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ISOMERIC RATIOS IN THE MASS RANGE $107 < A < 143$ FOR $(n, 2n)$ REACTIONS AT 14.8 MeV

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Based on the method of Huizenga and Vandenbosch, the calculations of the isomeric cross-section ratios for $(n, 2n)$ reactions at 14.8 MeV neutron energy were performed in the mass range $107 < A < 143$ using different nuclear level density approaches. Theoretical predictions were found to be in reasonable agreement with the experimental data. Present accuracy of experimentally determined isomeric ratios is, however, insufficient to prefer any of the four level density models used.

ИЗОМЕРНЫЕ ОТНОШЕНИЯ В ОБЛАСТИ МАСС $107 < A < 143$ ДЛЯ РЕАКЦИЙ $(n, 2n)$ ПРИ ЭНЕРГИИ 14.8 МэВ

В области масс $107 < A < 143$ проведены вычисления изомерных соотношений поперечного сечения для реакций $(n, 2n)$ при энергии нейтронов 14.8 МэВ, основанные на методе Хуйзенги и Ванденбоша с использованием разных приближений плотности ядерных уровней. Установлено, что теоретические предсказания находятся в разумном согласии с экспериментальными данными. Данная точность экспериментально определённых изомерных соотношений является однако недостаточной для того, чтобы предпочесть какую-либо из четырёх использованных моделей плотности уровней.

I. INTRODUCTION

The isomeric state 11/2 is known in several spherical odd-A nuclei in the mass range $107 < A < 143$. In these nuclei isomerism is caused by the position of the $h_{11/2}$ — single — particle state with respect to either the $s_{1/2}$, $d_{3/2}$ or $d_{5/2}$ — states requiring an E3 or M4 radiative transitions [1].

Such odd-A nuclei can be formed in an $(n, 2n)$ reaction with 14—15 MeV neutrons. Extensive data for cross-sections on these nucleides are found in literature [2]. On the basis of a statistical model Vandenbosch and Huizenga [3] have developed a method for calculating the theoretical isomeric cross-section ratios (ICSR). This method is restricted to compound type reactions only. The $(n, 2n)$ reaction is a good example of the compound process.

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The nuclear level density is one of the fundamental quantities in the calculation of ICSR. It seems of interest to carry out theoretical calculations with different models of level densities in order to test the ability of the model for describing the experimental results.

The aim of the present paper was to test the validity of the four theoretical nuclear level densities for calculations of the isomeric ratios in 25 nuclei ($^{107, 109}$Pd, $^{111, 115}$Cd, $^{117, 119, 121, 123}$Sn, $^{119, 121, 123, 127, 129}$Te, $^{129, 131, 133, 135}$Xe, $^{131, 133, 135, 137}$Ba, $^{137, 139}$Ce, ^{141}Nd, ^{143}Sm) having the same isomeric state 11/2 and compare them with the averaged experimental results for $(n, 2n)$ reactions at 14.8 MeV. No such systematic calculations through-out large region of mass numbers have been carried out.

II. ADOPTED MODELS OF NUCLEAR LEVEL DENSITIES

The dependence of the nuclear level density of the angular momentum I is expected to have the functional form [3]

$$\varrho(I) = (2I + 1) \exp[-I(I+1)/2\sigma^2]\varrho(0), \quad (1)$$

where the quantity $\varrho(0)$ is the density of levels with $I = 0$ and contains most of the dependence of the nuclear level density on the excitation energy. The quantity σ is the spin cutt-off parameter which is a model-dependent quantity [4]. In the present section we shall describe briefly four theoretical models of nuclear level densities: the shifted Fermi gas (SFG) [5—7], independent pairing (IP) [5, 7, 8] superconductivity (S) [5, 7, 9, 10] and the Gilbert—Cameron (GC) [11].

II.1. Shifted Fermi gas model (SFGM)

The simplest theoretical model is the Fermi gas model. The equation of state for this model is

$$U = at^2 - t, \quad (2)$$

where U is the excitation energy, t the thermodynamic temperature, d the level density parameter (equal to $\pi^2 g/\sigma$, g is the density of the single particle levels). In this model, the quantity σ is related to the rigid body moment of inertia B_r by

$$\sigma^2 = \frac{B_r t}{\hbar} = 0.01378 A^{5/3}\,(\mathrm{MeV})^{-1}\,t\,(\mathrm{MeV}). \quad (3)$$

From Eq. (2) and the relation (3) the angular momentum dependence of the level density at different excitation energies can be determined. This model was modified to take into account pairing interactions.

The effective excitation energy U is related with the excitation energy E by

$$U = E - \varepsilon\delta \tag{4}$$

where δ is the pairing energy and $\varepsilon = 0, 1, 2$ for odd-odd, odd mass number and even-even nuclei, respectively. We shall call this modified Fermi gas model a Shifted Fermi gas Model (SFGM).

