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THE VOLUME PINNING FORCE FOR COMBINED ATTRACTIVE-REPULSIVE FLUX-LINE DEFECT INTERACTION POTENTIALS

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It is suggested that the form of the interaction potential between defects and flux lines can play an important role for the pinning of flux lines in superconductors and thus for the value of the critical current density. In some cases (e. g. statistically distributed dense point defects, dislocations), the defect — flux line interaction potential can be of a very complicated form. We calculate the volume pinning force under the assumption that the elementary interaction force between the defect and the flux line is a combination of attractive and repulsive labels and we obtain large differences in the dependence on the order of the potentials (attractive-repulsive or repulsive-attractive, respectively). For the repulsive-attractive combination, the deviations from the quadratic dependence of the volume pinning force on the maximum elementary interaction force are considerable, mainly for smaller elementary interaction forces, whereas this dependence is nearly ideally quadratic for the attractive-repulsive combination of the interaction potentials.

The possibility of explaining the unexpected increase of the volume pinning force at small defect concentrations in superconductors with statistically distributed point defects is given.

ОБЪЁМНАЯ СИЛА ПИННИНГА ДЛЯ КОМБИНИРОВАННЫХ ОТТАЛКИВАЮЩЕ-ПРИТЯГИВАЮЩИХ ПОТЕНЦИАЛОВ ВЗАИМОДЕЙСТВИЯ МЕЖДУ ЛИНЕЙ ПОТОКА И ДЕФЕКТОМ

Предполагается, что форма потенциала взаимодействия между дефектами и линиями потока могут играть важное значение в пиннинге линий потока в сверхпроводниках, и вследствие того для значения критической плотности тока. В некоторых случаях (например, статистически распределённые плотные точеные дефекты, дислокации) потенциал взаимодействия между дефектом и линией потока может иметь очень сложный вид. В данном случае рассчитаны обёмные силы пиннинга в предположении, что элементарная сила взаимодействия между дефектом и линией потока является комбинацией притягивающего и отталкивающего типов. Мы получили большую разницу в зависимости от последовательности потенциалов (притягивающий-отталкивающий или отталкивающий-притягивающий). Для комбинации отталкивание — притяжение отклонения от квадратичной

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зависимости обёмной силы пиннинга на максимиме силы элементарного взаимодействия являются значительными, прежде всего для малых сил элементарного взаимодействия, в то время как эта зависимость близка к точно квадратичной для комбинации притягивающе-отталкивающих потенциалов взаимодействия. Приводится возможное объяснение непредвиденного роста обёмной силы пиннинга для малых концентраций дефектов в сверхпроводниках со статистически распределёнными точечными дефектами.

I. INTRODUCTION

The mixed state [1] of type II superconductors — consisting of quantized flux lines (flux tubes, flux threads, vortices), arranged in a regular lattice almost in the whole range of magnetic fields between the lower (H_{c1}) and the upper (H_{c2}) critical magnetic fields — is unstable against applied forces on the flux lines. Under the influence of applied forces (e. g. the Lorentz force in case of the transport current perpendicular to the flux lines) the flux lines begin to move. This motion is accompanied by energy losses (thus by heating the superconductor), which would destroy the superconducting state — the material would go normal.

It is proved beyond doubt that the ability of type II superconductors to carry high electric current with negligible losses is caused by the presence of defects and inhomogenities in the crystal lattice of the superconductor.

The crystal imperfections interact with the flux lines by various interaction mechanisms (from which the elastic interaction, the so-called $\Delta\varkappa$-interaction and the diamagnetic interaction are the most important), preventing thus the free motion of the flux line lattice (pinning). The most important defects, which can lead to very high critical current densities, are the dislocations and dislocation networks, phase and grain boundaries, precipitations, defect cascades (e. g. after neutron, light or heavy ion irradiation), etc.

All pinning mechanisms can be understood qualitatively (and some of them also quantitatively) by considering the flux lines as consisting of normal kernels of the dimension ξ (the coherence length of the superconductor), around which the flux quantum of the vortex is screened by the microscopic supercurrents in the distance λ (the penetration depth of the magnetic field).

However, the problem of calculating the volume pinning force — even at a known elementary interaction force between the flux lines and the defects — is a very complicated statistical problem. This is mainly caused by the fact that for most of the known interaction mechanisms between the defects and the flux lines the maximum elementary interaction force between the individual flux line K_m is smaller than the interaction force between the neighbouring flux lines.

The repulsive interaction forces between the flux lines of the flux line lattice are given in principle by the interaction of their electromagnetic fields (it is some kind of dipole-dipole interaction). This interaction causes the arrangement of flux lines

in the form of a regular lattice (the same arrangement is obtained for magnetic needles in a closed tube). As a consequence of the interaction between the flux lines, the flux line lattice has its elastic properties, which can be described by means of the elastic constants, as in the crystal lattice.

Since the flux line lattice spacing (i. e. the distance between the flux lines), as well as the interaction force between the flux lines are determined by the magnetic induction B in the superconductor, the elastic constants of the flux line lattice are also functions of B.

Through strong flux line — flux line interactions, each deformation of the flux line near the defect results in a complicated response of the flux line lattice near the defect. The force on the flux line lattice must involve therefore the function of the elastic constants c_{ii} of the flux line lattice.

Moreover, for statistically distributed defects, the individual forces on the flux lines are different (some of the defects do not even interact with any flux line), and thus the calculation of the force in a larger volume (i. e. the volume pinning force) is a very complicated statistical problem.

One can illustrate this fact by the following extreme cases. For completely flexible flux lines (i. e. by neglecting the elastic properties of the flux line lattice), the individual forces on the flux lines would be summed up and we would have

$$F_v = \Sigma K_m .$$

In case of very strong elastic properties of the flux line lattice (in the limit case: rigid, i. e. unflexible, flux lines) or for an ideally homogeneous distribution of dense defects, we would obtain no volume force on the flux line lattice. The forces on the single flux lines would cancel each other in a larger volume of the superconductor. We need in this case an infinitesimal force for moving the flux line lattice, as the energy of the lattice is the same independently of the coordinates of the flux lines.

The realistic situation is somewhere between these two extreme cases: the flux lines are not rigid, they can be deformed near the defects, but these deformations are strongly reduced by the elastic properties of the flux line lattice (and also by the line energy of the flux lines if the increase of the flux line length at this deformation is considerable — this happens only for very strong interaction forces defect — flux line, which we do not consider in the present paper).

