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ON THE DISPLACED MAXWELLIAN DISTRIBUTION FUNCTION AND ON THE PROBLEM OF WARM ELECTRONS IN SEMICONDUCTORS

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Starting from the displaced Maxwellian distribution function derived by Wassef and Kao the often used distribution function $f = a \exp[-(E(k) - \hbar k \cdot v_d) / k_B T_e]$ is derived under the assumption that the electron drift velocity v_d is much smaller than the electron thermal velocity. The electron temperature T_e is defined in such a way that the mean energy of an electron is $3k_B T_e/2$. In a more general case the quantity $1/k_B T_e$ in the above introduced displaced Maxwellian distribution function should be replaced by a rather complicated function of T_e and v_d, which is implicitly given by the relation (7) of this paper. Using an approximative relation following from (7), the warm electron coefficient b is found to be given by the relation $b = m^* \mu_0^2 / 3k_B T_L$, in which μ_0 is the low field electron mobility and T_L is the lattice temperature. This result is compared with experimental estimations of the coefficient b in Ge, Si, and InSb arrived at by other authors.

О ФУНКЦИИ СМЕЩЁННОГО МАКСВЕЛЛОВСКОГО РАСПРЕДЕЛЕНИЯ И ПРОБЛЕМЕ НАГРЕТЫХ ЭЛЕКТРОНОВ В ПОЛУПРОВОДНИКАХ

Исходя из функции смещённого максвелловского распределения, полученной Вассефом и Као, выведена часто используемая функция распределения $f = a \cdot \exp[-(E)(k) - \hbar k v_d) / k_B T]$ в предположении, что скорость дрейфа электронов v_d намного меньше, чем тепловая скорость электронов. Электронная температура T_e определена таким образом, что средняя энергия электронов равна $3k_B T_e/2$. В более общем случае величина $(k_B T_e)^{-1}$, входящая в упомянутую выше функцию распределения, должна быть заменена более сложной функцией параметров T_e и v_d, неявный вид которой даётся формулой (7) данной работы. Используя приближённое выражение, вытекающее из формулы (7), найден коэффициент нагревания электронов $b = m^* \mu_0^2 / 3k_B T_L$, где μ_0 представляет собой подвижность слабо возбуждённых электронов и T_L представляет собой температуру решётки. Полученный результат сравнивается с экспериментально определёнными коэффициентами b для Ce, Si и InSb, представленными другими авторами.

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I. INTRODUCTION

In many papers [1—8] on hot or warm electrons in semiconductors the displaced Maxwellian distribution function

$$f(k)=a \exp[-(E(k)-\hbar \boldsymbol{k} \cdot \boldsymbol{v}_d)/k_B T_e], \tag{1}$$

in which $E(k)$ is the energy of an electron with the momentum $\hbar \boldsymbol{k}$, $\boldsymbol{v}_d$ is the electron drif velocity and T_e is the electron temperature, is used. The type of this distribution function is considered to be reasonable in the case of a dominant electron-electron interaction. The aim of this paper is to arrive at the function (1) on the basis of more general principles. Wassef and Kao [9] have shown that the generalized Fermi-Dirac distribution function, which takes into account the effect of the applied electric field, can be deduced from the maximum entropy estimates if as a further restriction to the constant number and energy of electrons the constant value of the electric current is considered. Their distribution function has form

$$f=[1+\exp(\alpha+\beta E+\gamma e v_F)]^{-1}, \tag{2}$$

where α, β, γ are the Langrangian multipliers standing be for the number of electrons, their energy, and for the current. In the case of a zero electric field $\gamma=0$ and (2) becomes the Fermi-Dirac distribution function in which $\beta=1/k_B T_L$ (T_L is the lattice temperature). Wassef and Kao introduced also a special form of distribution function following from (2) in the case of a non-degenerate semiconductor with a parabolic dispersion law:

$$f(k)=\exp\left(-\alpha-\beta\frac{\hbar^2 k^2}{2m^*}-\gamma\frac{e\hbar}{m^*}k\cos\vartheta\right), \tag{3}$$

where ϑ is an angle between the $\boldsymbol{k}$ vector and the electric field $\boldsymbol{F}$.

