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SCATTERING OF COSMIC RAY PARTICLES OF AN ENERGY ABOVE 1 GeV IN THE INHOMOGENEOUS INTERPLANETARY MEDIUM

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The scattering of the charged cosmic ray particles in the stochastic magnetic field, in the presence of fluctuations of the solar wind velocity is investigated. A common form of the correlation tensor of the stochastic magnetic field $B_{\alpha\beta}$, the solar wind velocity $U_{\alpha\beta}$ and the crossing tensor $P_{\alpha\beta}$ is proposed. There are considered particles of an energy above 1 GeV, whose Larmour radius is larger than the characteristic size of the inhomogenities of the medium. In this approximation the equation is derived for the concentration and the current density of the particles. The radial diffusion coefficient for optimal values of the interplanetary parameters is computed.

РАССЕЯНИЕ ЧАСТИЦ КОСМИЧЕСКОГО ИЗЛУЧЕНИЯ С ЭНЕРГИЕЙ СВЫШЕ 1 Гэв В НЕОДНОРОДНОЙ МЕЖПЛАНЕТНОЙ СРЕДЕ

В работе исследуется рассеяние заряженных частиц космического излучения в стохастическом магнитном поле в присутствии флуктуаций скорости солнечного ветра. Предполагается, что общая форма тензора корреляции стохастического магнитного поля $B_{\alpha\beta}$, тензора скорости солнечного ветра $U_{\alpha\beta}$ и смешанного тензора $P_{\alpha\beta}$ одна и та же. Рассматриваются частицы с энергией свыше 1 Гэв, для которых радиус Лармора больше, чем характеристический размер неоднородностей среды. В этом приближении выведено уравнение для концентрации и потока частиц и рассчитан радиальный коэффициент диффузии для оптимальных значений параметров межпланетной среды.

I. INTRODUCTION

In the inhomogeneous interplanetary medium the scattering of the charged cosmic ray particles takes place. This can be investigated by the method of the kinetic equation [1]. The general solution of this equation in most real cases is difficult. Therefore it is necessary to use various approximations in accordance with the energy particles: the resulting equations can be simplified in the case of $R \ll L_c$, respectively $R \gg L_c$, where R is the Larmour radius and L_c is the autocorrelation

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length of the stochastic magnetic field. If the inequality $R \ll L_c$ holds, we must use the "drifting" approximation and it is necessary to consider a helical motion of particles in the magnetic field. The authors found the diffusion equations for this case in [2]. The opposite case $R \gg L_c$ was established in Dolginov's and Toptygin's work [1], which supposed that 1. the solar wind velocity was regular, 2. the distribution of the magnetic field irregularities had a Gaussian character.

The first assumption leads to the disappearance of both the correlation tensor of the stochastic components of the solar wind velocity and the crossing correlation tensor of the solar wind velocity and the magnetic field. These two tensors may strongly influence the form of the resulting equations. The second assumption is not valid, because at present it is already well-known that the spectrum of the magnetic field inhomogenities has a power form. To such a spectrum there corresponds in the case of the statistic isotropic magnetic field the correlation tensor of the magnetic field in the form

$$B_{\alpha\beta}(\mathbf{r}, \mathbf{x}) = \frac{1}{3} \langle H_1^2(\mathbf{r}) \rangle \left\{ \Psi\left(\frac{x}{L_c}\right) \delta_{\alpha\beta} - \Psi_1\left(\frac{x}{L_c}\right) \frac{x_\alpha x_\beta}{x^2} \right\}, \tag{1}$$

where

$$\Psi\left(\frac{x}{L_c}\right) = \left(\frac{x}{L_c}\right)^{(\nu-1)/2} K_{(\nu-1)/2}\left(\frac{x}{L_c}\right),$$

$$\Psi_1\left(\frac{x}{L_c}\right) = \Psi\left(\frac{x}{L_c}\right) - \frac{2L_c^2}{x^2} \int_0^{x/L_c} \mathrm{d}y\, \Psi(y)\, y,$$