Labusch [2] has contributed in a basic way to the solution of this problem. He obtained for the volume pinning force

$$F_v = NK_m^2 f(c_{ii}, B), \qquad (1)$$

where N is the density of "obstacles" for the flux lines, K_m the maximum elementary interaction force between the individual defect and the individual flux line and f is the function of the elastic constants and of the magnetic field.

The "obstacles" in the Labusch theory can act as repulsive potentials (barriers), as well as attractive potentials (wells). As noted by Labusch [2], the resulting volume pinning force is the sum of the volume pinning force from both types of the interaction (if they are present and they do not interact yet).

The somewhat surprising quadratic $F_v(K_m)$ dependence (and some consequences of this result [3], as well as another result of the Labusch calculation — the existence of the threshold value for the flux line deformation below which the defect cannot contribute to the volume pinning force) is proved experimentally [4—7]. The resulting volume pinning force is much smaller than the sum of the elementary interaction forces on the flux lines.

Both results of Labusch can be explained very convincingly by using model potentials for the interaction defect — flux line [8, 9]. The finite volume pinning force for randomly distributed defects comes from the asymmetric distribution of flux lines on both sides of the defects. We explain this by considering repulsive interaction defect — flux line (some of these explanations will be more understandable in Sections 3 and 4).

As the flux lines come to one side of the defects (they are pressed onto the defects in the direction of the applied force), some of them are held by the defects on this side, although they would be already on the other side of the defects without the presence of the defects. The defects act on more flux lines on one side of the defects than on the other one. Because the individual forces show in different directions on both sides of the defects, we obtain a net volume force on the flux line lattice.

Since the spatial interval of these asymmetric flux line positions d increases with the maximum elementary interaction force K_m and the force on the flux lines is nearly K_m for most of the flux lines lying in these asymmetric positions, the resulting volume pinning force $F_v \sim K_m^2$.

We obtain also the existence of the threshold value for the pinning: at a smaller K_m the defect does not cause any asymmetry, the flux lines are on the same sides of the defects, as they would be without the presence of the defects.

Generally in all papers dealing with problems of pinning, the interaction between defects and flux lines was supposed to be given by a simple potential barrier of a potential well for the flux lines. This can be sometimes a very rough assumption and can lead in some cases to quantitatively untrue results. We give some arguments for this in the next Section.

Besides, the identical contribution of defects with attractive and repulsive interaction potentials — which was considered to be fulfilled at the application of the Labusch result (1) — could be rejected by calculations with a model as well as realistic interaction potentials defect — flux line [9, 10, 11].

In the next Section we give some examples for the cases where the assumption of single attractive or repulsive interaction could be incorrect. In the two following Sections, we calculate the volume pinning force for combinations of attractive and

repulsive interaction potentials and in Section 5 we discuss the obtained results, mainly with respect to statistically distributed point defects.

II. THE RANDOMLY DISTRIBUTED POINT DEFECTS AS AN EXAMPLE

In our previous work [12], we were able to explain most features of the measured volume pinning forces in niobium with statistically distributed point defects (Frenkel pairs) by considering the defect density fluctuations as the main sources of the pinning. Namely, statistically distributed defects of small dimensions (with respect to the coherence length, or the penetration depth of the magnetic field, in dependence on the interaction mechanism between defects and flux lines) should not contribute to the volume pinning force, as mentioned above. In most cases there are many defects in the interaction range of each flux line, the number of them on both sides of the flux line is nearly equal, their action on the flux line cancels out, therefore the total force on the flux line should be zero.

If seen in this light, the possibility of measuring finite volume pinning forces on the flux line lattice in superconductors with statistically distributed point defects [12] came somewhat surprisingly. But the idea that in the spatial distribution of the point defects statistical deviations from the mean defect density $\bar{N}$ can be expected, gave the explanation of the linear dependence of the volume pinning force F_v on the mean defect density $\bar{N}$ at small defect concentrations.

Since the mean square deviation of the number of defects acting on a flux line element

$$\sqrt{\overline{n^2} - \bar{n}^2} \approx \sqrt{\bar{n}}$$

determines in principle the elementary interaction force between the flux line element and the volume element in which the flux line element is placed (these volume elements are supposed as elementary defects in the following [12]), and $F_v \sim K_m^2$, we obtain the desired $F_v \sim \bar{N}$ dependence.

In paper [12], quantitatively also the saturation and the following decrease of the volume pinning force at larger defect concentrations, as well as the magnetic field dependence of the volume pinning force at a given defect concentration, could be explained. However, there remained two regions of discrepancies between the theoretical and the experimental results.

a) The increase of the volume pinning force at small defect concentrations is stronger than calculated from the simple statistical considerations (mainly in smaller magnetic fields).

b) The volume pinning force drops to zero at magnetic fields smaller than the upper critical field B_{c2} (from theoretical calculations, $F_v \to 0$ at B_{c2} as $(1-b)^2$, where $b = B/B_{c2}$).

These deviations could not be explained by considering the cut-off of small interaction forces below the threshold value [2].

We tried to explain both these discrepancies by considering some correlations in the defect numbers of neighbouring characteristic volumes [13]. These correlations set the statistical theory of pinning on point defects on a more solid basis.

In these considerations, the elementary interaction force is given by the difference of the defect numbers on both sides of the flux lines, which is a very realistic assumption.

Moreover, other distributions of the defect numbers n (in the characteristic volumes) than the Gauss distribution, and the influence of the cut-off elementary interaction forces below the threshold value could be simply considered.

But, both the statistical correlations and the special defect distributions in the characteristic volumes, as well as the mechanism of the cut-off did not lead to the explanation of the effects which cause the discrepancies mentioned above.

The main contributions to the volume pinning force are given by the volume elements with $n' \approx \bar{n} \pm \sqrt{\bar{n}}$ defects (here $\bar{n}$ is the mean number of defects in the characteristic volume $\approx \xi^3$, which corresponds to the mean defect density $\bar{N}$). The elementary forces from volumes with $|n-\bar{n}| < \sqrt{\bar{n}}$ decrease rapidly with the difference $|n-\bar{n}|$, as well as the statistical weight of the numbers $|n-\bar{n}| > \sqrt{\bar{n}}$.