In this paper we shall determine the coefficients γ and β in the function (3) on the basis of definitions of the electron drift velocity and electron temperature. Then we shall restrict our considerations to the problem of warm electrons when $(T_e-T_L)/T_L \ll 1$, and in a simple way we shall determine the warm electron coefficient standing in the relation for electron mobility. The result will be compared with experiments of other authors on Ge, Si, and InSb.

II. DETERMINATION OF THE COEFFICIENTS α AND β

The electron drift velocity can be defined by the relation

$$v_d=\frac{\int \frac{\hbar k}{m^*}\cos\vartheta f(k)\,\mathrm{d}^3 k}{\int f(k)\,\mathrm{d}^3 k}. \tag{4}$$

Substituting for $f(k)$ according to (3) we get

$$\gamma=-\frac{\beta m^*}{e}v_d. \tag{5}$$

We define the electron temperature T_e by the relation

$$\frac{3}{2}k_B T_e=\frac{\int \frac{\hbar^2 k^2}{2m^*}f(k)\,\mathrm{d}^3 k}{\int f(k)\,\mathrm{d}^3 k}. \tag{6}$$

As it is shown in the Appendix we obtain

$$\frac{3}{2}k_B T_e=\frac{1}{\beta}\,\frac{\frac{1}{2}(\mathrm{e}^{1/2a}-1)+\frac{1}{2a}}{\mathrm{e}^{1/2a}-1}, \tag{7}$$

where

$$1/2a=\beta m^* v_d^2.$$

In the case of

$$\beta m^* v_d^2 \ll 1 \tag{8}$$

we get from (7)

$$1/\beta=k_B T_e. \tag{9}$$

The condition (8) is fulfilled when the electron drift velocity is small as compared with the electron thermal velocity. In a limit $v_d \to 0$, $1/\beta=k_B T_L$. Thus we can conclude that if the electron temperature is defined by the relation (6), the coefficient β in the distribution function (3) is given by the relation (9) when the condition (8) holds. In this case the distribution function (3) takes the form (1). In the more general case the parameter β in (3) is a rather complicated function of the electron temperature and of the electron drift velocity given by the relation (7).

Evidently, the relations (4) and (6) do not give the electric field dependences of the electron drift velocity and electron temperature. For this purpose the energy and momentum balance equations can be used. In general this problem can be solved only if the collision mechanisms are known. However, in the case of warm electrons we can find the electric field dependences of the electron temperature and electron drift velocity in a semiempirical way which does not necessitate the knowledge of collision mechanisms. They are included in a single empirical parameter, i. e. the low field electron mobility. This will be shown in the next section.

III. WARM ELECTRONS

The range of the warm electrons is usually defined by the condition

$$\frac{T_e - T_L}{T_L} \ll 1 . \qquad (10)$$

In this case the empirical relation for the drift velocity

$$v_d = \mu_0 (1 - bF^2) F \qquad (11)$$

is well fulfilled with the field independent mobility μ_0 and the warm electron coefficient b. The problem consists in the theoretical determination of the coefficient b. We shall do it in a very simple way.

In the case of $1/2a < 1$ the relation (7) gives

$$\frac{3}{2} k_B T_e \doteq \frac{3}{2} \frac{1}{\beta} + \frac{1}{2} m^* v_d^2 . \qquad (12)$$

Approximating $1/\beta$ by $k_B T_L$ we get

$$\frac{3}{2} k_B (T_e - T_L) = \frac{1}{2} m^* v_d^2 , \qquad (13)$$

and substituting for v_d according to the relation (11) we obtain

$$b = \frac{m^* \mu_0^2}{3 k_B T_L} \frac{T_L b F^2 (1 - bF^2)^2}{T_e - T_L} . \qquad (14)$$

