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CURRENT DENSITY AND DIFFUSION COEFFICIENT FOR COSMIC RAY PARTICLES

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The expression of current density for cosmic ray particles scattering in the interplanetary turbulent magnetic fields is found in the work. It is shown that the convectional component of the speed of the current particles does not correspond to the solar wind velocity, but it is determined by the "drifting" velocity of the charged particles in the solar wind. A tensor coefficient diffusion is computed in the case of $R \ll L_c$, where R is the Larmour radius. The longitudinal spatial diffusion coefficient and the mean free path are discussed in detail.

ПЛОТНОСТЬ ТОКА И КОЭФФИЦИЕНТ ДИФФУЗИИ ДЛЯ ЧАСТИЦ В КОСМИЧЕСКИХ ЛУЧАХ

В работе найдено выражение для плотности тока частиц в космических лучах, которые рассеиваются в межпланетных турбулентных магнитных полях. Показано, что обычная компонента скорости тока частиц не соответствует скорости солнечного ветра, а определяется скоростью дрейфа заряженных частиц в солнечном ветре. Тензорный коэффициент диффузии расчитан в случае $R \ll L_c$, где R обозначает радиус Лармора. Детально обсуждаются коэффициент продольной пространственной диффузии и средняя длина свободного пробега частиц.

I. INTRODUCTION

In the investigation of the scattering process of cosmic ray particles in an interplanetary irregular magnetic field many authors have been using the ready diffusion equation, resp. the Fokker—Planck equation. It cannot be doubted that in the case of many phenomena, where the mean free path of the cosmic ray particles is larger than the irregularities of the medium [1, 2], the application of a more general equation — the Boltzman kinetic equation for the mean distribution function $F(\boldsymbol{r}, \boldsymbol{p}, t)$ — is necessary. This equation has the form [1]:

$$\left(\frac{\partial}{\partial t} + \hat{L}_0\right) F(\boldsymbol{r}, \boldsymbol{p}, t) = \mathrm{Col}\, F,$$

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where

$$\mathrm{Col}\,F=\int_0^{\infty} d\tau \frac{\partial}{\partial p_\alpha} e^{-\hat{L}_0\tau} D_{\alpha\lambda}(\boldsymbol{r}, \boldsymbol{r}_1; \boldsymbol{p}, \boldsymbol{p}_1; \tau) \frac{\partial}{\partial p_\lambda} F(\boldsymbol{r}, \boldsymbol{p}, t),$$

$\hat{L}_0=\boldsymbol{v}\nabla+\frac{e}{c}[\boldsymbol{v}-\boldsymbol{u}, \boldsymbol{H}_0]\frac{\partial}{\partial \boldsymbol{p}}$, $\boldsymbol{u}$ is the velocity of the solar wind, $\boldsymbol{v}$, $\boldsymbol{p}$ is the velocity and the momentum of particles, $D_{\alpha\lambda}$ is the tensor of electromagnetic forces in the random magnetic field.

This equation provides the exact form of equation of diffusion in the case of the correct calculation of the diffusion approximation [3]. In the case of the high energy particles, when the Larmour radius $R \gg L_c$, where L_c is the autocorrelation length of a turbulent magnetic field, Dolginov and Toptygin [2] determined that the current density $\boldsymbol{I}$ of particles is

$$I_\alpha=-k_{\alpha\beta}\frac{\partial N}{\partial x_\beta}-u_\alpha \frac{p}{3}\frac{\partial N}{\partial p}, \tag{1}$$

where $N(\boldsymbol{r}, p, t)$ is the diffusion tensor, $k_{\alpha\beta}$ signify the concentration of particles.

If the magnetic field is strong, i.e. $R \ll L_c$, it is necessary to consider a helical motion of particles in space. The diffusion approximation provides in this case two complicated equations. The second — the vector equation has the form [3]

$$\frac{1}{v}\frac{\partial \boldsymbol{I}}{\partial t}+\frac{v}{3}\nabla N-\frac{1}{R}[\boldsymbol{uh}]\frac{p}{3}\frac{\partial N}{\partial p}+\frac{1}{R}[\boldsymbol{hI}]=-\frac{1}{3p}\frac{\partial N}{\partial p}(\boldsymbol{uh})^2\boldsymbol{h}\Phi_-'+$$