In [12, 13] we have assumed that these volumes act on the flux lines as given in Fig. 1a. The flux lines "feel" the potential barrier (or potential well) from the given volume, if $n > \bar{n}$ (or $n < \bar{n}$), whereas the neighbouring volume elements should have $n = \bar{n}$. This assumption is very rough, as from statistical reasons we can expect that the defects which are missed (or which are above the mean value) in some element, will be placed (or missed) most probably in the neighbouring volumes of this element.

The interaction energy with respect to the flux line positions (under flux line position or flux line coordinate we mean in the following always the coordinate of the flux line centre) near these volume and the force on the flux line will therefore look like the functions in Fig. 1c.

As an intermediate step between the case of Fig. 1a and 1c, we consider the potential form and the force in the form of Fig. 1b, which has the advantage that the force can be described by a simple quadratic function (the equation for the equilibrium position of the flux lines is then a quadratic equation and like the model potential [9]), the considerations and the calculations of the volume pinning force are very lucid and illustrative.

Since more complicated forms of the interaction potential can be constructed with the aid of the potential forms of Fig. 1, we expect that these potential forms (i. e. the combination of the attractive and repulsive labels, where at least for the statistically distributed point defects both "turns" are equally probable) can give

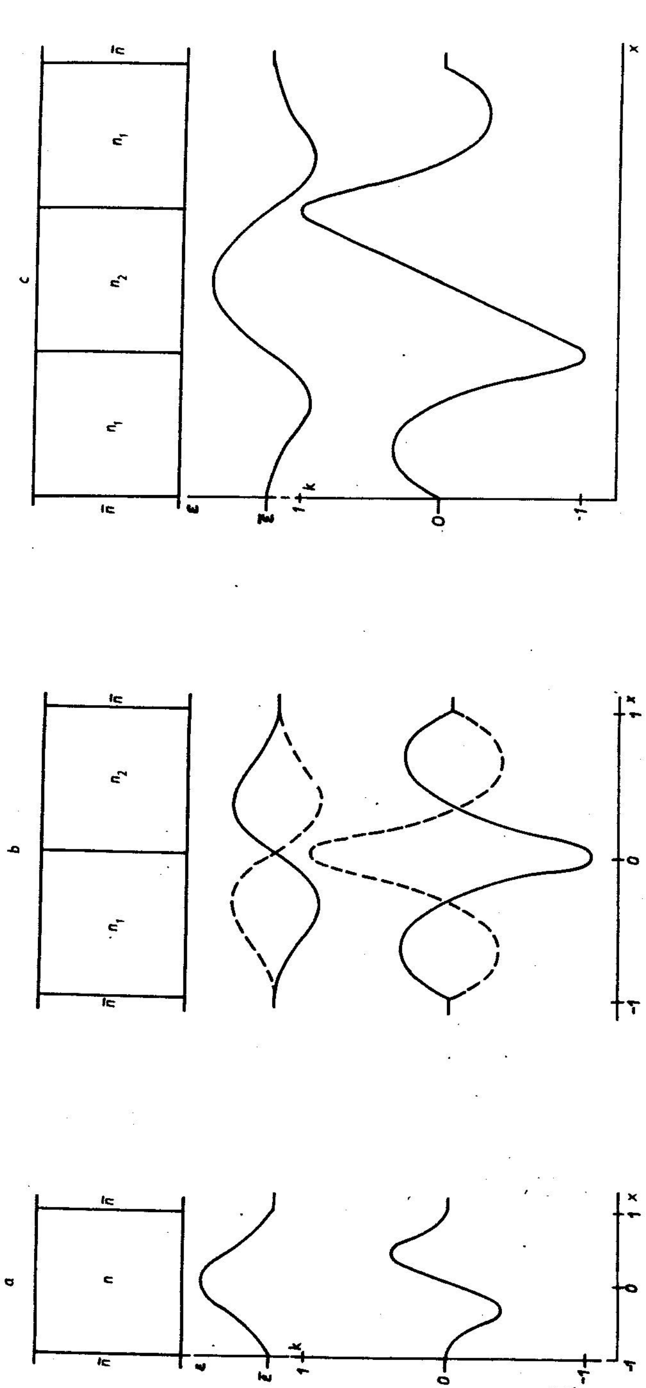


Fig. 1. The simple (a) and more complicated (b, c) interaction potentials (ε) and the corresponding interaction forces k between defects and flux lines. On the top of the figures the corresponding distributions of the defect numbers n in the volume elements $\approx \xi^3$ are given. $x = \zeta / A$, where ζ is the distance of the flux line centre from the centre of the defect, A — the interaction range of the defect. The case (a) demonstrates the volume element with $n \neq \bar{n}$, "embedded" in the superconductor with everywhere $n = \bar{n}$, $k = K/3K_m$ (K_m — the maximum elementary interaction force K). The case (b) demonstrates the neighbouring volume elements with more and less defect numbers than $\bar{n}$, respectively (we take for simplicity $n_1 - \bar{n} = -(n_2 - \bar{n})$ and again $n = \bar{n}$ in the neighbourhood of these elements). The full lines give the functions for the attractive-repulsive interaction potential (taken from the left), the dashed lines the repulsive-attractive combination of the potentials. In the case (c) the missed (or superfluous) defects of the characteristic volume element are placed (or missed) in the neighbouring volume elements.

some characteristic features of the statistical theory of pinning with more complicated interaction forms.

This form of the interaction potential is very probable also for dislocations (or possibly other defect structures), where, e. g. the electrostatic potential is without fail of this form (as on one side of the line dislocation there are more, on the other side fewer ions than the mean ion density in the crystal is).

The elementary interaction potential and the elementary interaction force are taken to be vanishing in the distance A, the interaction range of the volume element. Then we have

$$\varepsilon = \bar{\varepsilon}(1 + x - 2x^2 \operatorname{sign} x + x^3),$$

$$K = \pm K_m(1 + 3x^2 - 4x) = K_\pm,$$

where $x = \xi/A$, ξ is the distance of the flux line from the boundary between the volume elements (the origin of our coordinate system), K_m the maximum value of the interaction force ($K_m = 1$ for $x = 0$, the other two extremum values are $|K_m| = 1/3$ at $|x| = 2/3$, $\pm$ means the combination of the repulsive-attractive and attractive-repulsive potentials, respectively (dashed and full lines in Fig. 1b). $\bar{\varepsilon}$ is the energy of the flux line in volume with the mean defect density ($n = \bar{n}$).