II.2. Independent pairing model (IPM)

Ericson [4] and Lang and Le Couteur [8] have modified the Fermi gas model by including a simple form of pairing interaction. In this model the energy required to break a coupled pair of nucleons 2δ is taken to be independent of the excitation energy. We shall call this model the Independent Pairing Model (IPM) since it assumes that the pairing interaction for a particular pair is independent of other pairs. In accordance with this model the spin cut-off parameter is given by

$$\sigma^2 = \left(\frac{B_r}{\hbar^2}\right) t \exp(-0.874\delta/t). \tag{5}$$

The thermodynamic temperature t is given by the same equation (2) of the SFGM. The effective excitation energy U is related to the excitation energy E by

$$U = E + \frac{a\delta^2}{4.8} + (2 - \varepsilon)\delta, \tag{6}$$

ε has the same meaning as in the SFGM.

The moment of inertia implied by IPM is considerably less than the moment of the rigid body even at quite high excitation energies.

II.3. Superconductor model (SM)

In the superconductivity model of the level density the following expression for the spin cut-off parameter was derived by Lang [7]:

$$\sigma^2 = \left(\frac{B_r}{\hbar^2}\right) tA(U). \tag{7}$$

where the function $A(U)$ and the thermodynamic temperature t are tabulated in Ref. [9].

The effective energy U is related to the actual excitation energy E by the equation

$$U = E + \varepsilon\delta, \tag{8}$$

where $\varepsilon = 0, 1, 2$ for even-even, odd mass number and odd-odd nuclei, respectively.

II.4. Gilbert—Cameron model (GCM)

Gilbert and Cameron [11] give a special form of the nuclear level density $\varrho(U)$, namely a constant temperature level density at low excitation energies and a Fermi gas type level density at high excitation energies. If the ground state of the gas is approximated by the reference mass of odd-odd nuclei, then U is given by

$$U = E - \Delta \tag{9}$$

where Δ is the nucleon pairing energy, which may be subdivided into separate contributions from neutrons and protons

$$\Delta = P(N) + P(Z), \tag{10}$$

$P(Z)$ and $P(N)$ are pairing energies tabulated in Ref. [11]. The pairing correction has the practical justification of removing even-odd effects from the level density parameter a. The shell effects are accounted for by use of the shell correction parameters of Cameron's semi-empirical mass law. The total shell correction S is again subdivided into proton and neutron contributions

$$S = S(Z) + S(N), \tag{11}$$

$S(Z)$ and $S(N)$ are shell corrections tabulated also in Ref. [11]. Gilbert and Cameron have found that a linear correlation between a/A and S exists

$$a/A = 0.00917S - 0.142. \tag{12}$$

Thus the pairing corrections and shell corrections derived from the semi-empirical mass law remove the even-odd effects and shell effects from the level density parameter. The spin cut-off parameter σ^2 in the Gilbert—Cameron model is given by the formula

$$\sigma^2 = 0.0888(aU)^{1/2}A^{2/3}, \tag{13}$$

where A is the mass number of the compound nucleus.

III. CALCULATIONS

To simplify the calculations the following approximations were made:

1) The influence of competitive (n, p), (n, α) reactions has been neglected because their cross sections in the considered mass region $107<A<143$ are always much smaller than the $(n, 2n)$ reaction cross section [12].

2) As it follows from the evaporation theory [13] each neutron in the compound nucleus decay carries out an average energy

$$\langle \varepsilon_n \rangle = 2\tau, \tag{14}$$

where τ is the nuclear temperature which with a good approximation can be calculated from the relation between the excitation energy U and the nuclear temperature:

$$U = a\tau^2 - 4\tau. \tag{15}$$

The energy U was corrected by the pairing energy.

3) Other parameters important to the calculation are the number, the energy and the multipolarity of the gamma rays in the cascade. The method employed for estimating the number of gamma rays emitted from an excited nucleus of the energy E_c is based on the work of Mollenauer [14]. He has found experimentally that the average gamma energy ranged from 1—1.6 MeV.

The γ-cascade was assumed to consist of 1.3 MeV dipole γ-rays except for the last transition. The latter, not limited to dipole multipolarity, dissipated residual excitation energy and formed isomeric states [3, 5].

4) During γ-ray deexcitation the value of spin cut-off parameter σ was considered constant.