$$+\frac{v}{3p}\frac{\partial N}{\partial p}\{\boldsymbol{h}(\boldsymbol{uh})\Phi_- - \boldsymbol{u}\Phi_+ + [\boldsymbol{uh}]\Phi_s\}+$$

$$+\frac{v}{p^2}\left\{\boldsymbol{h}(\boldsymbol{Ih})\Phi_- - \boldsymbol{I}\Phi_+ - [\boldsymbol{uh}]\Phi_s - \frac{\boldsymbol{u}}{v}(\boldsymbol{Ih})\Phi_+' + \frac{1}{v}\boldsymbol{h}(\boldsymbol{uI})\Phi_-'\right\},$$

where the functions

$$\Phi_\pm=\frac{\langle H_1^2\rangle}{3}\left(\frac{e}{c}\right)^2\int d\tau\Phi\left(\frac{v\tau}{L_c}\right)(1\pm\cos\Omega\tau),$$

$$\Phi_s=\frac{\langle H_1^2\rangle}{3}\left(\frac{e}{c}\right)^2\int d\tau\Phi\left(\frac{v\tau}{L_c}\right)\sin\Omega\tau,$$

$$\Phi_\pm'=\frac{\langle H_1^2\rangle}{3}\left(\frac{e}{c}\right)^2\int d\tau v\frac{\partial\Phi(v\tau/L_c)}{\partial v}(1\pm\cos\Omega\tau),$$

$\Phi_c=\frac{1}{2}(\Phi_+-\Phi_-)$ arise from the collision integral Col F in a Boltzmann kinetic equation. Those are proportional to the Fourier component of the tensor of the random magnetic field in accordance with the circle velocity $\Omega=evH_0/cp$ of the particle in the mean part of magnetic field H_0. The function Φ determines the diagonal components of the tensor random magnetic field $\langle H_{1\alpha}H_{1\beta}\rangle\sim\Phi\delta_{\alpha\beta}$. The form of Φ is defined empirically. We shall use the following form of Φ in the concrete calculation:

$$\Phi\left(\frac{x}{L_c}\right)=\left(\frac{x}{L_c}\right)^{(\nu-1)/2}K_{(\nu-1)/2}\left(\frac{x}{L_c}\right)$$

whose Fourier component [5] is $\Phi(k)\sim 1/(k_0^2+k^2)^{\nu/2+1}$, $k_0^{-1}=L_c$. We denote by $\varkappa_{\alpha\beta}$ the diffusion tensor in distinction from $k_{\alpha\beta}$ in expression (1). The mean free path in the random magnetic field is denoted by Λ.

II. CURRENT DENSITY OF PARTICLES

From the diffusion approximation of the kinetic equation for the mean distribution function we have two equations: for the particle density $N(\boldsymbol{r}, \boldsymbol{p}, t)$ and current density $\boldsymbol{I}(\boldsymbol{r}, p, t)$ [3]. We shall investigate the case of the independence of the current density of time:

$$\frac{1}{I}\frac{\partial I}{\partial t}\ll\frac{v}{R}\sim\omega_c.$$

If we define the coordinate system by

$$\boldsymbol{n}_1=\frac{[\boldsymbol{uh}]}{|[\boldsymbol{uh}]|}, \quad \boldsymbol{n}_2=[\boldsymbol{n}_3, \boldsymbol{n}_1], \quad \boldsymbol{n}_3=\boldsymbol{h}=\frac{\boldsymbol{H}_0}{|\boldsymbol{H}_0|}, \tag{2}$$

from the vector equation of a diffusion approximation, we can obtain the explicit expression for the particle current density:

$$I_\alpha=-\varkappa_{\alpha\beta}\frac{\partial N}{\partial x_\beta}-w_\alpha\frac{p}{3}\frac{\partial N}{\partial p}. \tag{3}$$