We would like to mention that in our considerations, as well as in the discussion of the results, we suppose that the concrete form of the potential does not influence considerably the results, as in the case of the simple interaction potentials [9, 10].

III. THE COMBINATION OF THE REPULSIVE-ATTRACTIVE POTENTIAL

In the following considerations, we assume that under the influence of applied forces (e. g. the Lorentz force) the flux lines come to the defect (or to the defect structure) from the left, i. e. the defect begins to act on the flux line at $\xi = -A(x = -1)$. The coordinates of the flux lines at larger distances from the defect are signed Δ (or $a = \Delta/A$ in relative coordinates). These coordinates correspond to the flux line positions in which they would be without the presence of the defects.

In large volumes of superconductors, the coordinates a (i. e. the distance of the undeformed flux lines from the defect centre) are equally probable. This is very useful for the statistical summation of forces on the flux line lattice in larger volumes of the superconductor, i. e. also in calculating the volume pinning force (the resulting volume force on the flux line lattice from all defects in this volume).

In calculating the possible positions of the flux lines near the defect, it is more advantageous to work with the positions of the deformed flux lines x. These positions are determined by the equilibrium of the force from the defect and from the neighbouring flux lines, as mentioned in the Introduction.

The force from the defect is given by Eq. (2). The distortion of the flux line under the influence of this force causes the deformation of neighbouring flux lines in the flux line lattice (Fig. 2), and the interaction with these flux lines compensates the force from the defect.

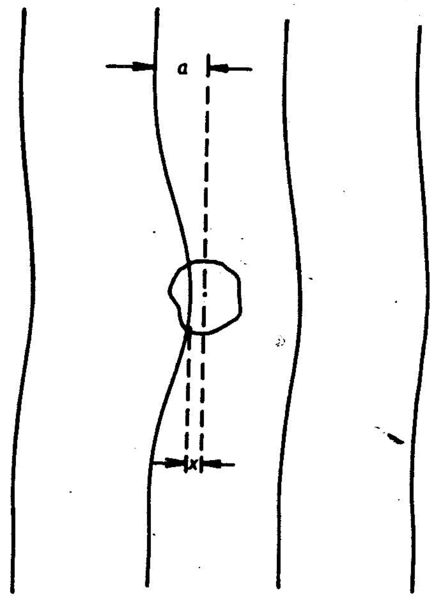


Fig. 2. The deformation of the flux lines near a defect (schematically), $a = \Delta/A$ — the "natural" (i.e. undeformed) position of the flux line, $x = \xi/A$ — the coordinate of the deformed flux line near the defect (A — the interaction range of the defect)

We assume that the flux lines are arranged in a nearly regular lattice in larger volumes of the superconductor (this is the so-called lattice approximation; other possible arrangements of the flux lines are, e. g. the fluid approximation and the rigid lattice approximation) and the response of the flux line lattice to the deformation is described by only one "effective" elastic constant σ which is a combination of the single elastic constants of the flux line lattice (so, e. g. for the GL parameter [14] $\varkappa \gg 1$ and $B \approx B_{c2}/2$, there is $\sigma \approx \sqrt{c_{44}c_{66}}$).

Our further considerations hold in the so-called dilute limit, i. e. the defects act "individually" on the flux lines (this condition is in our case of statistically distributed dense point defects fulfilled for magnetic fields not too close to B_{c2}, where the flux line kernels do not overlap strongly).

The equilibrium condition is then [9]

$$\pm(1 + 3x^2 - 4x) = \frac{A\sigma}{K_m}(x - a) = 2b(x - a), \qquad (3)$$

where the value $1/b = 2K_m/\sigma A$ is a very important quantity (besides the characterization of the defect parameters K_m, A, it is a function of the magnetic field, as $\sigma(B)$), as we shall see later.

Now, we turn our attention to the case K_+, which means the repulsive-attractive combination of the interaction potential. The solutions of Eq. (3) are then

$$x_- = \frac{b - 2 \overset{(+)}{-} (b^2 + 1 - 4b - 6ba)^{1/2}}{3} \qquad (4)$$

$$x_+ = \frac{b + 2 \overset{(+)}{-} (b^2 + 1 + 4b - 6ba)^{1/2}}{3} .$$

Here $x_- < 0$, $x_+ > 0$, and the "right" sign of the square roots is determined by the condition

$$x = -1 \leftrightarrow a = -1,$$

as for $x = -1$ the force on the flux line is zero.

The most important problem is now to find the intervals of the flux lines "natural" positions a, in which the individual solutions x_+ and x_- are possible. Thus, e. g. we have

$$x_+ = 0 \ \text{for} \ a = -1/2b = a_0,$$

therefore for x_+ only the values $a > a_0$ have to be considered.

As

$$x_-(a_0) = \frac{b - 2 - (b^2 + 1 - 4b + 3)^{1/2}}{3} = \begin{cases} 0 & \text{for } b > 2 \\ \dfrac{2b - 4}{3} & \text{for } b < 2 \end{cases}$$

there is a continuous transition between the solutions x_+ and x_- for $b > 2$, but no more for $b < 2$ (see Figs. 3).

We obtain for $b > 2$

$$x_+(+1) = +1,$$

therefore the flux lines are deformed continuously in the whole interaction range of the defect $(-A, +A)$.

For $b < 2$, the range of possible real x_- values is restricted by the condition of the non-negative term under the square root:

$$b^2 + 1 - 4b - 6ba \geq 0,$$

therefore only the undeformed flux line positions from the interval $a \in (-1, a_1)$ can lead to real x_- values, where

$$a_1 = (b^2 + 1 - 4b)/6b .$$

At $a = a_1$, we must have (for $b < 2$) an "abrupt" change of the flux line position, because then

$$x_+(a_1) = \frac{b + 2 - (b^2 + 1 + 4b - b^2 - 1 + 4b)^{1/2}}{3} = \frac{b + 2 - \sqrt{8b}}{3} > 0,$$

and the earlier position of the lux line was

$$x_-(a_1) = \frac{2b - 4}{3} < 0$$

(see Fig. 3b, c). Now, for $b > 1$ we have a continuous deformation of the flux line up to $a = +1$, as $x_+(+1) = +1$, but for $b < 1$ we obtain another abrupt change of the flux line position, because the flux line is "held" at the defect up to $b^2 + 1 + 4b - 6ba = 0$, i. e. for $a < a_2 = (b^2 + 1 + 4b)/6b$. The sudden motion of the deformed part of the flux line (this is now the release of flux line from the defect) takes place from $x_+(a_2) = (b + 2)/3$ to $x = a_2 > 1$ (Fig. 3c, d).