Calculations of ICSR were made with the following choice of parameters:

a) the transmission coefficients for neutrons were taken from Ref. [15]; b) the values of the pairing energy δ in SFG, IP and S models for correcting the energy U were calculated from an empirical expression given by Nemirovsky and Adamchuk [16]

$$\delta = \frac{11.56}{N^{0.552}}, \tag{18}$$

where N represents the neutron number; c) the level density parameter a appearing in Eq.(2) and Eq. (15) is expected on theoretical grounds to be related to the mass number A [17]. The calculations done in this work use $a = A/8$ MeV^{-1}; d) the rigid-body moment of inertia B_r was calculated using a nuclear radius parameter $r_0 = 1.2$ fm [5]. For SFGM lower values of B_r are sometimes used. Thus a reduced value of the moment of inertia equal to $0.65B_r$ was also used in our calculations [18].

Table 1

Isomeric cross-section ratios σ_m/σ_{tot} for $(n, 2n)$ reaction at 14.8—0.4 MeV neutron energy

No	Target nucleus	Cross-section for excitation of isomeric state σ_m (mb)	Total n, $2n$ cross-sect. σ_{tot} (mb)	Isomeric cross-section ratios: Experiment	Theory: SFGM $B=B_r$	Theory: SFGM $B=0.65B_r$	Theory: IPM	Theory: SM	Theory: GCM
1.	^{108}Pd	484± 27	1299± 70	0.37± 0.03	0.468	0.208	0.281	0.316	0.476
2.	^{110}Pd	515± 23	1299± 70	0.40± 0.03	0.429	0.222	0.279	0.394	0.497
3.	^{112}Cd	676± 52	1580±237	0.43± 0.07	0.475	0.192	0.277	0.329	0.483
4.	^{116}Cd	635± 31	1447± 56	0.44± 0.03	0.713	0.499	0.558	0.649	0.708
5.	^{118}Sn	942± 60	1650±247	0.57± 0.09	0.604	0.385	0.468	0.508	0.593
6.	^{120}Sn	1444±210*)	1735±260**)	0.83±017	0.614	0.383	0.482	0.531	0.600
7.	^{122}Sn	875±135	1794±269**)	0.49± 0.10	0.627	0.410	0.479	0.569	0.605
8.	^{124}Sn	562± 21	1810±270**)	0.31± 0.05	0.638	0.434	0.495	0.587	0.604
9.	^{120}Te	535± 85	1220±131	0.44± 0.08	0.677	0.434	0.587	0.546	0.706
10.	^{122}Te	670± 62	1473± 83	0.45± 0.05	0.596	0.399	0.484	0.466	0.608
11.	^{124}Te	980±100	1700±255**)	576 ± 0.10	0.616	0.395	0.485	0.526	0.608
12.	^{128}Te	949±150	1604±151	0.59± 0.11	0.640	0.441	0.504	0.588	0.612
13.	^{130}Te	811± 41	1455± 55	0.56± 0.03	0.648	0.481	0.517	0.609	0.610
14.	^{130}Xe	1435±130*)	1695±254**)	0.85± 0.15	0.632	0.446	0.492	0.553	0.610
15.	^{132}Xe	775± 65	1739±261**)	0.45± 0.07	0.643	0.449	0.510	0.588	0.618
16.	^{134}Xe	665± 80	1791±269**)	0.37± 0.07	0.655	0.474	0.529	0.613	0.605
17.	^{136}Xe	750± 50	1700±100	0.44± 0.04	0.666	0.502	0.539	0.641	0.533
18.	^{132}Ba	696±120	1685±253**)	0.41± 0.09	0.531	0.298	0.363	0.410	0.533
19.	^{134}Ba	833± 40	1720±258**)	0.48± 0.08	0.639	0.442	0.413	0.558	0.628
20.	^{136}Ba	1193± 67*)	1725±259**)	0.69± 0.11	0.643	0.449	0.500	0.565	0.616
21.	^{138}Ba	1948±100	1900±285	0.55± 0.10	0.659	0.487	0.528	0.617	0.616
22.	^{138}Ce	976± 66	1824±150	0.53± 0.06	0.640	0.464	0.53	0.56	0.624
23.	^{140}Ce	907± 44	1785± 47	0.51± 0.03	0.651	0.491	0.549	0.598	0.625
24.	^{142}Nd	646± 25	1646± 54	0.39± 0.02	0.640	0.488	0.534	0.559	0.630
25.	^{144}Sm	540± 20	1473± 32	0.37± 0.02	0.626	0.456	0.548	0.506	0.636

