-\omega_1)(\omega-\omega_2)}\,, \tag{6}$$

where

$$\omega_{1,2} = \frac{1}{2}\left(|\omega_c| \mp \sqrt{\omega_c^2 + 4\omega_p^2}\right), \quad \omega_1 < 0 < \omega_2\,.$$

The expression $G_R(\omega)$ for dispersion will be a basic relation to determine the helicon absorption $G_I(\omega)$. The integral correlations between dispersion and absorption, consequences of the principle of causality, have the following form [1]

$$-\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{G_R(\omega')\mathrm{d}\omega'}{\omega' - \omega} = G_I(\omega) \tag{7A}$$

$$\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{G_I(\omega')\mathrm{d}\omega'}{\omega' - \omega} = G_R(\omega)\,. \tag{8A}$$

The expressions (7A) and (8A) are derived for the function $G(\omega)$ not having poles on the real axis. With the aid of these expressions the authors of [6] calculate the imaginary part of permittivity from the real part and the extinction coefficient from the index of refraction. In our case the function $G(\omega)$ has poles on the real axis. Therefore we must start with more general relations given by the dispersion relations [2].

$$-\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{G_R(\omega')\mathrm{d}\omega'}{\omega' - \omega} = \operatorname{Im}\Sigma\ \text{Res. function}\ \frac{G(\omega')}{\omega'-\omega}\ \text{on the real axis} \tag{7B}$$

$$\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{G_I(\omega')\,\mathrm{d}\omega'}{\omega' - \omega} = \operatorname{Re}\Sigma\ \text{Res. of function}\ \frac{G(\omega')}{\omega'-\omega}\ \text{on the real axis}\,. \tag{8B}$$

The horizontal line across the integral sing in Eq. (7) and (8) means that the main values of integrals are to be considered. On the right-hand side of Eq. (7B) and (8B) are the imaginary and the real part of the sum of the residua of the function$G(\omega')/(\omega'-\omega)$, resp. It follows from the text that Eqs. (7A) and (8A) are special cases of Eqs. (7B) and (8B). The number of the residua summed up on the right-hand side of Eqs. (7B) and 8B) depends on the selection of ω. As it can be seen of Eq. (6), $G_R(\omega)$ has two poles on the frequencies ω_1 and ω_2. The number of poles of the function $G(\omega)/(\omega'-\omega)$ is therefore equal to three in a general case. In the special case $\omega = \omega_1$ or $\omega = \omega_2$ the number of poles is reduced to two. Further, it will be clear that just these particular cases are of extraordinary importance. Presuming that $G_R(\omega)$ is known we can calculate the left-hand side of Eq. (7B) and the right-hand side of (8B). It is to realize that the real part of the sum of the residua of the function $G(\omega')/(\omega'-\omega)$ is equal to the sum of the residua $G_R(\omega')/(\omega'-\omega)$. The right-hand sides of Eqs. (7B) and (8B) can be written as follows

$$\operatorname{Im}\Sigma\ \text{Res. function}\ \frac{G(\omega')}{\omega'-\omega}\ \text{on real axis} = \Sigma\ \text{Res.}\ \frac{G_I(\omega')}{\omega'-\omega} \tag{9}$$

$$\operatorname{Re}\Sigma\ \text{Res. function}\ \frac{G(\omega')}{\omega'-\omega}\ \text{on real axis} = \Sigma\ \text{Res.}\ \frac{G_R(\omega')}{\omega'-\omega}\,. \tag{10}$$

On substituting (6) for $G_R(\omega)$ we find that the expression in Eq. (10) is equal to zero not only in a general case but also the main value of the integral (8B) is always equal to zero. By calculating the integral in Eq. (7B) we obtain various cases

$$-\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{G_R(\omega')\mathrm{d}\omega'}{\omega' - \omega} = \Sigma\ \text{Res}\ \frac{G_I(\omega')}{\omega'-\omega} = 0 \quad \omega \neq \omega_{1,2} \tag{11}$$

$$-\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{G_R(\omega')\mathrm{d}\omega'}{\omega' - \omega_1} = \Sigma\ \text{Res}\ \frac{G_I(\omega')}{\omega'-\omega_1} = -\infty \tag{12}$$

$$-\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{G_R(\omega')\mathrm{d}\omega'}{\omega' - \omega_2} = \Sigma\ \text{Res}\ \frac{G_I(\omega')}{\omega'-\omega_2} = \infty\,. \tag{13}$$

For constructing the function $G_I(\omega')$ we fit the equations (11)—(13) and the Dirac δ-function can be applied. Its properties secure the zero value of $G_I(\omega)$ except at the points ω_1 and ω_2, which ais in accordance with (11). In (12) and (13) we obtain the sum of two residua, where the larger is always the one correspoding to a pole of the second order. The absorption $G_I(\omega')$ can be expressed with the aid of the Dirac δ-function in the form

$$G_I(\omega') \sim \delta(\omega'-\omega_1) - \delta(\omega'-\omega_2)\,. \tag{14}$$

In such a way helicon absorption is defined, except the multiplicative constant. For determining this constant we can use the rule of the f-sums. We apply the rule in the form [3]

$$-\int_{-\infty}^{\infty} \omega' \operatorname{Im}\left[\frac{1}{\varepsilon(\omega')}\right] d\omega' = \pi\omega_p^2 \tag{15}$$

where ω_p is plasma frequency.

