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ZEROS OF FORWARD π^-p AMPLITUDE AND LOW ENERGY PARAMETERS

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The location of zero of forward π^-p amplitude is calculated by the method based on the statistical approach of the representation of data by analytic functions. The result differs slightly from the value obtained previously by a different method. The correlation between the values of scattering lengths and the position of zeros is discussed.

НУЛЪ π^-p АМПЛИТУДЫ РАССЕЯНИЯ ВПЕРЁД И НИЗКОЭНЕРГЕТИЧЕСКИЕ ПАРАМЕТРЫ

В работе найдены положения нулей π^-p амплитуды рассеяния вперёд на основе метода, использующего статистический подход к представлению данных с помощью аналитических функций. Результат несколько отличается от значений, полученных раньше с помощью другого метода. Обсуждается связь между значениями длин рассеяния и положениями нулей.

I. INTRODUCTION

The analyticity of the scattering amplitude is one of the most frequently used tools in high energy physics. The analytic structure of the amplitude, its singularities and asymptotic behaviour characterize in a well-known way the scattering processes. The direct consequence of the analyticity — dispersion relations helps us to get some information about the scattering amplitude, especially its real part. When the standard dispersion method is used for the forward amplitude $f(E)$, we have to know the singularities and the behaviour of the amplitude at $E\to\infty$. However, in some cases it is more suitable to apply dispersion methods to the logarithm of the amplitude. Then the knowledge of the position of zeros is of great importance, since the zeros od the amplitude are singularities of the logarithm. So far the zero positions were calculated by Jorna, McClure [1] for the π^-p forward amplitude and by Dumbrajs [2] [3] for the $K^{\pm}p$, $p\bar{p}$, pp amplitudes. They have used the method of phase contours. Theoretically, the relation between the number of zeros and the behaviour of the phase at the threshold and infinity was

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determined by Sugawara and Tubis [4] via the phase representation. If the amplitude behaves as E^{α} at $E \to \infty$, then the number of zeros is given by

$$N = P + \alpha + \frac{1}{\pi}[\delta(\infty) - \delta(1)] - \frac{1}{\pi}[\delta(-\infty) - \delta(-1)], \tag{1}$$

where P is the number of poles and δ is the phase of the amplitude. The same relation can be obtained immediately using the principle of argument as shown by Zamiralov and Kurbatov [5]. For the case of the $\pi^- p$ forward amplitude they get the final result

$$N_- = P_- + \frac{1}{\pi}[\delta_-(-1) - \delta_-(1)]. \tag{2}$$

This formula does not depend on α and is valid even if the amplitude behaves as some power of the logarithm at $E \to \infty$.

Since $P = 1$ (nucleon pole) and the data give $\delta_-(1) = 0$ and $\delta_-(-1) = \pi$, the number of zeros of the forward $\pi^- p$ amplitude is equal to two.

The result of Jorna and McClure shows that the zeros are located near the treshold $E = 1$ and therefore the zero position is greatly influenced by the low energy parameters.

Recently there have been several discussions concerning the scattering lengths of the πN amplitude [6], [7].

In this paper we shall evaluate the location of zeros using different sets of scattering lengths. The result obtained will be compared with that of Jorna and McClure [1].

We shall use a different method which can be shortly characterized in the following way: i) data are treated statistically; ii) as an input we use only interpolated experimental data and we do not need any hypothesis regarding the asymptotic behaviour; iii) an error estimate is given. The method will be described briefly in part II. In part III the results will be presented together with the concluding remarks.

II. METHOD

We shall consider the forward $\pi^- p$ amplitude in the complex E-plane (E is the laboratory energy of the pion in $\hbar = c = m_\pi = 1$ units.) It has two symmetric cuts along the real axis $(1, \infty)$, $(-\infty, -1)$ and a pole at $-1/2M$, where M is a nucleon mass. Because of real analyticity $f_-(E) = f_-^*(E^*)$. In addition, the amplitude satisfies the crossing relation

$$f_-(-E^*) = f_+^*(E), \tag{3}$$

where $f_+(E)$ is the forward $\pi^+ p$ amplitude. In order to determine the location of zeros we shall work with the function

$$F_-(E) = \frac{1}{f_-(E)}. \tag{4}$$

The zeros of $f_-(E)$ are poles of $F_-(E)$ and vice versa, so that we can use method [8] to find the poles of $F_-(E)$.

The analytic structure of $F_-(E)$ is the same as that of the $f_-(E)$, except for the nucleon pole at $-1/2M$. Instead, it has two complex conjugate poles corresponding to zeros of $f_-(E)$.

We map the whole E-plane onto the unit disc using the following mapping

$$z = \frac{\sqrt{1+E} - \sqrt{1-E}}{\sqrt{1+E} + \sqrt{1-E}}. \tag{5}$$

The cuts $(1, \infty)$, $(-\infty, -1)$ are mapped onto the unit circle $\mathscr{C}$.

In the E-plane we have the following experimental information along the cuts:

1) In the region from the threshold up to $E = 1.226$ we use the effective range approximation $q^{2l+1} \operatorname{cotg} \delta_l = \frac{1}{a_l}$. In calculating the forward amplitude we take S- and P-wave contributions. The values of the scattering lengths will be taken from different sources and the results will be compared. This region is mapped onto the arc $(0°, 34.8°)$ in the z-plane.

2) The interval from $E = 1.1226$ up to $E = 19.986$. Here the values of $F_-(E)$ are calculated from the Saclay [9] phase shift analysis. In the z-plane it corresponds to arc $(34.8°, 87°)$.

