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QUANTUM NUMBER DISTRIBUTIONS AND FLUCTUATIONS IN MULTIPARTICLE PRODUCTION

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On the basis of available experimental data we show that global features of the electric and baryonic charge distributions in hadronic collisions at high energies are successfully reproduced in the quark-parton model of multiparticle production. Further we present some qualitative arguments, which indicate that this model might also describe quantum number fluctuations in multiparticle production.

Finally we investigate here predictions of the model in which final state particles in multiparticle production originate from decays of neutral clusters of hadronic matter. It is shown that the simplest version of the cluster model predicts the energy dependence of charge transfer probabilities, which contradicts the data.

РАСПРЕДЕЛЕНИЯ КВАНТОВЫХ ЧИСЕЛ И ФЛУКТУАЦИИ В МНОГОЧАСТИЧНОЙ ПРОДУКЦИИ

На основе доступных экспериментальных данных показано, что общий характер распределения электрического и барионного зарядов в адронных столкновениях при высоких энергиях успешно воспроизводит кварк-партонная модель многочастичной продукции. Приводятся некоторые качественные соображения в пользу того, что эта модель может описывать также флуктуации квантовых чисел при многочастичной продукции. В работе рассматриваются предсказания модели, в которой конечные состояния частиц в многочастичной продукции достигаются распадами нейтральных кластеров адронной материи. Показано, что наиболее простая кластерная модель предсказывает энергетическую зависимость вероятности передачи заряда, которая противоречит наблюдаемым данным.

I. INTRODUCTION

A great deal of our information about the hadron structure comes from the detailed analysis of the deep inelastic lepton-nucleon scattering [1]. The data are adequately described by the parton model [2] in which the hadron is supposed to consist of pointlike constituents called partons (these being often identified with hypothetical quarks of Gell-Mann and Zweig [3]). In this scheme the lepton-nucleon scattering is caused by the interaction of the lepton with a quasi-free parton in the nucleon. Thus the deep inelastic scattering provides us with information on the distribution of pointlike constituents inside free hadrons.

On the other hand, though multiparticle production is a much more complicated process, it should also hide some information on the structure of hadrons and particularly on the nature of their interactions. In this way multiparticle production and deep inelastic scattering are complementary in reflecting different aspects of hadronic interactions. It is then plausible to expect that both processes can be explained using a common basis provided by the internal structure of hadrons. The quark-parton model represents a possibility of such an approach: deep inelastic scattering is caused here by lepton-parton interactions, while multiparticle production is a result of mutual interactions of partons from colliding hadrons.

The multiproduction data indicate that (using the parton model language) interactions of partons are of a short range in rapidity. In this way the distribution of valence partons in a compound system formed during the hadron-hadron collision should be closely connected with their distribution before the collision (which can be extracted from leptoproduction data).

The purpose of this paper is to study the electric and baryonic charge distributions and charge fluctuations in multiparticle production and discuss them from the quark-parton model point of view and in the cluster model approach.

The paper is organized as follows:

In Sect. II. we briefly summarize results of two previous papers [4, 5]. We show that the electric and baryonic charge distributions in multiparticle production agree with simple qualitative predictions of the quark-parton model.

Charge fluctuations are investigated in the quark-parton model framework in Sect. III. We show that the model can probably reproduce general trends observed in the data.

In Sect. IV. we investigate predictions of a simple cluster model. This is shown to predict the energy dependence of charge transfer probabilities, which does not agree with the data. It is, however, not excluded that more refined versions of the model will be successful in this respect.

Conclusions are given in Sect. V.

II. ELECTRIC AND BARYONIC CHARGE DISTRIBUTIONS IN THE QUARK-PARTON MODEL OF MULTIPARTICLE PRODUCTION

In most quark-parton models (QPM) of multiparticle production [6, 7] final state hadrons are produced by the recombination of $Q\overline{Q}$ pairs, QQQ and $\overline{QQQ}$ triplets (neighbouring in rapidity) to mesons, baryons and antibaryons, respectively. The hadronic collision is then supposed to consist of two steps:

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A) the rapidity space is populated by quarks and antiquarks;

B) constituents nearby in rapidity recombine to final state hadrons.