However, for warm electrons the coefficient b should be field independent, and for $F \to 0$, T_e should approach T_L. These conditions are fulfilled when

$$T_e = T_L [1 + bF^2 (1 - bF^2)^2] , \qquad (15)$$

$$b = \frac{m^* \mu_0^2}{3 k_B T_L} . \qquad (16)$$

Condition (10) is then equivalent to the condition

$$bF^2 \ll 1, \text{ i. e. } F^2 \ll \frac{3 k_B T_L}{m^* \mu_0^2} . \qquad (17)$$

The relation (16) can be experimentally verified. According to [10] in relatively pure n-Ge $\mu_0 \sim T^{-1.66}$ in temperature range 100—300 K and in n-Si $\mu_0 \sim T^{-2.5}$ in temperature range 160—400 K. Using these temperature dependences of the low field electron mobilities we get from (16) $b_{Ge} \sim T^{-4.32}$ and $b_{Si} \sim T^{-6}$ in good accordance with experimentally found temperature dependence of the coefficient b in a given temperature range. According to Seeger [11] $b_{Ge} \sim T^{-4.27}$ and according to Kästner et al. [12] $b_{Si} \sim T^{-5.89}$. Due to the effective mass anisotropy in Ge and Si the value of b depends also on the sample orientation with respect to

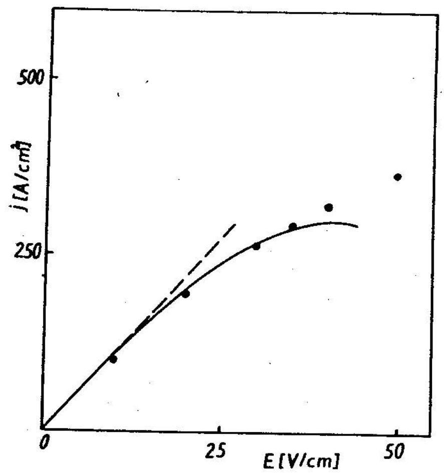

Fig. 1. Solid curve is computed from (18) with $\sigma_0 = 11\ \Omega^{-1}\text{cm}^{-1}$, $\mu_0 = 7 \times 10^5\ \text{cm}^2/\text{Vs}$, $T_L = 78$ K; circles correspond to experimental results of Bonek [13] sample (CO4FR).

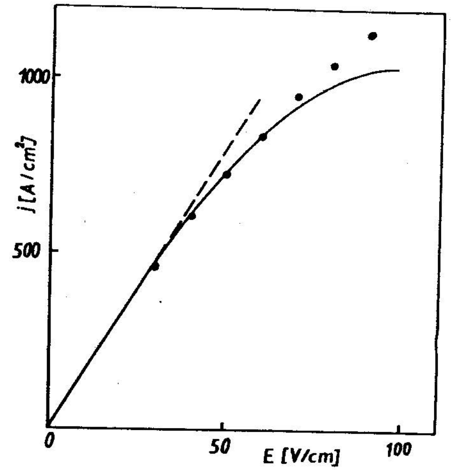

Fig. 2. Solid curve is computed from (18) with $\sigma_0 = 16\ \Omega^{-1}\text{cm}^{-1}$, $'_0 = 3 \times 10^5\ \text{cm}^2/\text{Vs}$, $T_L = 78$ K; circles correspond to experimental results of Bonek [13] (sample LEP).

the applied electric field. Kästner et al. found the following empirical relation for b in the $\langle 111 \rangle$ direction on the n-type Si crystal in the temperature range 194—273 K;

$$b = 3.0 \times 10^{-5} \left(\frac{T}{100\ \text{K}} \right)^{-5.89} \frac{\text{cm}^2}{\text{V}^2} ,$$

from which for $T = 273$ K we $b = 8.1 \times 10^{-8}\ \text{cm}^2/\text{V}^2$. Using the relation (16) we get the same value for b at 273 K taking $\mu_0 = 1800\ \text{cm}^2/\text{Vs}$ and $m^* = 0.34\ m_0$, which are reasonable values for silicon. Thus the relation (16) does not give only the right temperature dependence of coefficient b but also its value. This is especially well confirmed by the experimental results of Bonek [13] on two samples of InSb with various parameters at 78 K as can be seen from Figs. 1 and 2. In Fig. 1 we compare the relation