*) With contribution of inelastic scattering on neighbouring stable isotope

**) Evaluated using Pearlstein's procedure [18].

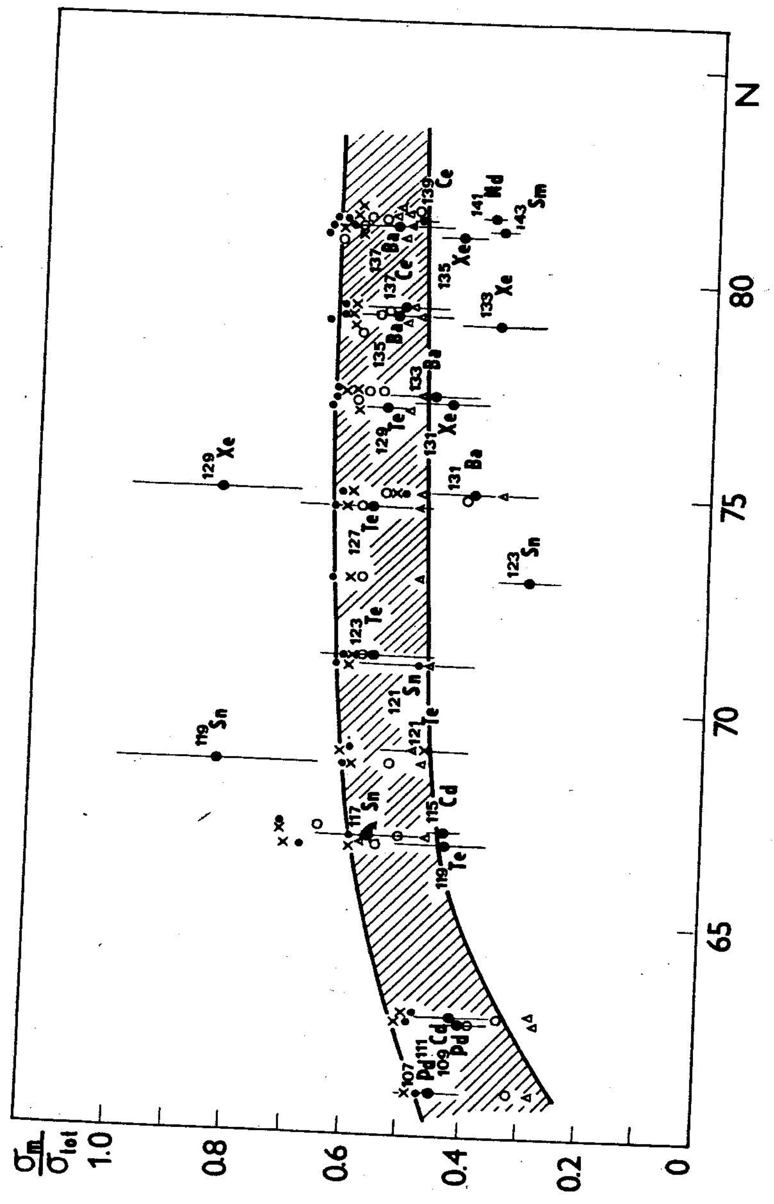

Fig. 1. Comparison of the weighted averaged experimental isomeric cross-section ratios σ_m/σ_{tot} for $(n, 2n)$ reactions at a 14.8—0.4 MeV neutron energy in the mass range $107 < A < 143$ with theoretical calculations using different level density models (● — SFGM, × — GCM, ○ — IPM, △ — SM. ● — experimental values), plotted versus the neutron number N.

IV. NUMERICAL RESULTS

Isomeric cross-section ratios $\sigma_m/(\sigma_m + \sigma_g)$ calculated with these models for 25 nuclei are listed in Table 1. The experimental values of isomeric ratios compiled for the 14.8—0.4 MeV neutron energy are also shown in Table 1 for comparison. From the measured values of the $\sigma_m(n, 2n)$ and $\sigma_{tot}(n, 2n)$ we have obtained the weighted means; the weights being related to the reported statistical error of each measurement. For the compilation we have made use of the CINDA 74 listing [2].

When only the σ_m cross-section was known through experiment the σ_{tot} was evaluated using the Pearlstein [19] theoretical values. The errors of these total cross sections have been estimated to be as large as 15 %. On the basis of the results of Table 1 we have plotted in Fig. 1 the experimental ICSR (with errors) and the theoretical ones predicated by different models versus the neutron number N.

We see that in Fig. 1 almost all the data are encompassable within an arbitrary drawn band whose half-width in the flat portion corresponds to a deviation of about 15 % from the mean equal to 0.5 and in the early increasing part to about 20 %. Fig. 1 may be therefore used as a rough guide for predicting unknown ICSR with an uncertainty of about 20 % for the lightest nuclei and 15 % for heavier nuclei.

The calculated ICSR for SFGM (with $B = B_{rigid}$) and GCM give nearly the same results. On the other hand differences between results of SM and IPM are rather small. We can only say that every one of the used models is able to explain satisfactorily the experimental ICSR within experimental errors.

We can conclude that the accuracy of the experimental ICSR is generally unsufficient to test the various nuclear density models.

At this point we must note that significant differences exist for experimental ICSR in which a contribution of the inelastic scattering cross section on neighbouring stable isotopes was not separated from the cross section for the excitation of isomeric activity σ_m on the $(n, 2n)$ reaction.

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