The first term in the expression describes the pure anisotropic diffusion of the particles in space, which arise in the presence of the gradient ∇N. For the diffusion tensor we obtain:

$$\varkappa_{\alpha\beta}=\varkappa_0\left\{\delta_{\alpha\beta}-\left[1-Q_1Q_3^2\alpha+(Q_1+Q_2)\frac{(\boldsymbol{uh})}{v}+Q_1Q_2\frac{|[\boldsymbol{uh}]|^2}{v^2}\right]\times\right. \tag{4}$$

$$\times h_\alpha h_\beta+Q_3\varepsilon_{\alpha\beta\gamma}h_\gamma+\frac{1}{v}(Q_2u_\beta h_\alpha+Q_1u_\alpha h_\beta)+$$

$$\left.+(Q_2Q_3\varepsilon_{\beta\gamma\delta}h_\alpha+Q_1Q_3\alpha^3\varepsilon_{\alpha\gamma\delta}h_\beta)\frac{u_\gamma}{v}h_\delta\right\},$$

where

$$\varkappa_0=\frac{p^2}{3Q_3}\Phi_+^{-1}; \quad \alpha=\left(\Phi_s+\frac{p^2}{vR}\right)\Phi_+^{-1}; \quad Q_1=\frac{\Phi_+}{2\Phi_c}\left(1-\frac{1}{v}(\boldsymbol{uh})\frac{\Phi_c'}{\Phi_c}\right); \tag{5}$$

$$Q_2 = -\frac{\Phi_-'}{2\Phi_c}; \qquad Q_3 = \left(1+\alpha^2+\frac{1}{v^2}\,|[\boldsymbol{uh}]|^2 Q_2\right)\alpha^{-1}.$$

The second term, convectional, arises in the particle scattering stochastic magnetic field, which moves in space with a velocity $\boldsymbol{u}$. The magnitude w_α is not equal to the solar wind velocity u_α, but it is

$$w_\alpha = u_\alpha + \frac{1}{v} Q_2 \left(1+\frac{(\boldsymbol{uh})}{v} Q\right) |[\boldsymbol{uh}]|^2 h_\alpha + \frac{1}{v} Q_2 (\boldsymbol{uh})^2 h_\alpha + \tag{6}$$

$$+\frac{1}{v} Q\alpha\varepsilon_{\alpha\gamma\delta} h_\gamma (\boldsymbol{uh}) u_\gamma + \frac{1}{v} Q(\delta_{\alpha\gamma} - h_\alpha h_\gamma)(\boldsymbol{uh}) u_\gamma,$$

where

$$Q = \left(1+\frac{(\boldsymbol{uh})}{v} Q_2\right)\left(1+\alpha^2+\frac{1}{v^2}\,|[\boldsymbol{uh}]|^2 Q_2\right)^{-1}. \tag{5a}$$

The expression (3) for the current density differs from expression (1) not in the modified convection term only, but in the different diffusion tensor, too.

III. CONVECTION COMPONENT OF THE SPEED OF CURRENT

It can be seen that owing to the strong magnetic field the convection term is proportional $\boldsymbol{w} \neq \boldsymbol{u}$. We can find out there is a difference between these quantities. From formulae (5, 5a) and the expressions for the functions Φ we obtain complicated expressions with the hypergeometric functions for the coefficients Q, Q_2. In the approximation $R \ll L_c$ the expressions for the coefficients are essentially simplified and we have

$$Q \approx \frac{1+\frac{1}{2}\frac{(\boldsymbol{uh})}{v}\left(\frac{L_c}{R}\right)^v}{1+\frac{1}{2}\frac{[\boldsymbol{uh}]^2}{v^2}\left(\frac{L_c}{R}\right)^v}; \qquad Q_2 \approx \frac{1}{2}\left(\frac{L_c}{R}\right)^v. \tag{7}$$

Let us consider the most probable values characterizing the interplanetary space: $L \approx 2\times 10^{11}$ cm, $H \approx 4.5\gamma$, $u \approx 4\times 10^7$ cms^{-1}, $1.5 \leqslant v \leqslant 2$, and a proton with the energy of 1 MeV. From the expressions (7) it follows that the second term in (6) reaches a value near to u (for an angle between the vector $\boldsymbol{u}$ and $\boldsymbol{h}$ larger than zero, for $v=1.5$). For a more energetic particle the term vanishes. When v increases, the situation becomes complicated, because for $v=2$ the second term in (6) is much greater than the first term describing the "drifting" of the particles in a solar wind, especially those of lower energy. If $v=2.5$, the deposit of the second term is $\approx u$, especially for the energy of particles of 100 MeV. A deposit increases disproportionately especially with the larger value v. In fact, the disproportionate increase is

scarcely realized, because we see in form $\Phi(k)$ that for large v the particles scatter mostly in large irregularities, the number of which is sizable. We suppose that in calculations the irregularities have the size L_c, although theoretically the size of the irregularities $L \gtrsim L_c$ for a value $v \lesssim 2.5$. The physical meaning is evident: for high energy particles and a small v the irregularities are small enough so that the particles do not interact with them. On the other hand the low energy particles deviate at a larger angle if v is high. We obtain a reasonable deposit for low energy particles and a small v. A high v and high energy particles with an indicated limitation provide a reasonable deposit as well.