$$j = \sigma_0 F (1 - bF^2) \qquad (18)$$

for the current density in which b is given by (16) with experimental values taken from Fig. 31 of [13] for a sample with $\sigma_0 = 11\ \Omega^{-1}\text{cm}^{-1}$ and $\mu_0 = 7.1 \times 10^5\ \text{cm}^2/\text{Vs}$ at 78 K. Taking $m^* = 0.0141\ m_0$ we get from (16) $b = 2 \times 10^{-4}\ \text{cm}^2/\text{V}^2$. Substituting this value of b and the given value of σ_0 into (18) we get the full curve in Fig. 1. This curve is in good agreement with experiment for $F < 35$ V/cm, i. e. for

$bF^2 < 0.15$. In Fig. 2 the computed field dependence of the current density is compared with Bonek's experiments on an InSb sample with $\sigma_0 = 16\ \Omega^{-1}\mathrm{cm}^{-1}$ and $\mu_0 = 3 \times 10^5\ \mathrm{cm^2/Vs}$ at 78 K. In this case get from (16) $b = 3.6 \times 10^5\ \mathrm{cm^2/V^2}$. The agreement with the experiment is good for $F < 65$ V/cm, i. e. for $bF^2 < 0.15$.

APPENDIX

Substituting the distribution function (3) with γ given by the relation (5) into (6) we get

$$\tfrac{3}{2} k_B T_e = \frac{\int_0^\infty \int_0^\pi \int_0^{2\pi} \frac{\hbar^2 k^2}{2m^*} \exp\left[-\alpha - \beta\left(\frac{\hbar^2 k^2}{2m^*} - \hbar k v_d \cos\vartheta\right)\right] k^2\, \mathrm{d}k\, \mathrm{d}\vartheta\, \mathrm{d}\varphi \sin\vartheta}{\int_0^\infty \int_0^\pi \int_0^{2\pi} \exp\left[-\alpha - \beta\left(\frac{\hbar^2 k^2}{2m^*} - \hbar k v_d \cos\vartheta\right)\right] k^2\, \mathrm{d}k\, \mathrm{d}\vartheta\, \mathrm{d}\varphi \sin\vartheta} = \tag{A1}$$

$$= \frac{a \int_0^\infty x^4 \mathrm{e}^{-ax^2} \frac{\mathrm{sh} x}{x} \mathrm{d}x}{\beta \int_0^\infty x^2 \mathrm{e}^{-ax^2} \frac{\mathrm{sh} x}{x} \mathrm{d}x} = -\frac{a}{\beta} \frac{\frac{\mathrm{d}I(a)}{\mathrm{d}a}}{I(a)},$$

where

$$I(a) = \int_0^\infty x^2 \mathrm{e}^{-ax^2} \frac{\mathrm{sh}\, x}{x} \mathrm{d}x,$$

$x = \beta \hbar k v_d$, $a = 1/2m^* \beta v_d^2$. If we expand the function $(\mathrm{sh}\, x)/x$ into the power series, then, using the formula [14]

$$\int_0^\infty x^{2n} \mathrm{e}^{-ax^2} \mathrm{d}x = \frac{(2n-1)(2n-3)\ldots 5.3.1}{2.2n} \left(\frac{\pi}{a^{2n+1}}\right)^{1/2}$$

we obtain

$$I(a) = \tfrac{1}{2}\left(\frac{\pi}{a}\right)^{1/2} \sum_{n=1}^\infty \frac{(1/2a)^n}{n!} = \tfrac{1}{2}\left(\frac{\pi}{a}\right)^{1/2} (\mathrm{e}^{1/2a} - 1). \tag{A2}$$

Substituting (A2) into (A1) we obtain the relation (7).

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