$K_\mu(y)$ is the McDonald function. We suppose the function $\Psi(x/L_c)$ to be known from measurements. $\mathbf{H}_1(\mathbf{r}, t)$ is the stochastic component of the magnetic field:

$$\mathbf{H}(\mathbf{r}, t) = \mathbf{H}_0(\mathbf{r}) + \mathbf{H}_1(\mathbf{r}, t), \tag{2}$$

where $\mathbf{H}_0$ is the regular component of the field, i.e. $\langle \mathbf{H} \rangle = \mathbf{H}_0$ holds. The square brackets are used to indicate the averaging over an ensamble of realizations of the random magnetic field. We will consider the fluctuations of the solar wind velocity analogically:

$$\mathbf{u}(\mathbf{r}, t) = \mathbf{u}_0(\mathbf{r}) + \mathbf{u}_1(\mathbf{r}, t), \tag{3}$$

$$\langle \mathbf{u}(\mathbf{r}, t) \rangle = \mathbf{u}_0(\mathbf{r}), \qquad \langle \mathbf{u}_1(\mathbf{r}, t) \rangle = 0.$$

The presence of the velocity $\mathbf{u}$ causes the adiabatic change of the particles' energy. The expressions (2) and (3) allow to establish the cross-correlation tensor $\langle H_{1\alpha} u_{1\beta} \rangle$. Let us note that in order to receive further information about the spectra of the fluctuations it is necessary to investigate simultaneously the correlations of many plasma parameters [3], for example the spectra of the interplanetary magnetic field, the velocity and the plasma density, and their corresponding cross-correlation tensors, etc.

II. KINETIC EQUATION IN THE CASE OF THE PRESENCE OF THE SOLAR WIND VELOCITY FLUCTUATIONS

We compile the kinetic equation for the mean distribution function $F(\mathbf{r}, \mathbf{p}, t)$ in the phase space if $R \gg L_c$ holds. This condition is true for the energy of the particles $T > 1$ GeV, because the value $L_c \cong 2 \times 10^{11}$ cm. We use the general form of the kinetic equation [4]. If the distance is of the order of the scale of the magnitude of the magnetic field correlation radius, we can put the change of the particle momentum $\Delta\mathbf{p}(\tau) \cong 0$ and the change of the particle radius vestor $\Delta\mathbf{r}(\tau) \cong \mathbf{v}\tau$. (For instance, the operator $\exp\left\{\left(-\mathbf{v}\frac{\partial}{\partial \mathbf{r}} - \frac{e}{c}[\mathbf{v}-\mathbf{u}, \mathbf{H}_0]\frac{\partial}{\partial \mathbf{p}}\right)\tau\right\}$ from a general kinetic equation [4] has no influence on the momentum). The stochastic component of the electromagnetic force acting on the particle is

$$\boldsymbol{\mu}_1 = \frac{e}{c}[\mathbf{v} - \mathbf{u}_0, \mathbf{H}_1] - \frac{e}{c}[\mathbf{u}_1 \mathbf{H}_0], \tag{4}$$

and then the kinetic equation has the form

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \frac{e}{c}[\mathbf{w}\mathbf{H}_0]\frac{\partial}{\partial \mathbf{p}}\right) F(\mathbf{r}, \mathbf{p}, t) = \tag{5}$$

$$= \frac{\partial}{\partial p_\alpha} \int_0^\infty \langle \mu_{1\alpha} \mu_{1\beta} \rangle \, \mathrm{d}\tau \frac{\partial}{\partial p_\beta} F(\mathbf{r}, \mathbf{p}, t),$$

where $\mathbf{w} = \mathbf{v} - \mathbf{u}_0$, $\mathbf{v} = c^2 E^{-1}(p)\mathbf{p}$, E is the energy. The correlation tensor $\langle \mu_{1\alpha} \mu_{1\beta} \rangle$ is

$$\langle \mu_{1\alpha} \mu_{1\beta} \rangle = \frac{e^2}{c^2} \varepsilon_{\alpha\gamma\delta} \varepsilon_{\beta\mu\nu} \{ w_\gamma w_\mu B_{\delta\nu} - w_\gamma H_{0\nu} P_{\delta\mu} - \tag{6}$$