While the second term in (6) is necessary to consider in the case of $[\boldsymbol{uh}] \neq 0$, the third term is proportional $(\boldsymbol{uh})$, i.e. when an angle between $\boldsymbol{u}$, $\boldsymbol{h}$ is near zero. Its value increases for particles (protons) with an energy of several MeV with the growth of v, and achieves a value larger then $\boldsymbol{u}$ for $v \lesssim 2$ only.

The fourth term in the expression (6) differs from zero for an angle between $\boldsymbol{u}$, $\boldsymbol{h}$ near to $\pi/4$ only, and achieves a value several percent of $\boldsymbol{u}$ (2 % for $v=1.5$; 15 % for $v=2.5$). The last term gives a deposit for $(\boldsymbol{uh}) \neq 0$ and $\alpha = 1$, 2 only. For $(\boldsymbol{uh}) \approx 0.8u$ the quantity of the last term is equal $\approx 0.1u$ (for $v=1.5$) and $\approx u$ (for $v=2.5$). If v is still increasing, the value is practically constant.

The second and the third terms in (6) describe the convection component of a current speed along the vector $\boldsymbol{H}_0$. The first and the last term describe a primary convection in the direction $\boldsymbol{u}$; the fourth term shows a negligible convection up to $\boldsymbol{u}$.

Remark

In case of $R \gg L$ the expressions for the coefficients (5, 5a) are essentially simplified and the exponent v dependence of the coefficients vanishes quite completely. The coefficients Q, Q_2 are small in this case and the expression (3) for the density current turns to the expression (1). In this case, the change of the particle momentum in a regular field may be neglected (at the distance $\sim L_c$).

IV. DIFFUSION TENSOR
COMPARISON WITH AN EXPERIMENT

We can find the components of the diffusion tensor from the expressions (4), (5). Let us write the component $\varkappa_{33}$ expressing the meaning of the longitudinal diffusion coefficient (concerning the regular magnetic field):

$$\varkappa_{33} = \varkappa_0 \left(\alpha Q_1 Q_3^2 + Q_1 Q_2 \frac{1}{v^2}\,|[\boldsymbol{uh}]|^2\right). \tag{8}$$

The coefficients α, $\Phi_\pm$, $\Phi_c = \frac{1}{2}(\Phi_+ - \Phi_-)$, Φ_-' in the case of $R \ll L_c$ acquire the form

$$\alpha = \frac{3}{\sqrt{\pi}}\,\frac{2^{(3-v)/2}}{\Gamma(v/2)}\,\frac{H_0^2}{\langle H_1^2\rangle}\,\frac{R}{L_c}; \tag{9}$$

$$\Phi_{\pm}=\frac{\langle H_1^2\rangle}{3}\left(\frac{e}{c}\right)^2\sqrt{\pi}\,2^{(\nu-3)/2}\Gamma\left(\frac{\nu}{2}\right)\frac{L_c}{v};$$

$$\Phi_c=\frac{\langle H_1^2\rangle}{3}\left(\frac{e}{c}\right)^2\sqrt{\pi}\,2^{(\nu-3)/2}\Gamma\left(\frac{3}{2}\right)\frac{L_c}{v}\left(\frac{R}{L_c}\right)^{\nu};$$

$$\Phi'_-=-\frac{\langle H_1^2\rangle}{3}\left(\frac{e}{c}\right)^2\sqrt{\pi}\,2^{(\nu-3)/2}\Gamma\left(\frac{\nu}{2}\right)\frac{L_c}{v}\left\{1-\left(\frac{R}{L_c}\right)^{\nu}\right\}.$$