To be more specific, let us use a simple quark-parton model in which quantum numbers of quarks are locally conserved in the sense that quantum number distributions of final state hadrons in (non-diffractive) multiparticle production are identical with distributions of quantum numbers of valence partons before the collision. A simplified version of this model was described in [4]. In this oversimplified picture we supposed that valence quarks are immersed into the (potentially) infinite sea of $Q\bar{Q}$ pairs which carry almost no momentum. In this case one immediately arrives at two conclusions:

A) the electric charge distribution in multiparticle final states is directly connected with the probability distribution functions of valence up and down quarks determined from lepto-production data, and we have [4]

$$\varrho_Q(x)=\frac{2}{3}u_v(x)-\frac{1}{3}d_v(x) \tag{1}$$

(x is the Feynman variable);

B) the electric and baryonic charge distribution have to be almost identical, as [5]

$$\varrho_B(x)=\frac{1}{3}u_v(x)+\frac{1}{3}d_v(x) \tag{2}$$

and

$$u_v(x)\approx 2d_v(x) \tag{3}$$

for the majority of values of x in most of currently used phenomenological distribution functions.

Of course, both these conclusions cannot be taken literally as the recombination of $Q\bar{Q}$ pairs and QQQ and $\overline{QQQ}$ triplets to hadrons and decays of hadronic resonances modify quantum number distributions. However, both processes are of a short range in rapidity and cannot change global features of the distributions.

The predictions A), B) were investigated in [4, 5]. In paper [4] we compared the distribution of the electric charge of valence partons in the proton (calculated using parton distribution functions of Kuti and Weisskopf and of McElhaney and Tuan [8]) with the charge distribution in multi-particle final states of proton-proton collisions at 53 GeV c. m. energy [9]. The result is satisfactory (Fig. 1) if we realize that the model of the local quark quantum number retention in Ref. [4] is severely simplified.

In paper [5] we extracted from the available experimental data [10] the electric and baryonic charge distributions in pp collisions at ISR energies at $p_T=0.4$ GeV/c

(Fig. 2). Both distributions are similar. As the value of $p_T=0.4$ GeV/c is of no particular physical significance, one can expect that also overall (i. e. integrated over p_T) distributions are similar.

In this section we have shown that even the simplest quark-parton model successfully describes basic features of quantum number distributions in multiparticle production. However, in order to reveal the underlying mechanism of the production of hadrons one has to study more detailed characteristics of the process, particularly quantum number fluctuations.

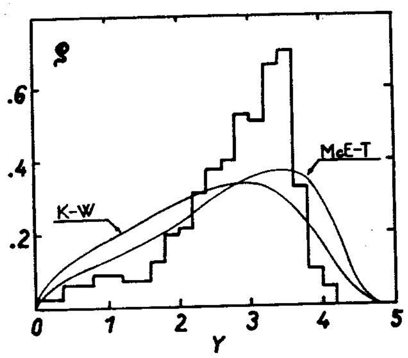

Fig. 1. The histogram [9] of the charge distribution in pp collisions at the CERN ISR ($E=53$ GeV, $p_T=0.4$ GeV/c) compared with the charge distribution of valence quarks obtained from Eq. (1) using parton distribution functions of Kuti and Weisskopf and of McElhaney and Tuan [8].

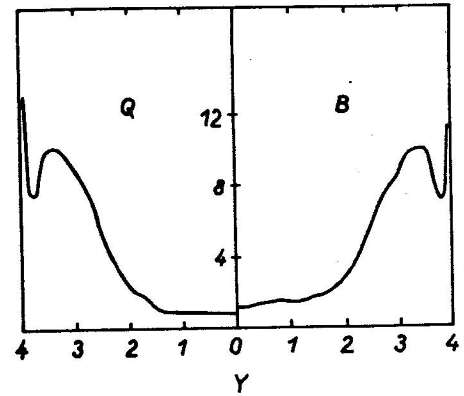

Fig. 2. The electric (Q) and baryonic (B) charge rapidity distributions in pp collisions at ISR energies at $p_T=0.4$ GeV/c (in mb/(GeV^2/c^3)). (Remark: The charge distribution in Figs. 1, 2 are a little different. This is obviously caused by the different procedure of extracting the distributions from data. E.g. we did not remove the diffraction peak at $y=y_{max}$.)