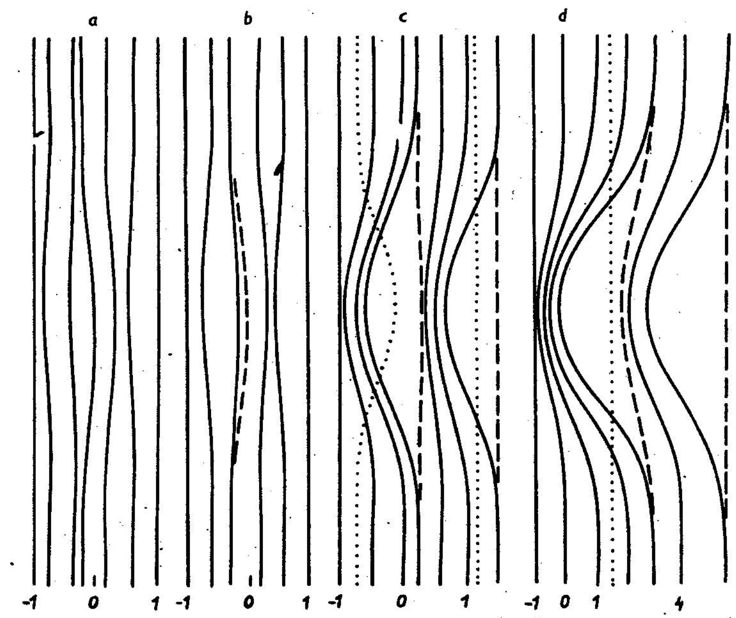


Fig. 3. The schematic pictures of the deformation of flux lines (i.e. the positions of one flux line while moving through the defect) at the combination of repulsive-attractive interaction potentials (dashed lines in Fig. 1b) for different elementary interaction forces; $b = 3$ (a), 1.5 (b), 0.5 (c), 0.1 (d). The numbers mean the undeformed coordinates a. The dashed lines are the positions after the sudden changes of the flux lines, the dotted lines the positions of the flux lines after reversing the force on the flux line lattice (see text).

These sudden changes of the flux line deformation cause the so-called hysteretic losses in type II superconductors.

Let the flux lines to be in equilibrium after applying some transport current in the superconductor (the so-called critical state). We begin now to increase the force applied to them (e. g. by applying slowly a small current to the current at which the flux lines are in equilibrium). Those flux lines which are deformed continuously are shifted slowly to the next equilibrium position, the deformation energy of the flux lines can be relaxed reversibly to the energy of the superconducting electrons (they have "obtained" the deformation energy from the superconducting electrons).

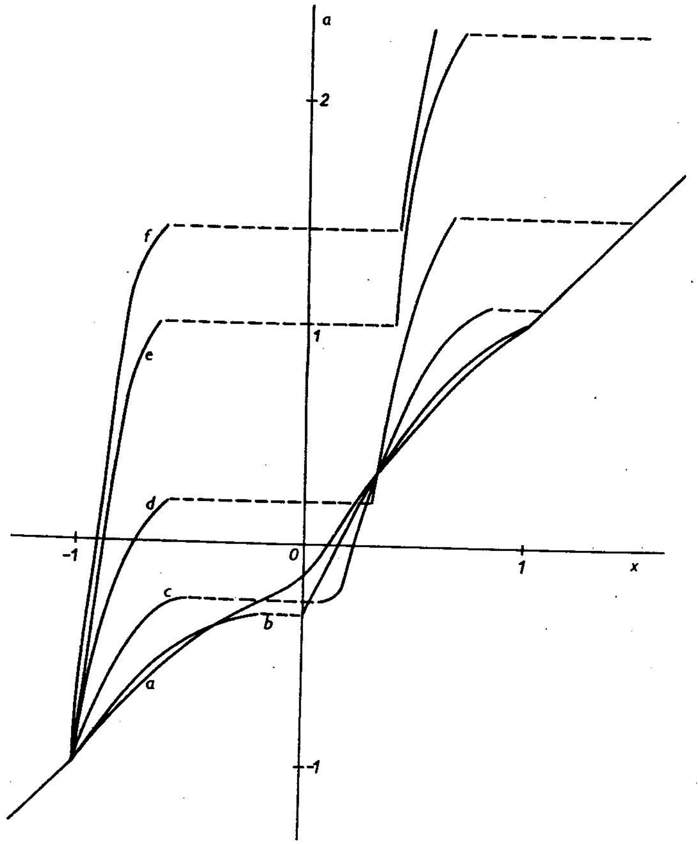


Fig. 4. To demonstrate the sudden changes (dashed lines) of the flux line positions near the defect with different elementary interaction forces, we have plotted the dependence of the deformed flux line positions x on the corresponding "natural" positions a for $b=3$ (a), 1.5 (b), 1 (c), 0.8 (d), 0.5 (e), 0.1 (f).

But this is not the case at the abrupt motion of the flux lines, where the processes are too rapid for this relaxation. The deformation energy changes to heat, the superconductor has losses. We can see this also from another point of view.

If we apply an additional force in the other direction (e. g., by reversing the direction of the current), the flux lines after the sudden change can be deformed continuously, reaching positions in which they were not before (dotted lines in Fig. 3). We have thus irreversible changes of the flux line deformations (and irreversible effects lead to losses).