$$- H_{0\delta} w_\mu P_{\gamma\nu} + H_{0\delta} H_{0\nu} U_{\gamma\mu} \}.$$

We have denoted by $B_{\delta\nu}(\mathbf{r}, \mathbf{w}\tau) = \langle H_{1\delta} H_{1\nu} \rangle$ — the autocorrelation tensor of the stochastic magnetic field, by $P_{\delta\mu}(\mathbf{r}, \mathbf{w}\tau) = \langle H_{1\delta} u_{1\mu} \rangle$ — the crossing correlation tensor of the stochastic components of the magnetic field and the solar wind velocity, by $U_{\gamma\mu}(\mathbf{r}, \mathbf{w}\tau) = \langle u_{1\gamma} u_{1\mu} \rangle$ — the autocorrelation tensor of the stochastic solar wind velocity, by $\varepsilon_{\alpha\beta\gamma}$ the tensor Levi-Civita. The form of $B_{\delta\nu}$ is given by (1). It can be shown that

$$\int_0^\infty B_{\alpha\beta}(\mathbf{r}, \mathbf{w}\tau)\, \mathrm{d}\tau = \frac{\sqrt{\pi}}{4\nu} \frac{\Gamma\left(\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu-1}{2}\right)} \frac{L_c}{w} \langle H_1^2 \rangle (\delta_{\alpha\beta} + b_{\alpha\beta}), \tag{7}$$

where $b_{\alpha\beta}$ is the symmetric tensor with zero diagonal components. Its nondiagonal components are proportional to $w_\alpha w_\beta / w^2$. The exact analytic form of the tensor $P_{\alpha\beta}$ and $U_{\alpha\beta}$ is unknown at present. However, the magnetic lines of force are carried by the moving fluid into which they are "frozen". Hydromagnetic turbulence may be generated owing to a high magnetic Reynolds number, although we will not discuss this subject here. Solar wind steadily flows out into interplanetary space from the coronal region. It seems that this wind interacts strongly with the magnetic field ambient in space to a distance of many astronomical units from the solar corona, and perhaps even beyond the solar system.

We assume the analytic forms of the tensors $B_{\alpha\beta}$, $P_{\alpha\beta}$ and $U_{\alpha\beta}$ to be similar, but their numerical coefficients are not quite accurate. The experimental values confirm this assumption. For example, the power spectrum of the radial solar wind velocity u_r in the 2×10^{-6} to 2×10^{-3} Hz frequency range is demonstrated in [5]. Jokipi [3] reported that extensive direct measurements of the properties of solar-wind plasma showed the solar wind to be highly turbulent, with a broad spectrum of fluctuations extending from wave numbers $k\sim10^{-11}$ to $k \gtrsim 10^{-6}$ cm^{-1}. The data consist mostly of a simple power-law dependence $k^{-\alpha}$ with the index $\alpha\sim1.5\div2$. Cross-correlation between the radial velocity u_r and the density n, and the strong coherence between B^2 and n have been found by Goldstein and Siscoe [6]. Pioneer 6 and Pioneer 10 observations [7—8] show that the frequency dependence in the $\sim10^{-4}$ to 10^{-3} Hz frequency range of the power spectra of the solar-wind-proton speed is similar to that of the power spectra of the solar-wind-proton number density. Further, the power spectra of the solar-wind-proton streaming speed and the power spectra of the interplanetary magnetic field are similar, too.

For illustration we can show a simplified example on the boundary of the magnetohydrodynamics theory: if the irregularities of the solar wind velocity **u** and the magnetic field **H** have a wave origin, the characteristics of **u**, **H** satisfy similar equations. Consequently, in the solar-wind-plasma

$$\frac{\partial \mathbf{H}}{\partial t} = \mathrm{rot}\,[\mathbf{uH}].$$

In the case of **u** and $\mathbf{H}_1 = \mathbf{H} - \mathbf{H}_0$ being small perturbations, we have

$$\frac{\partial \mathbf{H}_1}{\partial t} = (\mathbf{H}_0\nabla)\,\mathbf{u},$$

$$n\frac{\partial \mathbf{v}}{\partial t} = -\nabla\left(p + \frac{\mu}{4\pi}(\mathbf{H}_0\mathbf{H}_1)\right) + \frac{\mu}{4\pi}(\mathbf{H}_0\nabla)\,\mathbf{H}_1,$$