III. FLUCTUATIONS OF QUARK CHARGES

In this section we shall investigate charge fluctuations in multiparticle production from a simple quark-parton model point of view. Particularly, we shall follow up QPM predictions about the charge transfer between centre-of-mass hemispheres, which is defined by the following expression

$$u\equiv\frac{1}{2}(Q_R-Q_L)-\frac{1}{2}(Q_B-Q_T), \tag{4}$$

where $Q_R(Q_L)$ is the total charge of forward (backward) moving particles in the centre-of-mass system, $Q_B(Q_T)$ is the charge of the beam (target) particle.

Fluctuations of this variable could be expected to display the mechanism of multiparticle production [11]. In all models in which final state hadrons come from decays of intermediate objects (clusters, bags, etc.) one can expect smaller fluctuations. On the other hand, in quark-parton models, in which "sea" quarks act in an uncorrelated way, charge transfers should be large.

However, it is very difficult to calculate the charge transfer in the quark-parton model since — except for the short range character of the interaction — almost nothing is known about the dynamics of parton interactions. Nevertheless, we shall at least try to determinate charge transfer characteristics in the first stage of the hadronic collision (see Sect. II). This effort is motivated by the belief that charge fluctuations in this phase are decisive also for transfers in the final state, as all processes in the last stage are shortrange phenomena in rapidity.

In our calculation we shall suppose that valence quarks do not contribute to quantum number fluctuations since they tend to carry large momentum fractions and follow the direction of the parent hadron. Further we shall assume that sea quarks of both hadrons from a common "sea" in which n quarks and n antiquarks (n having the Poisson distribution) are uniformly distributed in rapidity. We shall denote by P_1 (P_2) — the probability that the quark or antiquark carries the charge $\pm 1/3$ ($\pm 2/3$): $P_1 + P_2 = 1$; $\langle N_Q \rangle = \langle N_{\bar{Q}} \rangle$ — the mean number of $Q\bar{Q}$ pairs in the compound "sea" of colliding hadrons.

Quark charge transfer characteristics will be calculated here using the generating function technique (see Appendix). A $Q\bar{Q}$ pair gives

with the probability	$P_1/4$	the charge transfer	$u = 1/3$,
—,,—	$P_1/4$	—,,—	$u = -1/3$,
—,,—	$P_2/4$	—,,—	$u = 2/3$,
—,,—	$P_2/4$	—,,—	$u = -2/3$,
—,,—	$1/2$	—,,—	$u = 0$.

The generating function of charge transfer for a $Q\bar{Q}$ pair is then given by the expression (see (A. 1))

$$P(x) = \frac{1}{4}\left[P_1(x^{1/3} + x^{-1/3}) + P_2(x^{2/3} + x^{-2/3}) + 2\right] \tag{5}$$

and the grand-generating function is

$$G(x) = \exp\left\{\frac{\langle N_Q \rangle}{4}\left[P_1(x^{1/3} + x^{-1/3}) + P_2(x^{2/3} + x^{-2/3}) - 2\right]\right\}. \tag{6}$$

Knowing $G(x)$ we obtain using Eqs. (A. 15—16)

$$\langle u \rangle = 0 \tag{7a}$$

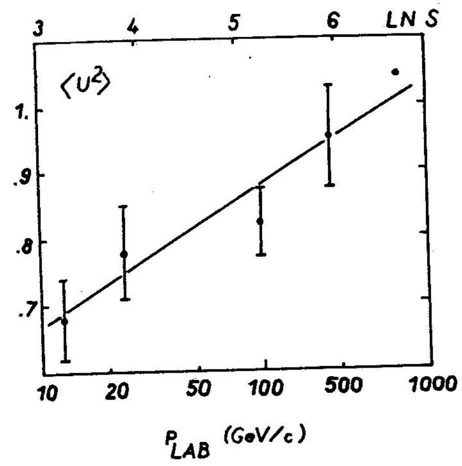

Fig. 3. The charge transfer dispersion in pp collisions as a function of the laboratory momentum P_{LAB}. The experimental points were determined from charge transfer cross sections (see [13]). The solid curve represents the fit $\langle u^2 \rangle = A \ln s + B$ with $A = 0.096$, $B = 0.377$.

$$\langle u^2 \rangle = \frac{1}{2}\lambda^2 \langle N_Q \rangle, \tag{7b}$$

where

$$\lambda^2 = \frac{1}{9}P_1 + \frac{4}{9}P_2$$

is the mean of the squared quark charge.

