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DESCRIPTION OF MULTIPARTICLE PRODUCTION BY MEANS OF CHAPMAN-KOLMOGOROV EQUATIONS

VLADIMÍR MAJERNÍK*, Nitra
JOZEF PROKEŠ**, Bratislava

It is pointed out that at least three models for multiparticle production lead to a set of Chapman-Kolmogorov equations in which the transition probabilities occur as the input parameters. Since the transition probabilities provide generally more information about the internal dynamics of the high-energy multiparticle process than the set of the final probabilities for the production of n particles, we have determined them from the multiparticle distribution. It is shown that these transition probabilities represent a simple function of the laboratory momentum which may help to determine the dynamical parameters of the multiparticle processes.

ОПИСАНИЕ МНОГОЧАСТИЧНОЙ ПРОДУКЦИИ С ПОМОЩЬЮ УРАВНЕНИЙ ЧЕПМЕНА – КОЛМОГОРОВА

В работе показано, что по крайней мере три модели для многочастичной продукции приводят к набору уравнений Чепмена – Колмогорова, в которых вероятности переходов выступают в качестве входных параметров. Так как вероятности переходов дают, в общем, о внутренней динамике многочастичного процесса при высоких энергиях большую информацию, чем набор конечных вероятностей для продукции N частиц, мы их определяем из многочастичного распределения. Показано, что эти вероятности переходов представляют простую функцию импульса в лабораторной системе координат, который может помочь определить динамические параметры многочастичных процессов.

I. INTRODUCTION

There are at least three phenomenological or semiphenomenological models for multiparticle production which lead to the same general system of linear differential equations (called Chapman—Kolmogorov) in spite of the fact that the physical arguments in them are different. The first model is the string model [1] within the framework of which the high-energy scattering process is described as an interac-

* Katedra fyziky Pedagogickej fakulty, CS-949 74 NITRA.
** Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta, CS-899 30 BRATISLAVA.

tion between two strings at their end-points. The input momentum by a high-energy collision propagates along the strings (representing general hadrons) to their ends whereby these strings decay into a number of fragmentary strings manifesting themselves as the outgoing hadrons. Under general conditions the fragmentation process can be described by means of the equations [2]:

$$\frac{\mathrm{d}}{\mathrm{d}x} P_1(x) = -p_1(x)P_1(x) \tag{1}$$

$$\vdots \qquad\qquad n = 2, 3, \ldots$$

$$\frac{\mathrm{d}}{\mathrm{d}x} P_n(x) = p_{n-1}(x)P_{n-1}(x) - p_n(x)P_n(x).$$

Here x means the distance from one end of the string to the point of the string where the string is divided. We see that instead of time we have the distance in the set of differential equations of the Chapman—Kolmogorov type.

The second model describing the high-energy multiparticle process that leads to a similar type of differential equations is the well-known jet model [3]. The Chapman—Kolmogorov equations in this model have the form

$$\dot{P}_n(t) = \sum_i [q_i(n-i/t)P_{n-i}(t) - q_i(n-i+1/t)P_{n-i+1}(t)] \tag{2}$$

$$n = 1, 2, \ldots$$

Here $q_i(n-i/t)$ represents the transition probability from the state with $(n-i)$-th particle to the state with n particles and $P_n(t)$ is the probability for the production of n particles.

The third model leading to the system of linear differential equations of the Chapman—Kolmogorov type is the so-called Markov one [4], [5], within the framework of which one assumes that the particle production is realized via the excited states of the hadrons (resonances) in two basic stages: (i) the interaction stage, in which the hadrons are excited into higher energy levels by means of the incoming particles; (ii) the decay stage in which the excited hadron decays producing secondaries. The Chapman—Kolmogorov equation of such a Markov system has the following form:

$$\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t} = L\boldsymbol{P} \tag{3}$$

where $\boldsymbol{P}$ represents the vector of the final probability distribution of the excited hadronic states $P_0, P_1, \ldots, P_n$ (the probability that the excited hadron decays into n secondaries) and

$$L = \begin{pmatrix} q_{00} & q_{01} & \ldots & q_{0n} \\ q_{10} & q_{11} & \ldots & q_{1n} \\ \vdots & & & \\ q_{n0} & q_{n1} & \ldots & q_{nn} \end{pmatrix}$$

where q_{ij} represent the transition probabilities between the corresponding states.