where p is the hydrodynamic pressure and n is the plasma density.
Putting $H_0 = H_{0x}$, we obtain equations, whose solution is in the form of transverse, plane waves expanding in the direction of $\mathbf{H}_0$. Let us take $H_1 = H_{1y}$. We assume the

dependence of the pressure only in the direction of the x axis, i.e. in the propagation direction of the waves. The resulting equations have the form

$$\frac{\partial H_1^2}{\partial t} = 2H_{0x}H_1\frac{\partial u_y}{\partial x},$$

$$\frac{\partial u_y^2}{\partial t} = \frac{\mu}{2\pi}\frac{H_0}{n}u_y\frac{\partial H_1}{\partial x}, \text{ etc.},$$

which give equations like the following one

$$\frac{\partial P}{\partial t} \doteq \frac{H_0}{2}\frac{\partial}{\partial x}\left(u - \frac{\mu}{4\pi}B\right), \text{ etc.},$$

(moreover ordinary wave equations for the Alfven waves). The equations for $\langle H_1^2\rangle$, $\langle u^2\rangle$, $\langle uH_1\rangle$ are the same as regards the accuracy of the numerical coefficients. In accordance with this circumstance we can expect a high correlation between the magnetic field **H** and the solar wind velocity **u** in waves (and perturbations, respectively) moving away from the sun [4]. (In a community, the fluctuations are not only pure transverse Alfven waves). The author gives magnitudes of the correlation coefficient ϱ of the solar wind velocity and the field strength, which were measured on the Explorer 43. These magnitudes are cca 0.7, frequently even 1.0.

The diagonal components of the tensors $P_{\alpha\beta}$ and $U_{\alpha\beta}$ (as in the case of the tensor $B_{\alpha\beta}$) *give*

$$\int_0^\infty P_{\alpha\beta}(\mathbf{r}, \mathbf{w}\tau)\,\delta_{\alpha\beta}\,d\tau = \gamma_P\frac{L_P}{v}\langle u_1 H_1\rangle, \quad (8)$$

$$\int_0^\infty U_{\alpha\beta}(\mathbf{r}, \mathbf{w}\tau)\,\delta_{\alpha\beta}\,d\tau = \gamma_u\frac{L_u}{v}\langle u_1^2\rangle,$$

where L_P, L_u are the correspondent radii of the correlation, γ_P, γ_u are the

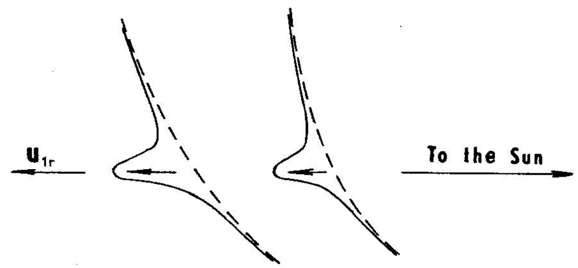

Fig. 1. A deformation of the magnetic line of force is caused by the disturbance of the solar wind velocity.

numerical constants (with the value ~ 1). The non-diagonal components are not written, because these give a zero deposit in the following calculation, like $b_{\alpha\beta}$.

This reality will be understood if we assume that the regular magnetic field form of Archimedean spirals (the dashed curves in Fig. 1.), i.e. $\mathbf{u}=\mathbf{u}_0$ holds. If there appears the fluctuation $\mathbf{u}_1(\mathbf{r}, t)$ (for example owing to the disturbance on the Sun), the magnetic line of force begins to drift in the direction $\mathbf{u}_1$. Let the vector $\mathbf{u}_1$ have a direction from the Sun, i.e. a radial direction. Evidently, most changes arise in the radial component of the field and we suppose — with a certain inaccuracy — that the other components are negligible. We have an analogical case as regards the direction vertical to the radius vector. Consequently, the tensor $P_{\alpha\beta}$ is approximately proportional to $\delta_{\alpha\beta}$.