Since quarks have fractional values of charge, also fractional transfers are possible at this stage. The probability of $u = k/3$ (k integer) is given by the expression (see (A. 17))

$$p\left(u = \frac{k}{3}\right) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos k\varphi \exp\left\{\frac{\langle N_Q \rangle}{2}(P_1 \cos\varphi + P_2 \cos 2\varphi - 1)\right\} d\varphi. \tag{8}$$

The data [12] show that

$$\langle u^2 \rangle = A \ln s + B \tag{9}$$

($A \doteq 0.096$, $B \doteq 0.377$ from the data in [13], see Fig. 3). The fluctuations are caused not only by charge transfers in initial quark configurations, but also in the process of the recombination of partons and decays of resonances. Due to the short-range character of these processes their contribution should be roughly constant (or at least approach a constant value at high energies). Thus the logarithmic rise of $\langle u^2 \rangle$ can only be attributed to the same rise of $\langle N_Q \rangle$. If we for simplicity neglect the contribution of recombinations and decays and assume that up, down and strange quarks appear in the "sea" with equal probabilities (i.e. $P_1 = 2/3$, $P_2 = 1/3$), we obtain

$$\langle N_Q \rangle \doteq 0.864 \ln s + 3.393. \qquad (10)$$

However, as we shall show in the next section also other models are capable to reproduce the observed energy dependence of $\langle u^2 \rangle$. Therefore it is necessary to

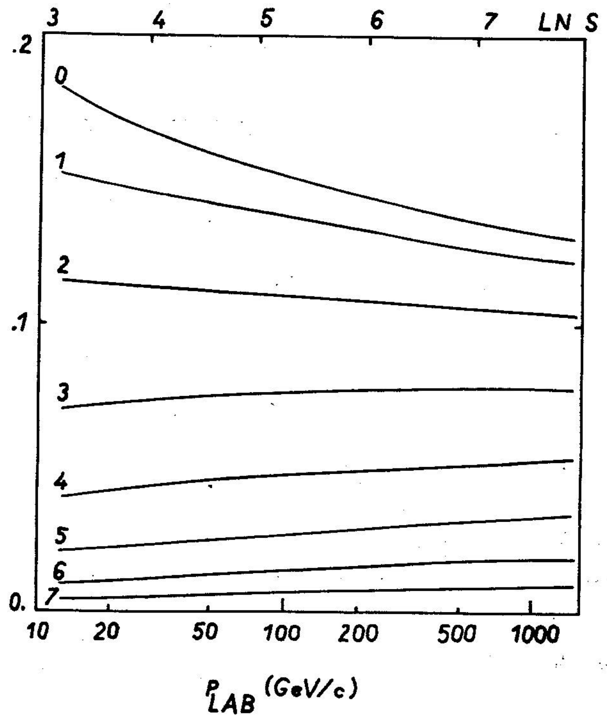

Fig. 4. The energy dependence of quark charge transfer probabilities $p(u = k/3)$ $(k = 0, 1, \ldots, 7)$ calculated using Eq. (8).

look at even more detailed characteristics, particularly at charge transfer probabilities. Hence we used the mentioned values of $\langle N_Q \rangle$, P_1, P_2 and calculated the energy dependence of quark charge transfer probabilities (Fig. 4). The result cannot however be compared with experimental data [13] (see Fig. 5), as the relation between quark charge fluctuations before the recombination to hadrons and the charge transfer in multiparticle final states is not evident. Nevertheless the trend of curves in Fig. 4 is promising. We believe that it is possible to find such rules of the recombination of partons to hadrons which lead to the observed energy

dependence of charge transfer probabilities (in particular to the decrease of $p(u = 0)$ and $p(|u| = 1)$ and the rise of other probabilities, see Fig. 5). It would thus be very useful to study the charge transfer in a QPM of multiparticle production in which plausible rules of the parton recombination are included (e.g. [7]).