Comparing Eqs. (2) and (15) it can be seen that in our case the rule of the f-sums can be written as

$$-\int_{-\infty}^{\infty} \omega' G_I(\omega')\, d\omega' = \pi\omega_p^2 . \tag{16}$$

From this follows directly how to select the constant, not defined yet, in the expression of the helicon absorption and to transcribe Eq. (14) into the final form

$$G_I(\omega') = -\frac{\pi\omega_p^2}{\omega_1 - \omega_2}\left[\delta(\omega' - \omega_1) - \delta(\omega' - \omega_2)\right] . \tag{17}$$

III. DISCUSSION

The helicon dispersion given by Eq. (6) and the calculated absorption, expressed by Eq. (17) are causal, correlated by the Kramers-Kronig dispersion relations and fitting (7B) and (8B).

The described procedure for determining the helicon absorption, based on the correlations (7B) and (8B) was proved on non-magnetic collisionless plasma. For the real component of permittivity of such plasma the known correlation is valid

$$\varepsilon_R(\omega) = 1 - \omega_p^2/\omega^2 \tag{18}$$

and for the function of the dielectric response, given by Eq. (2), we obtain

$$G_R(\omega) = \frac{\omega_p^2}{\omega^2 - \omega_p^2} . \tag{19}$$

The imaginary part, corresponding to this real part of G(ω), calculated from (7B) and (8B) is

$$G_I(\omega) = -\frac{\pi\omega_p}{2}\left[\delta(\omega - \omega_p) - \delta(\omega + \omega_p)\right] . \tag{20}$$

With the aid of absorption, expressed in such a way, we can construct also the frequency dependence of energetic losses in plasma. These losses are given by the expression — ω Im $[1/\varepsilon(\omega)]$ [3], but in our case

$$-\omega \operatorname{Im}\left[\frac{1}{\varepsilon(\omega)}\right] = -\omega G_I(\omega) . \tag{21}$$

The result we obtained is in accordance with that given in [3] from where also Fig. 1 is taken showing the frequency dependence of losses in plasma. The full lines in Fig. 1 correspond to a long-wave approximation and are in accordance with our results (20) and (21), respectively. The dashed line represents the short-wave case when $k \gg k_D$ (k_D — the Debye wave number). In this case the losses are caused by the Landau attenuation. If we knew the dispersion $G_R(\omega, k)$ from the Kramers-Kronig relations, we could determine the Landau attenuation, too. It is to be noted, however, that the Landau attenuation fits the rule of the f-sums (15) and (15) as well as $G_I(\omega)$ given by (17) and (20). It means that the areas in Fig. 1 bounded with full lines and the frequency axis are equal to the area under the dashed line expressing the losses due to the Landau attenuation. The fact that in a long-wave approximation the Landau attenation does not appear is comprehensible as in this case only the particles with infinite velocity are attenuated by the Landau attenuation.

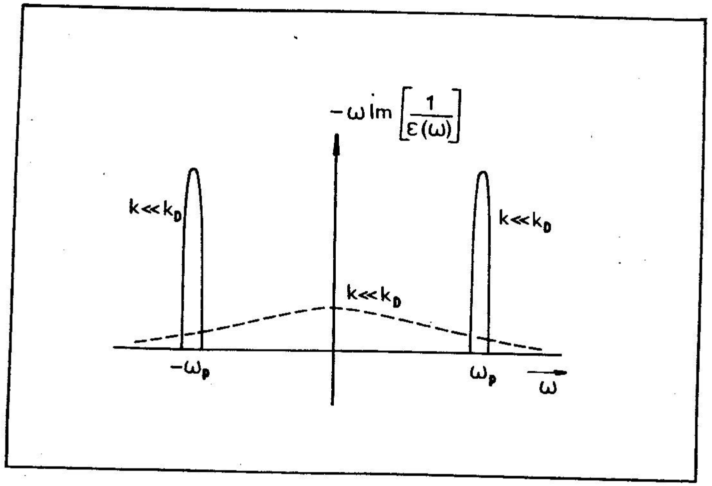

Fig. 1. Spectral energy loses dependence of longitudinal waves in plasma. Taken from [3].

REFERENCES

[1] Nussenzveig, H. M.: *Pričinnosť i dispersionnyje sootnošenija*. Mir, Moskva 1976.
[2] Landau, L. D. Lifšic, E. M.: *Statističeskaja fizika*. Nauka, Moskva 1964.
[3] Išimaru, S.: *Osnovnyje principi fiziki plazmy*. Atomizd., Moskva 1975.
[4] Steele, M., Vural, B.: *Vzajmodejstvijė voln v plazme tvjordovo tela*. Mir, Moskva 1973.
[5] Platzman, P. M. Wolf, P. A.: *Waves and Interactions in Solid State Plasmas*. Moskva 1975.
[6] Altshuler, N. S., Basoon, S. A., Karamyon, A. A.: Phys. Stat. Sol., (b) *70* (1975), K127.

Received October 24th, 1977