III. DIFFUSION EQUATION FOR THE CURRENT DENSITY OF PARTICLES

An application of the method of the calculation of diffusion equations [1—2] to our case gives the vector equation for the first and second momenta of the distribution function, i.e. the equations for the concentration $N(\mathbf{r}, p, t)$ and the current density of the particles $\mathbf{J}(\mathbf{r}, p, t)$. On the right-hand side of equation (5) there arise expressions consistent with the component of the tensors $P_{\alpha\beta}$, $U_{\alpha\beta}$, $B_{\alpha\beta}$, which are of a higher order in the decomposition according to the parameter (u/v). We put $(u/v)^2 \to 0$ in these expressions. The non-diagonal component, for example $b_{\alpha\beta}$, gives a zero deposit in the resulting diffusion equation owing to the integration in an angular space with a polar axis along the momentum vector.

If the energy change of the current is small in comparison with the diffusion change, we obtain the equation

$$\frac{\Lambda_c}{R_{H_0}}[\mathbf{hJ}]+q_1\mathbf{J}-\mathbf{q}(\mathbf{hJ})+q_2\mathbf{h}\left(\mathbf{J}\frac{\mathbf{u}}{u}\right)= \tag{9}$$

$$=-\varkappa_0\frac{\partial N}{\partial \mathbf{r}}-\frac{p}{3}\frac{\partial N}{\partial p}\left\{\mathbf{u}_0+\frac{\Lambda_c}{R_{H_0}}[\mathbf{hu}]+g\mathbf{h}\right\}-\frac{\Lambda_c}{R_{H_0}}\frac{\partial \mathbf{J}}{\partial t},$$

where we denoted

$$\mathbf{q}=\frac{12}{5}\beta^2\frac{L_u\gamma_u}{L_c\gamma_c}\frac{H_0^2}{v^2}\frac{\langle u_1^2\rangle}{\langle H_1^2\rangle}\mathbf{h}-\left(6-\frac{18}{5}\beta^2\right)\frac{L_p\gamma_p}{L_c\gamma_c}\frac{H_0}{v^2}\frac{\langle u_1H_1\rangle}{\langle H_1^2\rangle}\mathbf{u}_0, \tag{10}$$

$$q_1=1-\frac{24}{5}\beta^2\frac{L_u\gamma_u}{L_c\gamma_c}\frac{H_0^2}{v^2}\frac{\langle u_1^2\rangle}{\langle H_1^2\rangle}-\left(6-\frac{3}{5}\beta^2\right)\frac{L_p\gamma_p}{L_c\gamma_c}\frac{u_0H_0}{v^2}\frac{\langle u_1H_1\rangle}{\langle H_1^2\rangle},$$

$$q_2=\frac{12}{5}\beta^2\frac{L_p\gamma_p}{L_c\gamma_c}\frac{u_0H_0}{v^2}\frac{\langle u_1H_1\rangle}{\langle H_1^2\rangle},$$

$$g=6\frac{L_p\gamma_p}{L_c\gamma_c}H_0\frac{\langle u_1H_1\rangle}{\langle H_1^2\rangle},\qquad \gamma_c=\frac{\sqrt{\pi}}{4v}\frac{\Gamma(v/2)}{\Gamma(v-1/2)},$$

$$\varkappa_0=\frac{v}{3}\Lambda_c,\qquad R_{H_0}=\frac{cp}{eH_0},\qquad \mathbf{h}=\frac{\mathbf{H}_0}{|\mathbf{H}_0|},$$

$\Lambda_c=\dfrac{3c^2p^2}{e^2\gamma_cL_c\,\langle H_1^2\rangle}$ is the mean free path of the particles. The vector equation (9) contains the parameters which characterize both the stochastic magnetic field and the fluctuations of the solar wind velocity. This equation is more exact than the equation found in [1] for the high energy particles and shows the current dependence on the gradient of concentration and the power spectra of cosmic ray particles. The last term in (9) may usually be neglected if we investigate the diffusion process in a time scaling $t \gtrsim \Lambda_c/v$, i.e. when the use of the diffusion approximation is applicable.