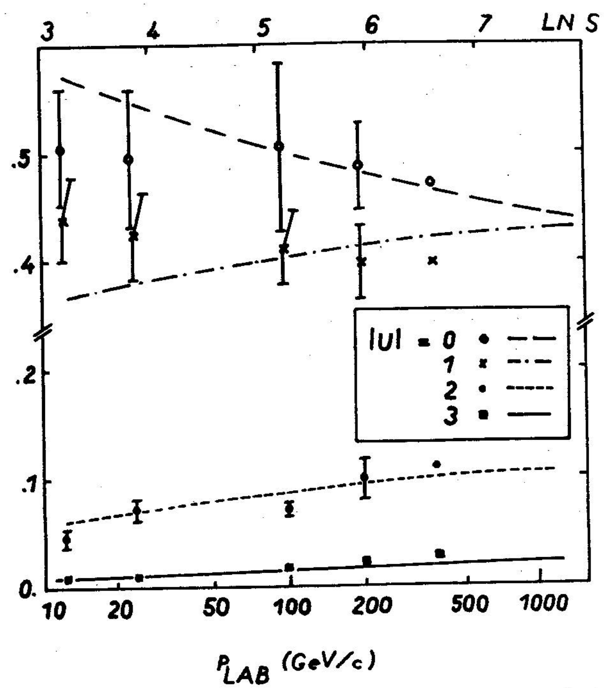

Fig. 5. The energy dependence of charge transfer probabilities in pp collisions [13]. The curves show predictions of the simple "ϱ" cluster model.

IV. CHARGE TRANSFER IN A SIMPLE CLUSTER MODEL

An alternative approach to the quark-parton description of multiparticle production is represented by the independent cluster emission model [14]. In this model it is supposed that during the hadronic collision independent clusters of hadronic matter are produced, which decay without any final state interaction to final state hadrons. A simple one-dimensional model in connection with this was proposed by Quigg and Thomas [14]. The model is based on the following assumptions:

1. neutral clusters are produced independently and uniformly in the effective rapidity interval $\langle -Y/2, Y/2 \rangle$;[1])

[1]) We do not include leading clusters into the discussion assuming that their contribution to the charge transfer can be neglected since they predominantly follow directions of incident hadrons.

2. cluster decay without mutual interactions to $\pi^+ \pi^-$ pairs ("ϱ" model).

In the original formulation of Quigg and Thomas [14] a simplified scheme of cluster decays was assumed (a cluster with the rapidity y decaying to pions with rapidities $y \pm \Delta$, $\Delta = \text{const}$) and the model was used to determine the charge transfer dispersion, charge transfer fluctuations at fixed topologies and at fixed numbers of negative particle tracks in both hemispheres, etc. In this section we will use a modified version of the model based on a more realistic cluster decay rule, and will use it also to predict the energy dependence of charge transfer probabilities.

The cluster is produced in the interval $(y, y+\mathrm{d}y)$ with the probability

$$\varrho(y)\,\mathrm{d}y = \mathrm{d}y \begin{cases} 1/Y & \text{for } |y| \leq Y/2 \\ 0 & \text{for } |y| > Y/2. \end{cases} \tag{11}$$

If the decay of a cluster with the rapidity y is isotropic in its rest frame, its decay products have rapidities $y+\delta$, $y-\delta$, $\delta \in (D, D+\mathrm{d}D)$, with the probability[2)]

$$g(D)\,\mathrm{d}D = \frac{1}{\cos h^2 D}\,\mathrm{d}D\,. \tag{12}$$

It can easily be verified that a single cluster contributes to $u=0$, $u=1$, $u=-1$ with the probabilities $2I$, $1/2 - I$, $1/2 - I$ respectively, where

$$I = \int_0^{Y/2} \mathrm{d}y\varrho(y) \int_0^{y} \mathrm{d}Dg(D)\,. \tag{13}$$

The generating function of charge transfer by a single cluster is then given by the expression

$$P(x) = (1-p) + \frac{p}{2}\left(x + \frac{1}{x}\right), \quad p = 1 - 2I\,. \tag{14}$$

The grand-generating function of charge transfer is simply (see (A. 14))

$$G(x) = \exp\left\{\langle N\rangle\, p\left[\frac{1}{2}\left(x+\frac{1}{x}\right) - 1\right]\right\} \tag{15}$$

where $\langle N\rangle$ is the mean number of the produced clusters.