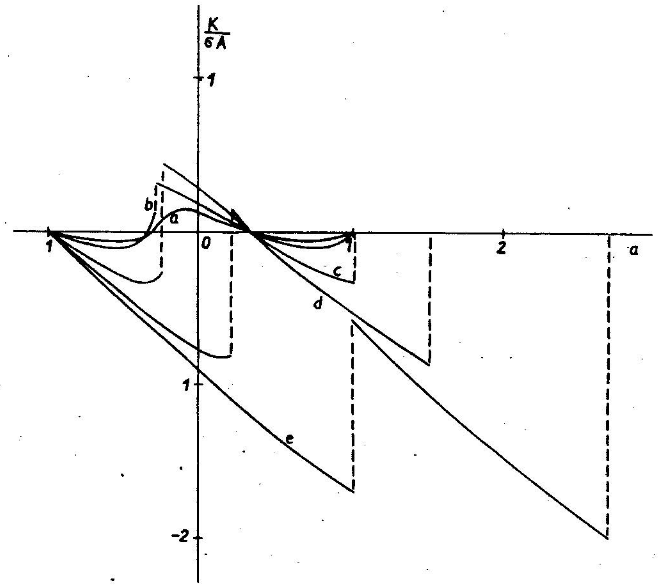


Fig. 5. The force on the flux lines vs. the flux lines "natural" positions for $b=3$ (a), 1.5 (b), 1 (c), 0.8 (d), 0.5 (e). From these figures, the appearance of the force asymmetry (resulting in non-zero volume pinning force) for $b<2$ is more clear.

To have another picture of the flux line positions near the defect, we have plotted the coordinates of the flux lines in dependence on the undeformed flux line positions in Fig. 4.

We can see the asymmetry of the flux line positions, caused by the defect, also from the force on the flux lines (Fig. 5).

Knowing the possible intervals of the undeformed flux line positions a (for the individual solutions x_+, x_-), we can calculate the volume pinning force on the flux line lattice. As mentioned above, all positions of a are equally probable, therefore

$$F_{v+}=N\left\{\int_{-1}^{a_1}\sigma A^2(x-a)\,da+\int_{a_1}^{a_2}\sigma A^2(x-a)\,da\right\},\qquad (5)$$

where

$$\alpha_1=a_0,\quad \alpha_2=1 \text{ for } b>2;$$
$$\alpha_1=a_1,\quad \alpha_2=1 \text{ for } 2>b>1;$$
$$\alpha_1=a_1,\quad \alpha_2=a_2 \text{ for } b<1.$$

After integrating (5) we obtain

$$F_{v+}=\frac{N(K_m/3)^2}{2}\begin{cases}0 & \text{for } b>2\\ \left(\frac{64}{3}b-64b^2+\frac{128}{3}\sqrt{2}\,b^{5/2}-16b^3\right) & \text{for } 2>b>1\\ \left(1+\frac{56}{3}b-62b^2-\frac{b^4}{3}+\frac{128}{3}\sqrt{2}\,b^{5/2}\right) & \text{for } b<1.\end{cases}$$

The results are plotted in Fig. 6. Similarly as in the case of the simple attractive and repulsive interaction potential, we have a threshold value for 1/b, below which the defect configuration does not contribute to the volume pinning force. This threshold is given by the reversible-irreversible change of the flux line position near the defect, as mentioned above. It is worthwhile to note that this threshold value is smaller than for the single attractive or repulsive interaction potential (see Fig. 6).

Much more interesting in our case are the deviations from the $F_v(K_m^2)$ dependence. These deviations could be possibly explained by the fact that at smaller maximum interaction forces both the barrier and the well contribute to the volume pinning force (and act in the same way, i. e. "both" volume pinning forces are in the same direction), whereas at larger maximum elementary interaction forces the barier dominantly determines the volume pinning force. Therefore, the total volume pinning force is larger at smaller K_m than one would expect (the maximum in Fig. 6 for the combined potential is approximately twice the maximum of the single potential barrier).

IV. THE COMBINATION OF THE ATTRACTIVE-REPULSIVE POTENTIAL

The calculations in the case of the attractive-repulsive potential are analogous to those in the preceding Section.

The flux lines should come to the defect from the left and the defects begin to act on them at $x=-1$. The flux line position is determined by the equilibrium equation

$$(1+3x^2-4x)=-\frac{\sigma A}{K_m}(x-a)=-2b(x-a).\qquad (6)$$

The solutions of this equation are

$$x_-=\frac{-2-b+(b^2+1+4b+6ba)^{1/2}}{3},\qquad (7)$$
$$x_+=\frac{2-b+(b^2+1-4b+6ba)^{1/2}}{3},$$

where again $x_-<0$, $x_+>0$. The "right" sign of the square roots is now given by the condition

$$x=-1/3\leftrightarrow a=-1/3,$$

as for $x=-1/3$ we have $K=0$.

We have again some characteristic cases. For $b>2$, we obtain solutions of the Eq. (6) for all x, i. e. x_- for $a\in(-1,a_0)$, x_+ for $a\in(a_0,+1)$, with

$$x_-(a_0)=x_+(a_0),\qquad a_0=1/2b.\qquad (8)$$

The deformation of the flux lines in this case is continuous and reversible (see Fig. 7a).

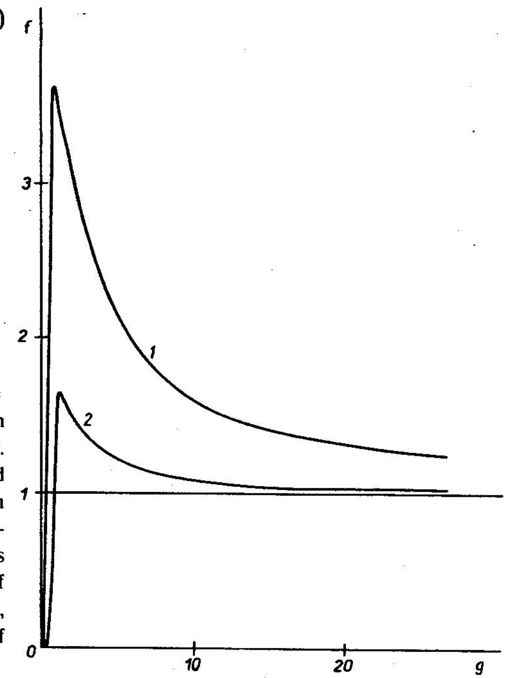


Fig. 6. The relative volume pinning force $f=F_v/(NK_m^2/2\sigma)$ in dependence on the maximum elementary interaction force K_m ($g=K_m/\sigma A$). The "ideal" quadratic $F_v(K_m)$ dependence would give $f=1$ (independent of g). K_m is the maximum elementary interaction force, with which the defect on the flux lines at large interaction forces acts. 1 — the repulsive-attractive combination of the interaction potentials (dashed lines in Fig. 1b), 2 — the single repulsive interaction potential of a barrier (Fig. 1a).