3) The rest of the right-hand cut $E = (19.986, \infty)$ is mapped onto the arc $\varphi = (87.1°, 90°)$. The imaginary part of the amplitude can be calculated from the data on the total cross section [10], via the optical theorem. The real part is determined from the measurements of $\varrho = \mathrm{Re}f/\mathrm{Im}f$ [11] with the help of the Coulomb interference. The number of data points is 18, so that the density of data along this part of the circle is even higher than in the phase shift region.

The left part of the semicircle $\varphi = (\pi/2, \pi)$ is covered by the data on f_+p in a similar way.

Because of real analyticity the lower part of the circle is a complex conjugate of the upper part.

Following [8] we construct Q_K coefficients

$$Q_K = \frac{1}{2\pi} \oint_{\mathscr{C}} \frac{Y(z)}{g(z)} z^n (\mathrm{d}z), \tag{6}$$

where $Y(z)$ is a smooth interpolation of data points and $g(z)$ is a weight function constructed from experimental errors. (For details see [12]). As shown in [13] Q_K are Gaussian random variables with the mean

$$Q_K = \frac{1}{2\pi} \oint_{\mathscr{C}} \frac{F(z)}{g(z)} z^K \, (dz). \qquad (7)$$

$F(z)$ is a function analytic inside the unit disc except for two symmetric poles at $z=\lambda$ and $z=\lambda^*$.
Thus we can write

$$F(z)=\sum_0^\infty b_n z^n + \frac{\alpha}{z-\lambda} + \frac{\alpha^*}{z-\lambda^*}.$$

The quantity

$$\chi^2=\sum_1^N |Q_K - a_K|^2 = \sum_1^N |Q_K - 2\mathrm{Re}\, \bar{\alpha}\lambda^{K-1}|^2 \qquad (8)$$

has the chi-squared distribution with N degrees of freedom. The values $\bar{\alpha}$, λ are calculated by minimizing (8).

III. RESULTS

The number N in (8) was taken equal to 30. One can take even a lower N since starting from a certain value $(N \sim 5)$ χ^2 is independent of N.

The results are listed in Table 1.

The second and the third column gives the values of the real and imaginary part of the zero position. The value obtained in [1] is $E = 0.946 \pm i\, 0.669$.

Table 1

Data on scatt. lengths	E_R	$\pm E_I$	χ^2/N
Pilkuhn [14]	0.963±0.018	0.665±0.02	1.189
Bugg et al. [15]	0.967±0.015	0.663±0.02	1.193
Samaranayke [16]	1.00 ±0.002	0.665±0.003	1.061
Höhler [6]	0.953±0.127	0.664±0.05	1.228
Lichard [7]	1.101±0.03	0.633±0.02	1.128

IV. CONCLUSIONS

i) As seen from the Table 1 the real part is higher than the one obtained previously [1]. The value which deviates most (Samaranayke) gives the lowest χ^2/N.

However, the results obtained from Bugg's or Höhlers scattering lengths could be more reliable since their calculation is based on more recent and accurate data

[17]. The point is that the Saclay data which we have used in the region $E=(1.12, 19.9)$ may not be consistent with analyticity when combined with the low energy values based on different analysis. One can conclude that the present data give the real part of the zero position between $E_R = 0.95$ and $E_R = 1.1$. The imaginary part is $E_I \sim 0.66$.

ii) One of the aims of this paper was to test whether the method which was originally used to determine the singularities of the Δ_{33} resonance [8] and the $\pi\pi$ resonances [18] is able to detect the zeros of the forward amplitude. The above example of $\pi^- p$ zeros shows that the method gives even more reliable results, since we have no problems with left-hand cuts and high energy behaviour as in the case of the πN partial amplitude.

One can expect that the method can successfully work also in determining the zero position of other amplitudes, including the case of fixed t different from zero.

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REFERENCES

[1] Jorna, S., McClure, J. A.: Nucl. Phys. B *13* (1969), 68.
[2] Dumbrajs, O. V.: Nucl. Phys. B *38* (1972), 600.
[3] Dumbrajs, O. V.: Nucl. Phys. B *46* (1972), 164.
[4] Sungawara, M., Tubis, A.: Phys. Rev. *130* (1963), 2127.
[5] Zamiralov, V. S., Kurbatov, A. F.: J. nucl. Phys. *23* (1976), 900.
[6] Höhler, G., Jakob, H. P., Strauss, R.: Nucl. Phys. B *39* (1972), 237.
[7] Lichard, P.: Acta phys. slov. *26* (1976), 152.
[8] Nogová, A., Pišút, J. Prešnajder, P.: Nucl. Phys. B *61* (1973), 438.
[9] Ayed, R., Bareyre, P., Lemoigue, Y.: preprint CEN-Saclay, 1972.
[10] Carroll, A. S.: Palermo Conf. 1975;
Foley, K. J.: Phys. Rev. *19* (1967), 330.
[11] Foley, K. J.: Phys Rev. *181* (1969), 1775;
Apkenbrandt, C.: FERMILAB Conf. 75/61 — EXP.
[12] Nogová, A., Pišút, J.: Nucl Phys. B *61* (1973), 445.
[13] Pázman, A. et al.: Nucl. Phys. B *64* (1972), 637.
[14] Pilkuhn, N. et al.: Nucl. Phys. B *65* (1973), 460.
[15] Bugg, D. V. et al.: Phys Lett. *44B* (1975), 278.
[16] Samaranayke, V. K., Woolcock, W. S.: Nucl. Phys. B *48* (1972), 205.
[17] Bussey, P. J. et al.: Nucl. Phys. B *58* (1973), 363.
[18] Boháčik, J., Kühnelt, H.: Viena preprint, March 1977.

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