Using (A. 15—16) we obtain

$$\langle u\rangle = 0 \tag{16a}$$

$$\langle u^2\rangle = \langle N\rangle\, p\,. \tag{16b}$$

2) The formula is just approximate and can be used in the case when the cluster is much heavier than its decay products.

The probability that $u = k$ (k integer) is

$$p(u=k) = I_K(\langle N\rangle\, p)\exp(-\langle N\rangle\, p) = \\ = I_K(\langle u^2\rangle)\exp(-\langle u^2\rangle), \tag{17}$$

where $I_k(z)$ is the k-th modified Bessel function of the 1st order[3)].

Finally, let us here show the correspondence between the above developed version of the "ϱ" model and that of Quigg and Thomas [14]. From Eqs. (11—13) there follows

$$p = 1 - \frac{2}{Y}\ln\left(\cos h\,\frac{Y}{2}\right), \tag{18}$$

which has the asymptotic form for high energies, i.e. large Y

$$p \doteq \frac{2\ln 2}{Y}\,. \tag{19}$$

In the Quigg-Thomas formulation

$$p = \frac{2\Delta}{Y}, \tag{20}$$

and so both formulations are equivalent for high energies if Δ is taken equal to ln 2.

It is evident from Eqs. (16, 19) that the logarithmic rise of $\langle u^2\rangle$ with energy (see the previous section) can be explained by supposing that the density of clusters in the central region of rapidities (proportional to $\langle N\rangle\, p$) grows also logarithmically. In that case, however, we do not obtain the correct energy dependence of charge transfer probabilities. As seen in Fig. 5, the model predicts a too fast decrease of $p(u=0)$ and the rise of $p(|u|=1)$ with energy, while the data indicate a mild decrease even for $p(|u|=1)$.

Consequently, the simplest cluster model of Quigg and Thomas does not simultaneously reproduce the energy dependence of the charge transfer dispersion and of charge transfer probabilities. Still, the contradiction cannot be considered as an argument against cluster models. It might be caused by the simplicity of the model, which

3) The result (17) was obtained from (15) using the formula

$$\exp\left\{\frac{z}{2}\left(x+\frac{1}{x}\right)\right\} = \sum_{k=-\infty}^{\infty} x^k I_k(z)$$

(see [15], Eqs. 8.406.1, 8.511.1).

By the way, after completing this work we obtained a preprint by Argyres and Lam [16] who derived the same result in a simple "ω" model of Quigg and Thomas.

— does not take conservation laws and transverse momenta into account,
— assumes that charge transfer comes exclusively from decays of neutral clusters, while the data indicate the production of charged clusters even in the central region of rapidities [17].

It is therefore quite possible that cluster models contrived in a more sophisticated way can lead to the correct energy dependence of charge transfer probabilities.

V. CONCLUSIONS

In this section we shall summarize results of our investigation of quantum number distributions and fluctuations in the multiparticle production in this paper and in papers [4, 5]:

1. We have shown that the quark-parton model, which arose from the data on the deep inelastic lepton-nucleon scattering, successfully describes also general features of the charge distribution in multiparticle production at high energies.
2. Analysing the data on the inclusive particle production we have shown the similarity of the electric and baryonic charge distribution in multiparticle final states. This fact is in agreement with simple qualitative predictions of the QPM.
3. On the basis of the data on charge fluctuations in multiparticle final states of hadronic collisions and of the results in Sect. III. we judge that the QPM is capable to reproduce also the observed features of the charge transfer.

All these facts support the idea of using the quark-parton model as a common basis for the description of both the deep inelastic lepton-nucleon scattering and the multiparticle production. We believe that both processes should (and have to) be described from a common view of the structure of hadrons and their interactions.