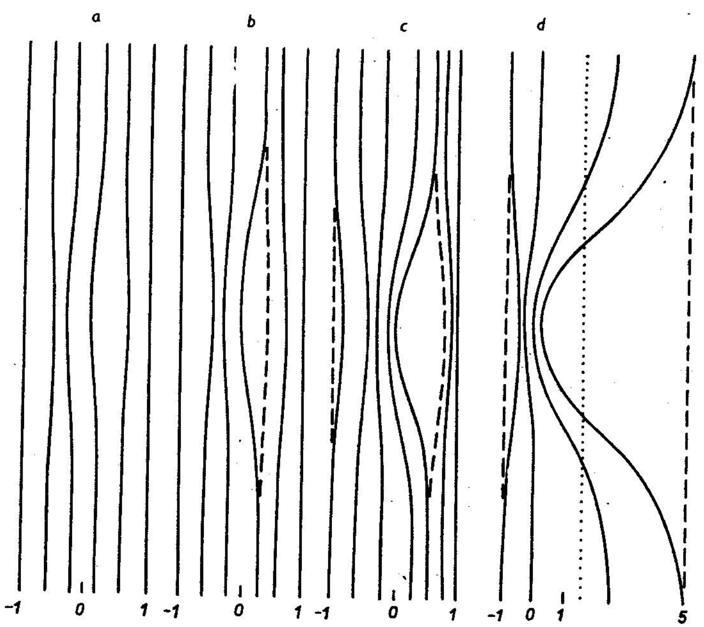


Fig. 7. The same schematic pictures as in Fig. 3, but for the attractive-repulsive combination of potentials (full line of Fig. 1b) for $b = 3$ (a), 1.5 (b), 1 (c), 0.1 (d).

One cannot any more combine the solutions x_- and x_+ continuously for $2 > b > 1$, as in this case Eqs. (8) are not true.

We have then

$$x_-(a_0) = 0, \qquad x_+(a_0) = (4 - b)/3 = x_1$$

and we obtain no solution for x_+ in the interval $x \in (0, x_1)$. The flux line at the position $a = a_0$ changes its form abruptly (Fig. 7b), the deformation energy of the flux line is lost in heat.

For $b < 1$, the first sudden change of the flux line deformation is at $a = -1$, as in this case the force from the defect is already so strong that the nearest possible coordinate of the deformed flux line is nearer to the defect centre.

Since

$$x_-(-1) = \frac{-2 - b + (b^2 + 1 - 2b)^{1/2}}{3} = \begin{cases} -1 & \text{for } b > 1, \\ -\dfrac{1 + 2b}{3} & \text{for } b < 1, \end{cases}$$

the interval of the impossible real x_- solution is

$$x \in (-1, x_2), \qquad x_2 = -(1 + 2b)/3$$

and the sudden change of the deformed flux line position is from $x = -1$ to $x = x_2$ (Fig. 7c and 7d).

For $b < 1/2$, we obtain $x_-(+1) < 0$ and x_+ has no solutions in the intervál (0, 1) for $a > 1$. The flux line is held on the defect up to the value $x_- = 0$ and springs then to the coordinate $a_1 = 1/2b$, as $x_-(a_1) = 0$ (Fig. 7d).

In Fig. 7d (as in Fig. 3) we see very illustratively the irreversible effects of the defects, which contribute to the volume pinning force. By changing the direction of the applied force on the flux line lattice, the flux line which is just released from the defect, can be displaced some distance backwards without comig into the interaction range of the defect. Meanwhile, the flux line occupies such positions in which it was not before — the cause of the irreversible effects, mentioned above.

The sudden changes of the flux line positions and the asymmetry of the force action (leading to the pinning force) are illustrated also in Figs. 8 and 9, which are the analogous dependences to the Figs. 4 and 5 for the repulsive-attractive potential of the preceding Section.

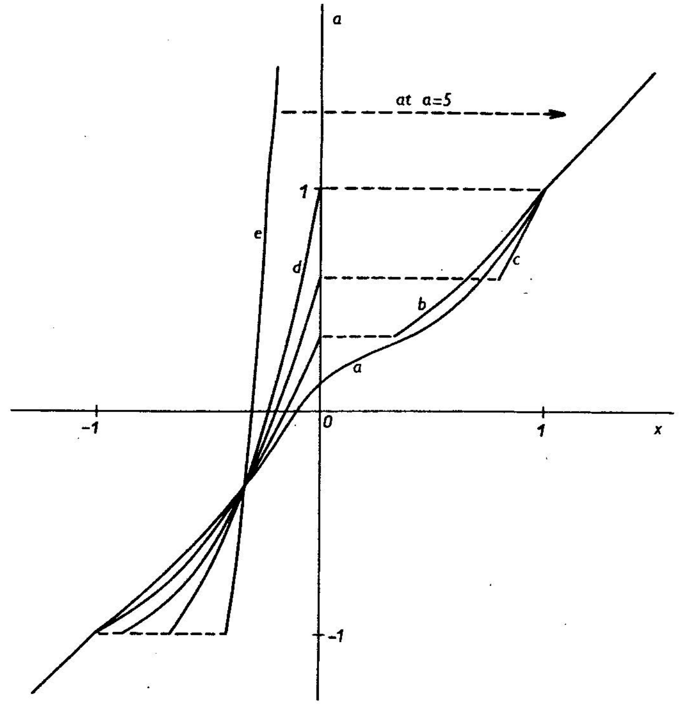


Fig. 8. The same dependence as in Fig. 4, but for the attractive-repulsive combination of the interaction potentials (full line of Fig. 1b), $b = 3$ (a), 1.5 (b), 1 (c), 0.5 (d), 0.1 (e).