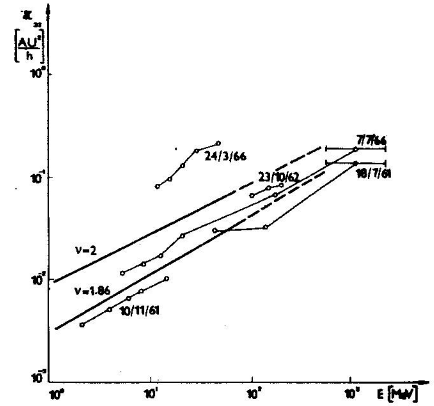

Fig. 1. A selection of the experimental magnitude of the radial diffusion coefficient, which describes the propagation of solar particles in the period 1961—1966 (the magnitude is denoted by circles). The theoretical curves (full curves) describe the energy dependences of κ_{33}. The treated experimental values are taken over from [4].

From the expression (5) and (9) we obtain the longitudinal diffusion coefficients $\varkappa_{//}=\varkappa_{33}$

$$\varkappa_{//}=v\frac{2^{(1-\nu)/2}H_0^2}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)\langle H_1^2\rangle}\frac{\left[1+\frac{1}{2}v^{-2}[uh]^2\left(\frac{L_c}{R}\right)^{\nu}\right]}{\left[1+\frac{1}{2}v^{-1}(uh)\left(\frac{L_c}{R}\right)^{\nu}\right]}\left(\frac{L_c}{R}\right)^{\nu-2}L_c \qquad (10)$$

The fraction in brackets may be neglected. In Fig. 1 some experimental values taken over from [4] are compared with the theoretically calculated energy dependence of $\varkappa_{//}=\varkappa_{//}(E)$.

It can be seen that the experimental values are straggled near the theoretical curves for $\nu=1.86$ and $\nu=2$ (in our approximation). A scattering of the experimental values may be due to the time change of the exponent ν, because the diffusion tensor is probably most sensitive to the change of ν.

V. RIGIDITY DEPENDENCE OF THE MEAN FREE PATH

The mean path of the particles in a turbulent magnetic field can be calculated from the components of the diffusion tensor. Let us denote

$$\varkappa_{11}=\varkappa_{22}=\frac{v}{3}\Lambda_{\perp}; \qquad (11)$$

$$\varkappa_{33}=\frac{v}{3}\Lambda_{//}$$

$$\varkappa_{\alpha\beta}=\frac{v}{3}\Lambda_{\alpha\beta};\qquad \alpha,\beta=1,2,3;\qquad \alpha\neq\beta;$$

$$\varkappa_{12}=-\varkappa_{21}.$$

We substitute the coefficients (9) and (5) for the expressions in (4). Then we obtain for the mean free path:

$$\Lambda_{12}=\frac{3\cdot 2^{(3-\nu)/2}H_0^2}{\sqrt{\pi}\Gamma(\nu/2)\langle H_1^2\rangle}\frac{R}{L_c}R; \qquad (12)$$

$$\Lambda_{13}=\frac{3^4\cdot 2^{5-2\nu}}{\pi^2[\Gamma(\nu/2)]^4}\left(\frac{H_0^2}{\langle H_1^2\rangle}\right)^4\left(\frac{R}{L_c}\right)^{4-\nu}R;$$

$$\Lambda_{23}=\frac{3^2\cdot 2^{2-\nu}}{\pi[\Gamma(\nu/2)]^2}\left(\frac{H_0^2}{\langle H_1^2\rangle}\right)^2\frac{|[uh]|}{v}\left[1+\frac{|[uh]|^2}{v^2}\frac{1}{2}\left(\frac{L_c}{R}\right)^{\nu}\right]^{-1}\left(\frac{L_c}{R}\right)^{\nu-2}R;$$

$$\Lambda_{31}=\frac{3\cdot 2^{(1-\nu)/2}}{\sqrt{\pi}\Gamma(\nu/2)}\frac{H_0^2}{\langle H_1^2\rangle}\frac{|[uh]|}{v}\left(\frac{L_c}{R}\right)^{\nu-2}L_c;$$

$$\Lambda_{32}=\frac{3^2\cdot 2^{2-\nu}}{\pi[\Gamma(\nu/2)]^2}\left(\frac{H_0^2}{\langle H_1^2\rangle}\right)^2\frac{|[uh]|}{v}\left[1+\frac{1}{2}\frac{|[uh]|^2}{v^2}\left(\frac{L_c}{R}\right)^{2}\right]^{-1}\left(\frac{L_c}{R}\right)^{\nu-2}R$$