4. Further we have shown that the simple cluster model of multiparticle production predicts the energy dependence of charge transfer probabilities contradicting the data. However, the final decision between quark-parton and cluster models can be reached only after a more complete analysis of all available data.

ACKNOWLEDGEMENTS

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APPENDIX

PROBABILITY GENERATING FUNCTIONS OF DISCRETE RANDOM VARIABLES

When calculating quantum number fluctuations in a simple cluster model of multiparticle production Quigg and Thomas [14] successfully used the so called probability generating functions (PGF) of random variables. Since this technique is useful and has frequently been utilized in the previous text, we shall give here a brief list of definitions and formulae for PGFs of single random variables, systems of random variables and of their simple functions.

1. PGF of a discrete random variable

Definition: Let t be a discrete random variable which takes the values $t_1, t_2, t_3, \ldots, t_n$ with the probabilities $p(t_1), p(t_2), p(t_3), \ldots, p(t_n)$ respectively $\left(\sum_i p(t_i) = 1\right)$. Its PGF is defined by the expression

$$P(x) = \sum_{i=1}^{n} p(t_i)\, x^{t_i}, \qquad \text{(A. 1)}$$

where x is a complex argument.

Some useful formulae:

1)
$$P(1) = 1. \qquad \text{(A. 2)}$$

2) The mean $\langle t \rangle$ and the variance $D^2(t)$ of the discrete random variable t are given by the following expressions

$$\langle t \rangle = x \frac{d}{dx} \ln P(x)|_{x=1} \qquad \text{(A. 3)}$$

$$D^2(t) = \left(x \frac{d}{dx}\right)^2 \ln P(x)|_{x=1}. \qquad \text{(A. 4)}$$

3) Knowing $P(x)$ one can extract the probability $p(t_k)$ using the formula

$$p(t_k) = \frac{1}{2\pi i} \oint_C \frac{P(x)}{t_k + 1}\, dx, \qquad \text{(A. 5)}$$

where C is a closed contour around $x = 0$.

2. PGF of a system of discrete random variables

Definition: Let a, b be two discrete random variables[4] which take the values $a_1, a_2, \ldots, a_m$; $b_1, b_2, \ldots, b_n$, and let $p(a_i, b_j)$ be the probability that simultaneously $a = a_i$ and $b = b_j$ $(1 \leqslant i \leqslant m,\ 1 \leqslant j \leqslant n)$. The function

$$P(x, y) = \sum_{i=1}^{m} \sum_{j=1}^{n} p(a_i, b_j)\, x^{a_i} y^{b_j} \qquad \text{(A. 6)}$$

is called the probability generating function of the random variables a, b.

Calculating of mean values and variances:

1) The mean values of the variables a, b can simply be calculated using the expression

$$\langle a \rangle = \left(x \frac{\partial}{\partial x}\right) \ln P(x, y)\Big|_{\substack{x=1\\y=1}} \qquad \langle b \rangle = \left(y \frac{\partial}{\partial y}\right) \ln P(x, y)\Big|_{\substack{x=1\\y=1}}. \qquad \text{(A. 7)}$$

2) The variances of a-and b are given by the formulae

$$D^2(a) = \left(x \frac{\partial}{\partial x}\right)^2 \ln P(x, y)\Big|_{\substack{x=1\\y=1}} \qquad D^2(b) = \left(y \frac{\partial}{\partial y}\right)^2 \ln P(x, y)\Big|_{\substack{x=1\\y=1}}. \qquad \text{(A. 8)}$$

3) The correlation moment of the random variables a, b defined as

$$C(a, b) = \langle (a - \langle a \rangle)(b - \langle b \rangle) \rangle = \langle ab \rangle - \langle a \rangle \langle b \rangle$$

equals

$$C(a, b) = \left(y \frac{\partial}{\partial y}\right)\left(x \frac{\partial}{\partial x}\right) \ln P(x, y)\Big|_{\substack{x=1\\y=1}}. \qquad \text{(A. 9)}$$

3. PGF of a function of discrete random variables

Definition: Let the discrete random variable t be a function of N discrete random variables $t^{(1)}, t^{(2)}, \ldots, t^{(N)}$

$$t = \varphi[t^{(1)}, t^{(2)}, \ldots, t^{(N)}]$$

and let $t^{(i)}$ have the values $t_1^{(i)}, t_2^{(i)}, \ldots, t_{n_i}^{(i)}$ $(1 \leqslant i \leqslant N)$.