In all the mentioned models there is a direct connection between the multiplicity distribution and some kind of transition probabilities. Many authors have tried to find a phenomenological description of the multiplicity distribution for the accelerator energy in order to determine the multiplicity distribution for higher energies by extrapolating the found regularities, e.g. [6]. The multiplicity distribution represent, however, the final products of some processes taking place in the high-energy collision [10]. We can expect that the set of the transition probabilities stands closer to the dynamical processes occurring at these high energies than the final multiplicity distribution, therefore the set of transition probabilities in the Markov model might represent a link to the inner dynamics of the considered hadronic object, at least at the level of its statistical description [7]. It seems, therefore, physically reasonable to determine the set of the transition probabilities occurring in the Chapman—Kolmogorov equation in order to find their dependences on the momentum of colliding particles. This enables one (by using these dependences) to find the set of transition probabilities above the region of the accelerator energies. Having ascertained this set one can determine the final multiplicity in the region of the very high energies by means of the corresponding Chapman—Kolmogorov equation. In order to find the transition probabilities mathematically we have to eliminate them from general Chapman—Kolmogorov equations.

II. THE INVERSE PROBLEM FOR THE CHAPMAN—KOLMOGOROV EQUATIONS

In the common calculation within the probability theory one puts the transition probabilities (by the description of the multiparticle production with some assumptions about the multiparticle process) in the corresponding Chapman—Kolmogorov equations and from these one determines the probabilities $P_1, \ldots, P_n$. Since, as said above, the transition probabilities may contain generally more information about the internal dynamics of a hadronic high-energy process, the problem to find them from the Chapman—Kolmogorov equations (by putting the experimental final probabilities into them) seems to be physically well-motivated. We therefore assume that the mechanism of the multiparticle production can be described by means of a system of Chapman—Kolmogorov equations (as it is required by the above-mentioned models) — and thus the set of transition probabilities will be determined. In other words, we solve the inverse problem indicated by the Chapman—Kolmogorov equations. In the general case, the problem would be mathematically very complicated, therefore we confine ourselves to a simple form of Chapman—Kolmogorov equations, namely, the following ones

$$\begin{aligned} \dot{P}_1 &= -q_1 P_1 \\ \dot{P}_2 &= q_1 P_1 - q_2 P_2 \\ &\vdots \\ \dot{P}_n &= q_{n-1} P_{n-1} - q_n P_n \end{aligned} \tag{4}$$

where we assume that generally $q_1 \neq q_2 \neq \ldots \neq q_n$. Within the framework of the Markov model this means that a transition exists only between the nearest excited states. According to [5] the general solution of Eq. (4) has the following form

$$P_k = \exp(-q_k t)\left(\prod_{i=1}^{k} q_i\right)\int_0^t \int_0^{\xi_{k-1}} \ldots \int_0^{\xi_2} \exp \sum_{i=0}^{k-1} [(q_{i+1} - q_i)\xi_i] \times \mathrm{d}\xi_1\, \mathrm{d}\xi_2 \ldots \mathrm{d}\xi_{k-1}. \tag{4a}$$