IV. RADIAL DIFFUSION COEFFICIENT

In a general case, the correct form of the diffusion tensor can be obtained from equation (9) if the vector of the current density $\mathbf{J}(\mathbf{r}, p, t)$ is expressed by the particles concentration and the parameters of the medium. However, obtained in this way the expression is very complicated. In order to compare with an experiment it is useful to calculate the "radial diffusion coefficient" only, which we can find from the expression for $\mathbf{J}$ written in the spherical coordinate system with the Sun in the centre. We will propose the following optimal values of the medium parameters near the orbit of the Earth, when the angle between $\mathbf{H}_0$ and the radial direction from the Sun $\Psi \doteq \pi/4$: $H_0=4.5\gamma$, $\langle H_1^2\rangle=4.2\gamma^2$, $u_0=4\times10^7$ cms^{-1}, $\langle u_1^2\rangle=5\times10^{14}$ cm^{2}s^{-2}, $L_p \doteq L_u \doteq L_c=2\times10^{11}$ cm, $v=2$.

The numerical calculation gives

$$\varkappa_{rr}=\begin{cases}0.24\ \ (\text{A.U.})^2/h \text{ for protons } T=1\text{ GeV},\\ 1.02\ \ (\text{A.U.})^2/h \text{ for protons } T=10\text{ GeV}.\end{cases} \tag{11}$$

The found values of the radial diffusion coefficient $\varkappa_{rr}(T)$ in the range of energy $T \lesssim 1$ GeV are represented in Fig. 2 (the full curve 1a).

In this figure there are illustrated some experimental values for $T \leqq 1$ GeV, [10]. (The case $v=2$ corresponds to the values dentod by a). For the higher solar activity period, when the index spectrum v of the inhomogenities is smaller, we obtain the theoretical curve 1b for the corresponding experimental values of b. We see the theoretically predicted dependence of $\varkappa_{rr}(T)$ link up with the dependence of $\varkappa_{rr}(T)$ from the low energy region. In Fig. 2. also theoretical predictions obtained by other methods are illustrated. The dependences 2a, 2a', 2b [10] were found from the Fokker-Planck equation. Analogically the value [11] was obtained — curve 3. The values of $\varkappa_{rr}$ calculated by many authors are usually smaller than those obtained from the diffusion approximation of the kinetic equation.

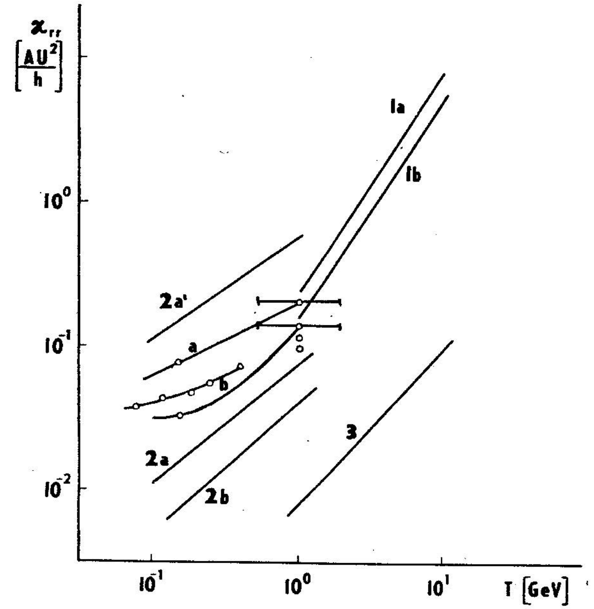

Fig. 2. A comparison of theoretically predicted dependences of $\varkappa_{rr}(T)$ with experimental values [10].

REMARK

The asymptotical dependence of $\varkappa_{rr}$ on the particle energy T, the characteristic size inhomogenities L_c and the index spectra v in the energy region $T \gtrsim 1$ GeV are as follows:

$$\varkappa_{rr}(T, v, L_c) \sim T^{3/2} v L_c^{-1}, \tag{12}$$

so that the radial diffusion coefficient $\varkappa_{rr}$ decreases if the index spectra of the magnetic field irregularities decrease. However, in this case there increases in the interplanetary space the number of inhomogeneities of large sizes, i.e. the value of L_c increases. We see that there is a close connection between the last two dependences.

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