[4] Just for simplicity we shall speak about a system of only two random variables, but the given expressions can be generalized in a simple and straightforward way for systems of more variables.

PGF of the variable t is given by the expression

$$P(x) = \sum_{i_1=1}^{n_1} \sum_{i_2=1}^{n_2} \cdots \sum_{i_N=1}^{n_N} p(t_{i_1}^{(1)}, t_{i_2}^{(2)}, \ldots, t_{i_N}^{(N)})\, x^{\varphi[t_{i_1}^{(1)}, t_{i_2}^{(2)}, \ldots, t_{i_N}^{(N)}]} \qquad \text{(A. 10)}$$

where $p(t_{i_1}^{(1)}, t_{i_2}^{(2)}, \ldots, t_{i_N}^{(N)})$ is the probability that simultaneously

$$t^{(j)} = t_{i_j}^{(j)} (1 \leqslant i_j \leqslant n_j,\ 1 \leqslant j \leqslant N).$$

It is obvious that characteristics of t can again be calculated using $P(x)$ and Eqs. (A. 3—5).

Now we shall pay attention to two particular cases of functions of discrete variables which are useful in evaluating quantum number fluctuations in cluster and quark-parton models.

3.1 PGF of a sum of N independent random variables

Let t be a sum of independent discrete variables $t^{(1)}, t^{(2)}, \ldots, t^{(N)}$

$$t = \sum_{i=1}^{N} t^{(i)}$$

and let each of the variables $t^{(i)}$ take the values $t_{k_i}^{(i)}$ with the probabilities $p(t_{k_i}^{(i)})$ $(1 \leqslant k_i \leqslant n_i,\ 1 \leqslant i \leqslant N)$. Then the probability that simultaneously $t^{(i)} = t_{k_i}^{(i)}$ is

$$p(t_{k_1}^{(1)}, t_{k_2}^{(2)}, \ldots, t_{k_N}^{(N)}) = \prod_{i=1}^{N} p(t_{k_i}^{(i)})$$

and the PGF of t is simply

$$P_N(x) = \prod_{i=1}^{N} P_{(i)}(x),$$

where $P_{(i)}(x)$ is the PGF of the discrete random variable $t^{(i)}$ $(1 \leqslant i \leqslant N$, see Eq. (A. 1)).

If all the $t^{(i)}$ $(1 \leqslant i \leqslant N)$ are distributed in the same way we have

$$P_{(1)}(x) = P_{(2)}(x) = \ldots = P_{(N)}(x) \equiv P(x)$$

$$P_N(x) = [P(x)]^N. \qquad \text{(A. 12)}$$

3.2 PGF of the sum of a random number of independent random variables

Let t again be a sum of N independent random variables $t^{(i)}$

$$t=\sum_{i=1}^{N} t^{(i)}$$

but let the number of terms be a random variable equal to N with the probability $p(N)$. In this case the PGF of t is given by the formula

$$G(x)=\sum_N p(N)\, P_N(x). \qquad \text{(A. 13)}$$

If all $t^{(i)}$ have the same distribution with the PGF $P(x)$,

$$G(x)=\sum_N p(N)[P(x)]^N. \qquad \text{(A.14)}^{5)}$$

Knowing $G(x)$ we can, e.g., calculate

$$\langle t\rangle=\left(x\frac{\mathrm{d}}{\mathrm{d}x}\right)\ln G(x)|_{x=1} \qquad \text{(A.15)}$$

$$D^2(t)=\left(x\frac{\mathrm{d}}{\mathrm{d}x}\right)^2\ln G(x)|_{x=1} \qquad \text{(A.16)}$$

$$p(a)=\frac{1}{2\pi\mathrm{i}}\oint_C \frac{G(x)}{x^{a+1}}\,\mathrm{d}x \qquad \text{(A.17)}$$

where C is a closed contour around $x=0$.

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5) In this case $G(x)$ was called the "grand generating function".

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