In the special case of $q_1 = q_2 = \ldots = q_n$ we get the well-known Poisson probability distribution

$$P_k(t) = \frac{(qt)^{k-1}}{(k-1)!}\exp(-qt), \qquad k = 1, 2, \ldots$$

The solution (4a) of Eq. (4) can be rewritten in this form

$$P_1(t) = \exp(-q_1 t) \tag{5}$$

$$P_2(t) = \left[\frac{\exp(-q_1 t)}{q_2 - q_1} + \frac{\exp(-q_2 t)}{q_1 - q_2}\right] q_1$$

$$\vdots$$

$$P_k(t) = \left(\prod_{i=1}^{k-1} q_i\right)\left\{\frac{\exp(-q_1 t)}{(q_k - q_1)(q_{k-1} - q_1)\ldots(q_2 - q_1)} + \right.$$
$$+ \frac{\exp(-q_2 t)}{(q_k - q_2)(q_{k-1} - q_2)\ldots(q_3 - q_2)(q_1 - q_2)} + \ldots +$$
$$\left. + \frac{\exp(-q_k t)}{(q_1 - q_k)(q_2 - q_k)\ldots(q_{k-1} - q_k)}\right\}.$$

After a rearrangement, Eqs. (5) can be written in the form of a system of the following equations suitable for our purpose

$$q_1 + \ln P_1 = 0 \tag{6}$$

$$P_1 + P_2 - P_2 q_2 / q_1 - \exp(-q_2 t) = 0$$

$$P_1 + P_2 + P_3 - \left[P_2/q_1 + \frac{P_3(q_1 + q_2)}{q_1 q_2}\right] q_3 + P_3 \frac{q_3^2}{q_1 q_2} - \exp(-q_3 t) = 0$$

$$\vdots$$

From Eq. (6) one can determine the transition probabilities $q_1, q_2, \ldots, q_n$ if the final probabilities $P_1, P_2, \ldots, P_n$ are known. By means of successive substitutions one can easily prove that a certain root of one of these equations represents the corresponding root of all the following ones, i. e. if the solution of the first equation of this system exists, then the solution of the whole system exists, too. We calculated the transition probabilities numerically by using a simple computer program. The final probabilities given in Table 1 were determined from the experimental topological cross-section of the high energy $p-p$ scattering [8]. Inserting the final probabilities from Table 1 into Eqs. (6) we got the values of transition probabilities, in Table 2.

III. THE PHENOMENOLOGICAL ANALYSIS OF THE OBTAINED RESULTS

We see from Table 2 that the value for the transition probability increases with the increasing momentum of the colliding particle. It is to be expected that the analytical expression of the function will possess a simple form that can be read from the plot of the transition probabilities versus the laboratory momentum. As shown in Fig. 1 this function can be approximately written in the following form

$$q_i = a_i + b_i \ln p_{\mathrm{lab}} \tag{7}$$

where a_i and b_i are constants, the values of which are given in Table 3. We can see from Table 3 that with the increase of the momentum p_{lab} the constants a_i decrease, whereas the constants b_i increase. The slope of the straight line b_i in Fig. 1 can be approximately expressed in the following form

$$b_i = 0.25 + 0.26i \tag{8}$$

where i denotes the indices of the transition probabilities.

Within the framework of our phenomenological analysis one can next try to find the dependence of the transition probabilities on the index i for the given values of the laboratory momentum

$$q_i = f(i, p_{\mathrm{lab}}). \tag{9}$$

From the linear and logarithmic plot of this dependence, showed in Figs. 2 and 3, we can see that the empirical function of type (9) can be approximately expressed as follows

$$q_i^p = w^p + y^p i \tag{10a}$$

Table 1.

Multiplicity	p_{lab} GeV/c	probability	p_{lab} GeV/c	probability
1		0.397		0.345
2		0.449		0.432
3	12.88	0.134	18	0.175
4		0.019		0.044
5		0.001		0.004
1		0.328		0.302
2		0.402		0.398
3	21.08	0.198	24.12	0.215
4		0.060		0.074
5		0.012		0.009
6		0.295		0.002
1		0.375		0.192
2		0.216		0.302
3		0.084		0.257
4	28.5	0.027	50	0.061
5		0.003		0.065
6		0.000		0.015
7		0.154		0.006
8		0.275		0.001
1		0.252		0.137
2		0.174		0.250
3		0.088		0.235
4	69	0.041	102	0.180
5		0.012		0.107
6		0.003		0.061
7		0.001		0.021
8				0.006
9				0.003
1		0.107		
2		0.170		
3		0.212		
4		0.177		
5		0.135		
6	205	0.105		
7		0.052		
8		0.026		
9		0.009		
10		0.005		
11		0.002		
12				
13				