and analogically for the longitudinal $\Lambda_{//}$ and the transverse $\Lambda_{\perp}$ mean free path:

$$\Lambda_{//}=\frac{3\cdot 2^{(1-\nu)/2}}{\sqrt{\pi}\Gamma(\nu/2)}\frac{H_0^2}{\langle H_1^2\rangle}\frac{\left[1+\frac{1}{2}\frac{|[uh]|^2}{v^2}\left(\frac{L_c}{R}\right)^{\nu}\right]}{\left[1+\frac{1}{2}\frac{(uh)}{v}\left(\frac{L_c}{R}\right)^{\nu}\right]}\left(\frac{L_c}{R}\right)^{\nu-2}L_c; \qquad (13a)$$

$$\Lambda_{\perp}=\frac{3^2\cdot 3^{3-\nu}}{\pi[\Gamma(\nu/2)]^2}\left(\frac{H_0^2}{\langle H_1^2\rangle}\right)^2\left[1-\frac{|[uh]|^2}{v^2}\frac{1}{2}\left(\frac{L_c}{R}\right)^{\nu}\right]^{-1}\left(\frac{R}{L_c}\right)R. \qquad (13b)$$

The quotient $H_0^2/\langle H_1^2\rangle\approx 5$ near the orbit of the Earth. If the second term in brackets in (13b) is small, the transverse mean free path $\Lambda_{\perp}$ is proportional to $H_0^{-2}\sim R^2$, and $\Lambda_{\perp}\ll\Lambda_{//}$ (In the strong field H_0 the low energy particles are concentrated in the power-tube).

Let us suppose the existence of the anisotropic current along $\boldsymbol{h}$. Although in this case $\partial N/\partial x_2$ may be (resp. $\partial N/\partial x_1$ as well) great in the coordinate system (2), the diffusion currents I_1, I_2 are small.

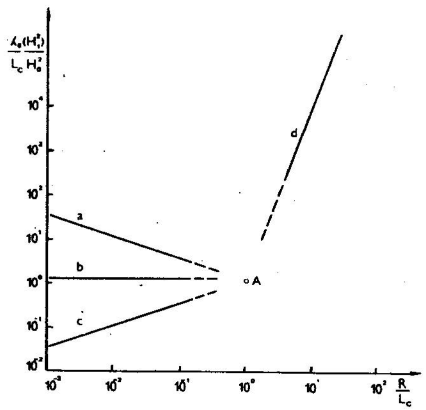

Fig. 2. The quotient R/L_c dependence of the "reduce" mean free path for some magnitude of the exponent ν: The curve a) corresponds to $\nu=2.5$, the curve b) corresponds to $\nu=2$, the curve c) corresponds to $\nu=1.5$. The curve d) holds for the approximation $R \gg L_c$.

Let us examine the expression (13a) for the mean free path of a particle along the magnetic field. In our approximation we obtain

$$\Lambda_{//}=\frac{3\,.\,2^{(1-\nu)/2}}{\sqrt{\pi}\Gamma(\nu/2)}\frac{H_0^2}{\langle H_1^2\rangle}\left(\frac{L_c}{R}\right)^{-2}L_c\,. \tag{14}$$

This value is practically equal to the value

$$l=\frac{4(\nu+2)\Gamma((\nu-1)/2)}{\nu\Gamma(\nu/2)\sqrt{\pi}}\frac{H_0^2}{\langle H_1^2\rangle}\left(\frac{L_c}{R}\right)^{\nu-2}L_c\,, \tag{15}$$

which in Galperin's, Toptygin's and Fradkin's [5] opinions expresses the meaning of the mean free path of particles in the strong magnetic field.

In Fig. 2 there is schematically illustrated the quotient R/L_c dependence of the "reduce" longitudinal mean free path. The meaning of this quotient becomes evident if we write down the quotient in the form

$$\frac{R}{L_c}=\left(\frac{pc}{ze}\right)\frac{1}{H_0L_c}\,,$$

where the term in brackets is the rigidity of particles, which increases proportionally with the momentum p.

We see the lingitudinal mean free path is proportional to $p^{2-\nu}$, i.e. for $\nu>2$ the mean free path decreases if the particles scatter in the large irregularities of the magnetic field, whose number (especially for great ν) increases.