In calculating the volume pinning force on the flux line lattice, we have the following integrals

$$F_{v-} = N\sigma\left[\int_{-1}^{\alpha_1} A^2(x_- - a)\,da + \int_{\alpha_1}^{\alpha_2} A^2(x_+ - a)\,da\right],$$

where $\alpha_1 = 1/2b$ and $\alpha_2 = \begin{cases} 1 & \text{for } b > 1/2 \\ 1/2b & \text{for } b < 1/2. \end{cases}$

After integrating we get

$$F_{v-} = \frac{NK_m^2}{2\sigma}\begin{cases} 0 & \text{for } b > 2 \\ 16/27(8b - 12b^2 + 6b^3 - b^4) & \text{for } 2 > b > 1 \\ 16/27(9b - 15b^2 + 9b^3 - 2b^4) & \text{for } 1 > b > 1/2 \\ [1 + 16/27(b - 3b^2 + 3b^3 - b^4)] & \text{for } b < 1/2. \end{cases}$$

The results are plotted in Fig. 10.

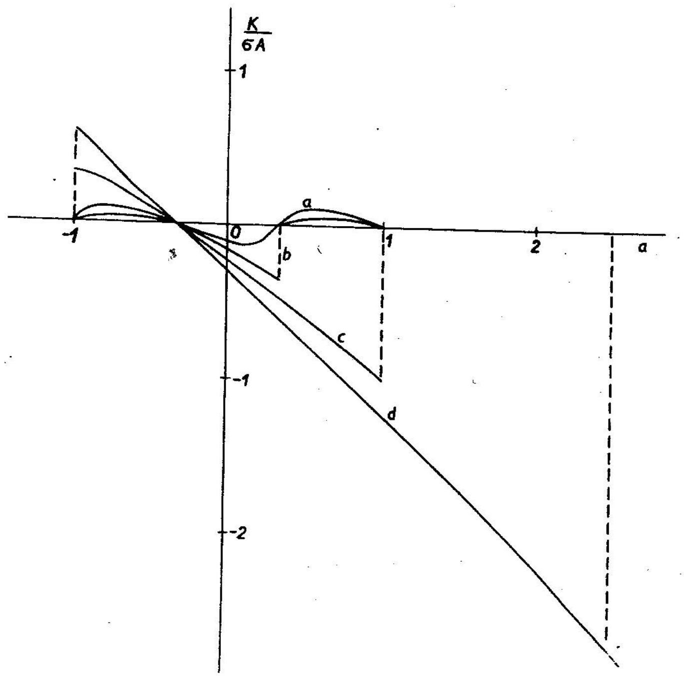
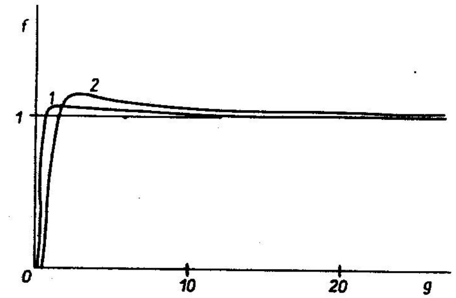


Fig. 9. The same dependence as in Fig. 5, but for the attractiverepulsive combination of the interaction potentials (full line of Fig. 1b), $b = 3$ (a), 1.5 (b), 0.5 (c), 0.1 (d).

Fig. 10. The volume pinning force in dependence on the maximum elementary interaction force (like in Fig. 6), for the attractive-repulsive combination of the interaction potentials (full lines in Fig. 1b), noted as (1), and the single (2) attractive potential (the reverse case of the potential in Fig. 1a).



In contradistiction to the results of the previous Section, the attractive-repulsive combination of the interaction potentials gives a nearly ideal quadratic dependence of $F_v(K_m)$, which was not the case for any other forms of the interaction potential [8—11]. Though the pinning forces from the barrier and the well were additive in case of the repulsive-attractive combination of the potentials, now the barrier and the potential well act in the opposite direction (the flux lines are attracted by the well — the force is to the right, but they are repulsed from the following barrier — the force is to the left). Therefore, the increase of the volume pinning force at smaller elementary interaction forces is absent.

V. DISCUSSIÓN

The difference in the $F_v(K_m)$ dependence are smeared out at larger elementary interaction forces, as then the first barrier or well is dominant for the interaction with the flux lines, respectively. The volume pinning force in case of K_- (i. e. attractive-repulsive combination — full line in Fig. 1b) is therefore much larger than in case of K_+, because the elementary interaction force has the maximum $K = K_m$, whereas for K_+ the corresponding maximum of the barrier is $K = K_m/3$.

The contribution of K_- to the total volume pinning force (with the same probability of both cases) is therefore much larger in the case of stronger elementary interaction forces, and in the limit $b \to 0$ (i.e. very strong interaction forces), we have $F_{v-} = 9F_{v+}$.

It is just this strong increase of F_{v+} at smaller elementary interaction forces which is supposed to explain the experimentally observed results for statistically distributed point defects [12] at smaller defect densities. This effect was more pronounced at smaller magnetic fields [12]. This is not surprising, as for larger magnetic fields the flux line distances are smaller, the interaction between the flux lines stronger, the flux line distortions therefore smaller. The role of "simple" potential barriers or wells will then increase to the prejudice of more complicated interaction potentials (e. g. our combined interaction of more volume elements with the individual flux lines).

As stated already in [13], at very small magnetic fields another effect can be important, since the Ginzburg-Landau parameter $\varkappa$ was not much larger than $1/\sqrt{2}$. In such superconductors, the interaction between the neighbouring flux lines can turn to the attractive one at some flux line distances, and this effect leads to a further increase of the volume pinning force.

Concluding, we would like to make some comments on the cut-off of small elementary interaction forces, which are below the threshold value [2]. As expounded in Sect. II, this effect failed to explain the decrease of the volume pinning force to zero at magnetic fields smaller than the upper critical magnetic field of the superconductor [13], although this effect seemed to be a very probable reason for this decrease. Looking at the form of Fig. 6, we can now understand a little the reason for this failure.

The maximum elementary interaction force K_m decreases with the increasing magnetic field (for all interaction mechanisms defect — flux line). The number of the cut-off volume elements (i.e. those with the maximum interaction force smaller than the threshold value, e.g. K_{om}) will therefore increase with the increasing magnetic field. At the same time, the number of volume elements increases, which come closer to the "peak" in the $F_v(K_m)$ dependence. These elements contribute more to the total volume pinning force than expected from the pure statistical point of view.

The increased contribution of these elements can compensate to some degree the contribution of the cut-off volume elements.

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