Table 2.

p_{lab} GeV/c	q_1	q_2	q_3	q_4	q_5	q_6
12.88	0.9238190	0.5319489	0.3824219	0.1900391	0.00000	
18	1.0641998	0.742870	0.6626953	0.3110352	0.00000	
21.08	1.1147217	0.9280273	0.8463867	0.6242188	0.00000	
24.12	1.1973283	1.0084358	0.8973450	0.4924011	0.9500000	0.000
28.5	1.2207799	1.1403976	1.1500000			
50	1.6502599	1.7470428	1.7769531			
69	1.8708027	2.1038132	2.1984375	2.2960938	2.7734375	
102	1.9877744	2.1613556	2.5064148	2.5664063	2.6968750	
205	2.2349264	2.9611641	3.1032105	3.6526856	3.9021484	

Table 3.

	a_i	b_i
1	−0.41	0.51
2	−1.77	0.83
3	−2.3	1.03
4	−3.41	1.31
5	−3.21	1.33

Table 4

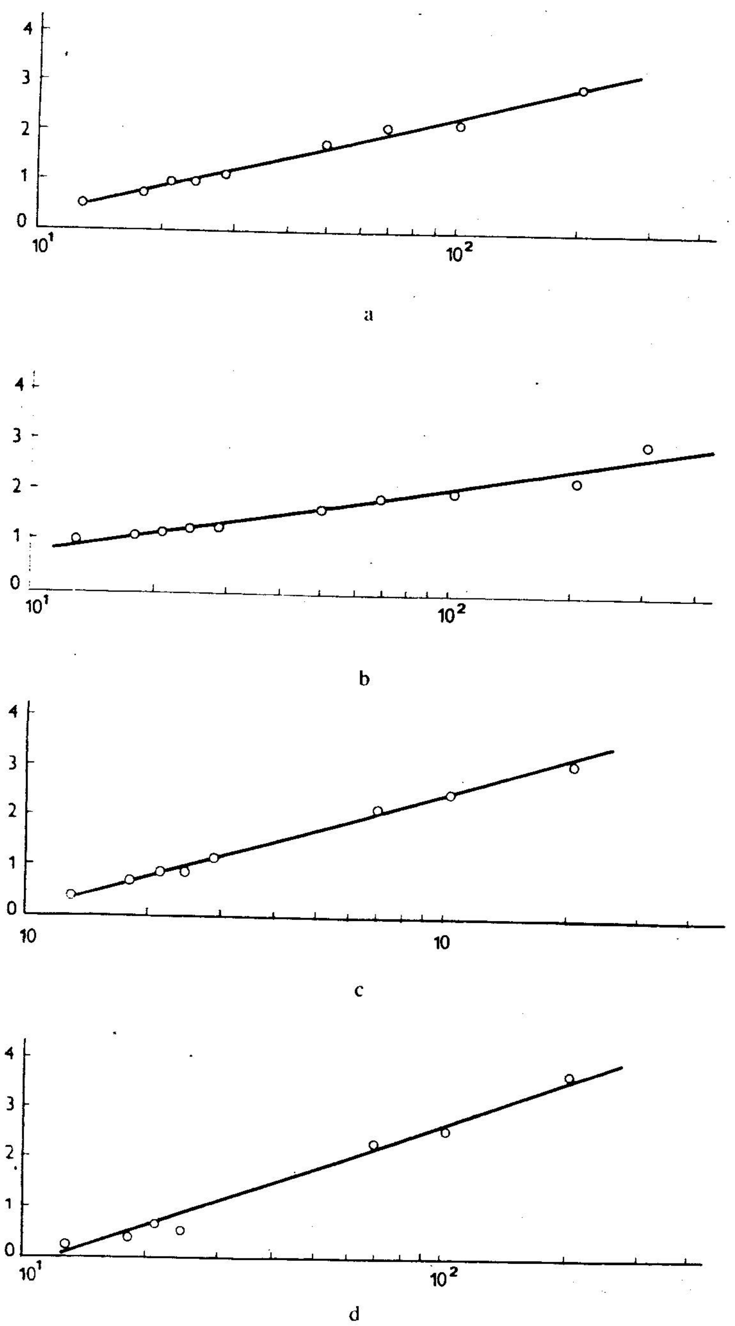
p_{lab}	w^p and $\bar{w}^p$	y^p	$\bar{y}^p$
12.88	0.923	−1.4	−2.87
18	1.064	−1.0	−2.8
21.08	1.114	−0.83	−0.9
24.12	1.197	−0.49	−0.9
28.5	1.220	−0.2	−0.2
50	1.650	+0.25	+0.25
69	1.870	+0.7	+0.41
102	1.987	+1	+0.42
205	2.234	+2	+0.7