If $\nu=2$, the mean free path $\Lambda_{//}$ does not depend on the monumentum; in case of $\nu<2$ we see that $\Lambda_{//}$ decreases for the high energy of particles (the number of the small irregularities is greater than for $\nu>2$).

The asymptotic momentum dependence of the "reduce" mean free path in case of $R \gg L_c$ is independent of the exponent ν (curve d in Fig. 2). The motion of the charged particles in the interplanetary space is defined by the rough characteristics of the interplanetary medium. In Fig. 2 we see that the curves a, b, c, d may be interpolated to the point A (near to $R \approx L_c$).

The numerical value of $\Lambda_{//}$ for the particle energy 1 MeV equals 9×10^{11} cm (for $\nu=1.5$) and 4×10^{12} cm (for $\nu=2$). Let us remark that the quotient $H_0^2/\langle H_1^2\rangle$ is approximately constant. For example, in the region over the Earth's orbit, this presumption has been verified by the measuring of the "Mariner-4" [6]: If the distance from the Sun is enlarged from 1 AU to 1.43 AU, the value $\langle H_1^2\rangle$ decreases 2.4-times and the value H_0^2 decreases 2.5-times. The magnitudes of the values Λ_{23}, Λ_{32} are small in comparison with $\Lambda_{//}$; the magnitudes Λ_{12}, Λ_{23}, Λ_{32} are near to R and $\Lambda_{23}, \Lambda_{32}\to 0$ for the angle between the vectors $\boldsymbol{u}$, $\boldsymbol{h}$ near to zero or $\pi/2$; further there holds $\Lambda_{13} \ll R$. The magnitude Λ_{31} tends to zero for $|[\boldsymbol{uh}]|\to 0$; if $|[\boldsymbol{uh}]|=u$, then $\Lambda_{31}=3\varkappa_{31}/v<L_c$.

VI. DISCUSSION

We mentioned in Chapt. I. that many authors begin from the Fokker—Planck equation in solving the particle propagation in an irregular magnetic field in the interplanetary space. Klimas A. and Sandri G. have calculated the complicated expression for the parallel diffusion coefficient and one expanded in a series. The authors used small parameters $1/\varepsilon=P/\lambda_p\langle H\rangle$, where $\langle H\rangle$ is the mean field strength, P is the particle rigidity, λ_p has the meaning of $\Lambda_{//}$. Let us remark that the case of $\lambda_p\langle H\rangle \gg P$ corresponds to the case of $L_c \gg R$.

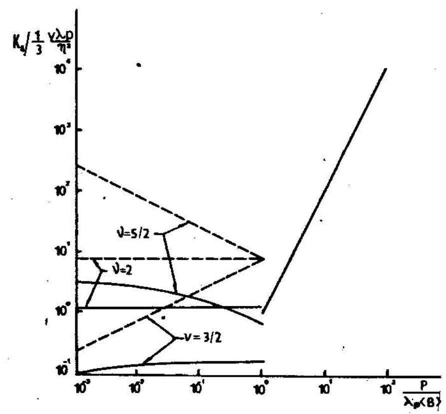

Fig. 3. The quotient $P/\lambda_p\langle B\rangle$ dependence of the "reduce" mean free path for $\alpha\nu=2.5$; 2; 1.5 [7]. The dashed curves are results of the first, the solid curves are results of the second approximation. The exponent $\alpha\nu$ corresponds to 2ν in [7].

The first and the second approximations of the series were calculated by the authors [7] the resonant and nonresonant interaction particle-fields. The results are illustrated in Fig. 3, where η = the value of the random field/$\langle H \rangle \ll 1$. The dashed curves are the results of the case where only the resonant interaction was included, while the solid curves are the result of including both interactions. In this way, the authors [7] expect that for a very low rigidity (≈ 100 keV) the mean free path will be independent of rigidity.

We see that the first approximation in [7] is very like the dependences in Fig. 2., which are the results of the first (diffusion) approximation of a Boltzmann kinetic equation. In the first approximation a kinetic equation gives the results identical with the results obtained by using the method in [7]. The method of our work, in addition, provides all the components of $\varkappa_{\alpha\beta}$ simultaneously.

In addition the authors in [7] used the power spectra $k^{-\nu}$ instead of (16), which have been used in our work.

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