and

$$\log q_i^p = \bar{w}^p + \bar{y}^p i \quad (10b)$$

respectively, where w^p, $\bar{w}^p$ and y^p, $\bar{y}^p$ are given in Table 4.

From Table 4 we can see that the values of $\bar{y}^p$ and y^p increase proportionally with the increasing laboratory momentum. In order to extrapolate our results it is necessary to know the functions $y^p = f(p_{lab})$ and $\bar{y}^p = f(p_{lab})$, respectively, in the



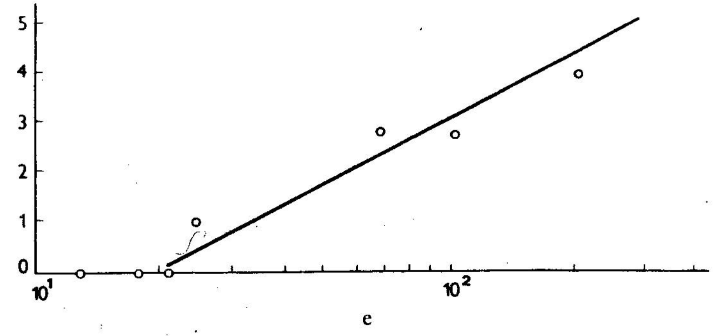


Fig. 1. The plots of the transition probabilities q_1, q_2, q_3, q_4 and q_5 versus the laboratory momentum (curves a, b, c, d and e).

whole region of the accelerator energies. The values of y^p and $\bar{y}^p$ as functions of the laboratory momentum are plotted in Fig. 4 and 5. We can see from Fig. 4 that one can write the function $y^p = f(p_{lab})$ in the form of a linear function

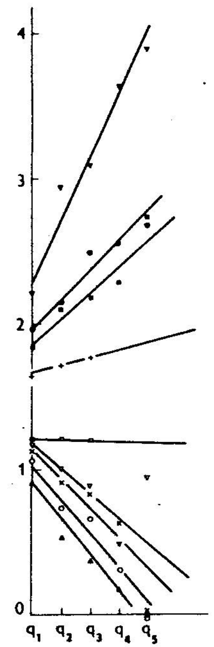


Fig. 2. The values of the transition probabilities for the increasing index i in the linear-linear scale. The parameter is the laboratory momentum.

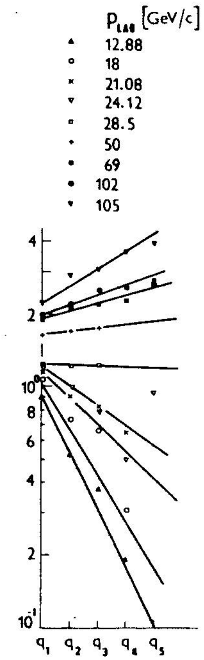


Fig. 3. The values of the transition probabilities for the increasing index i in the log-linear scale. The parameter is the laboratory momentum.

$$y^p = y_0^p + 0.9 \log p_{lab} \tag{11}$$

whereas (according to Fig. 5) the function $\bar{y}^p = f(p_{lab})$ can be divided into two parts

$$\bar{y}^p = \bar{y}_0^p + 3 \log p_{lab} \qquad p_{lab}\ 12.8,\ 30\ \text{GeV/c}$$

$$\bar{y}^p = \bar{y}_0 + 0.25 \log p_{lab} \qquad p_{lab}\ 40,\ 202\ \text{GeV/c}$$

For the values of the laboratory momentum $p_{lab} = 31$ GeV/c and 38.5 GeV we get $y^p = 0$ and $\bar{y}^p = 0$, respectively, i. e. for these momenta the transition probabilities are independent of the indices i. In other words, we have

$$q_1 = q_2 = \ldots = q_n$$

and the multiplicity distribution becomes the Poisson one. The existence of the simple phenomenological functions [7], [8], [10], [11] makes it possible to describe the multiparticle process in terms of transition probabilities and shows that these exhibit a great regularity which can be used by the determination of the internal dynamics of a hadronic collision. Further we can find the multiplicity by the

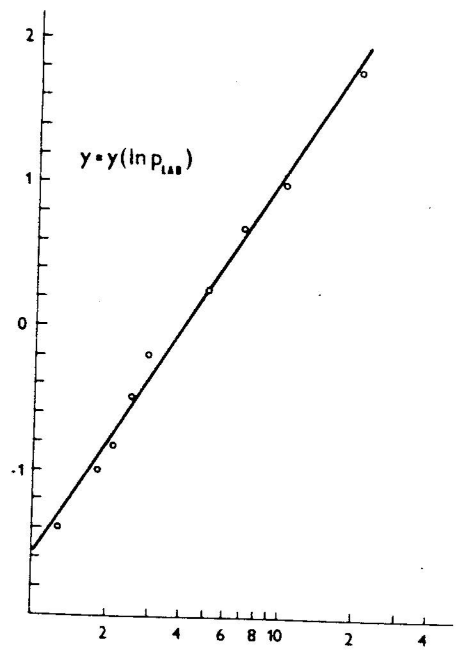


Fig. 4. The value of slope y from Eq. (10a) as a function of the laboratory momentum.

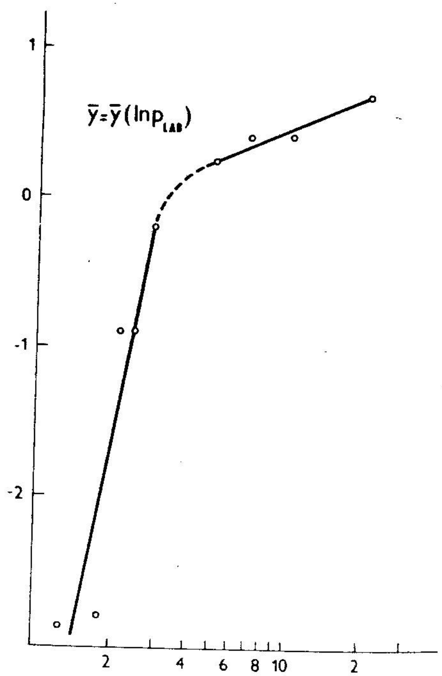


Fig. 5. The value of the slope y from Eq. (10b) as a function of the laboratory momentum.

extrapolation of the distributions for such high laboratory momenta for which they have not been experimentally determined yet.

Note that in the range of the laboratory momentum from 21 GeV/c to 102 GeV/c, the transition probabilities can be written in the following approximative form

$$q_i = q_1 + \varepsilon_i,$$

where ε represents a small quantity. Inserting this into the formula for the final probabilities we get multiplicity distributions which fit the experimental ones quite well [9].

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