acta phys. slov. 26 (1976), No. 4

MEASUREMENT OF THERMAL QUANTITIES IN SOME INSULANTS BY THE ULTRASONIC METHOD

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This paper refers to the possibilities of using thermal ultrasound methods for measurements of some acoustic and thermal quantities.

ИЗМЕРЕНИЕ ТЕРМИЧЕСКИХ ВЕЛИЧИН НЕКОТОРЫХ ИЗОЛЯЦИОННЫХ МАТЕРИАЛОВ С ПОМОЩЬЮ УЛЬТРАЗВУКА

В работе сообщается о возможностях использования термического ультразвукового метода для измерения некоторых акустических и термических величин.

I. INTRODUCTION

Basic thermal ultrasonic methods using for measurements of basic ultraacoustic quantities heat energy, which accompanies the ultrasound propagation in some solids with a relatively high absorption coefficient, are described in [1]. We can use the thermal ultrasonic method not only for measurements of basic ultraacoustic quantities but for measurements of some thermal quantities as well.

II. HEAT-FLOW EQUATION AND ITS SOLUTION

The basis of all methods dealing with measurements of thermal quantities is the knowledge of the temperature distribution in a sample. We can find the temperature distribution by applying definite conditions and by the solution of the heat-flow equation [2].

$$\gamma\varrho \frac{\partial T}{\partial t} = \mathrm{div}\,[\lambda \,\mathrm{grad}\, T] + F(x, y, z, t)\,, \qquad (1)$$

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where γ is the specific heat, ϱ the density, λ the thermal conductivity, T the temperature and $F(x, y, z, t)$ is the density of the heat sources determining the amount of heat generated in a unit volume per unit of time. When a substance is homogeneous and isotropic, then γ, ϱ and λ are constants and Rel. (1) may be written in the following form

$$\frac{\partial T}{\partial t}=a^2\Delta T+f(x, y, z, t)\,, \tag{2}$$

where $a^2=\lambda/\gamma\varrho$ and $f(x, y, z, t)=F(x, y, z, t)/\gamma\varrho$. When a sample is not heat-isolated from its environment we have to take into account the heat drained off by the environment in equation (2). Then we can write for the density of the heat source [2]

$$F_1(x, y, z, t)=F(x, y, z, t)-\alpha(T-T_0) \tag{3}$$

and Eq. (2) may be written in the form

$$\frac{\partial T}{\partial t}=a^2\Delta T-\varepsilon T+f_1(x, y, z, t)\,, \tag{4}$$

where α is the coefficient characterising the transfer of heat from the area delimiting a unit volume (α is connected with the heat transfer coefficient α_0 for the cylindrical sample of a radius r_0 with the relation $\alpha_0=r_0\alpha/2$), where T_0 is the temperature of environment, T temperature of the sample, $\varepsilon=\alpha/\gamma\varrho$ and $f_1(x, y, z, t)=F_1(x, y, z, t)/\gamma\varrho+T_0(x, y, z, t)$.

The different methods of solution of the differential Eq. (4) in some concrete cases can be found in [3] and [4]. With regard to simplicity we can assume that the sample has the shape of a cylinder having the length l and the diameter $d \ll l$ and its axis of symmetry coincides with the coordinate axis x. It is assumed further that the temperature $T(x, t)$ may be taken as equal at all points of the cross section of the sample, with the same coordinate x and at the same time t. Then Eq. (4) may be written in the form

$$\frac{\partial T}{\partial t}=a^2\frac{\partial^2 T}{\partial x^2}-\varepsilon T+f_1\,, \tag{5}$$

where T and f_1 are only the functions of the variable x and time t.

When we want to solve Eq. (5) we have to find the concrete form of the function determining the density of the heat sources. When we suppose that the heat in the sample is generated due to the absorption of the ultrasound in the mode of the stationary waves we may write [5]

$$f(x, t)=A+B\cos\frac{4\pi x}{\Lambda}\,, \tag{6}$$

where A and B are quantities depending especially on the mechanical properties of the material of the sample and Λ is the wavelength of the ultrasound in the sample. Let the initial and the boundary conditions be given according to (6) in the form

$$T(x, 0)=T_0, \qquad T(0, t)=T_0\,, \qquad T(l, t)=T_0\,. \tag{7}$$

Equation (5) complemented by conditions (7) will be solved by defining the function $v(x, t)$, which expresses the deviation from some known function $U(x, t)$ for which there is adopted [3]

$$T(x, t)=U(x, t)+v(x, t)\,. \tag{8}$$

By introducing this into Eq. (5) for $v(x, t)$ we get the equation

$$\frac{\partial v}{\partial t}-a^2\frac{\partial^2 v}{\partial x^2}+\varepsilon v=\bar{f}(x, t)\,, \tag{9}$$

where

$$\bar{f}(x, t)=f(x, t)-\left(\frac{\partial U}{\partial t}-a^2\frac{\partial^2 U}{\partial x^2}\right). \tag{10}$$

If we choose the function $U(x, t)$ in the form

$$U(x, t)=T_0\,,$$

the solution of equation (5) is transferred into the solution of equation (9) with zero boundary conditions.

Since the function (6) is periodical and even, we can find the solution of the function (9) in the form of the Fourier series

$$v(x, t)=\sum_{n=0}^{\infty} v_n(t)\cos\frac{n\pi x}{l}\,. \tag{11}$$

If we distribute the function $\bar{f}(x, t)$ into series

$$\bar{f}(x, t)=\sum_{n=0}^{\infty}\bar{f}_n\cos\frac{n\pi x}{l} \tag{12}$$

where

$$\bar{f}_n=\frac{2}{l}\int_0^l f(\xi, t)\cos\frac{n\pi\xi}{l}\,d\xi \tag{13}$$

and the assumed solution (11) is substituted into equation (9), we obtain the condition for the solution of equation (9) in the form

$$\frac{dv_n(t)}{dt}+\left[\left(\frac{n\pi a}{l}\right)^2+\varepsilon\right]v_n(t)-\bar{f}_n(t)=0 \tag{14}$$

and from that

$$v_n(t)=\int_0^t \bar{f}_n(t)e^{-[(n\pi a/l)^2+\varepsilon](t-\tau)}d\tau\,. \tag{15}$$

Thus for the solution of Eq. (9) we can write

$$v(x,t)=\sum_{n=0}^{\infty}\int_0^t e^{-[(n\pi a/l)^2+\varepsilon](t-\tau)}\bar{f}_n(t)\cos\frac{n\pi a}{l}\,d\tau\,. \tag{16}$$

From comparison of (6) and (12) it follows that the relation (16) may be written in the form

$$v(x,t)=\int_0^t\left\{A\,e^{-\varepsilon(t-\tau)}+B\,e^{-[(4\pi a/\Lambda)^2+\varepsilon](t-\tau)}\cos\frac{4\pi x}{\Lambda}\right\}d\tau\,. \tag{17}$$

When Eq. (17) is integrated and ε substituted on the basis of the boundary conditions we have

$$T(x,t)=T_0+\frac{Ad\gamma\varrho}{4\alpha}\left(1-e^{-(4\alpha/d\gamma\varrho)t}\right)+ \tag{18}$$

$$+\frac{B}{\left(\frac{4\pi a}{\Lambda}\right)^2+\frac{4\alpha}{d\gamma\varrho}}\left\{1-e^{-[(4\pi a/\Lambda)^2+(4\alpha/d\gamma\varrho)]t}\right\}\cos\frac{4\pi x}{\Lambda}\,.$$

In the case of the steady state, i.e. $t\to\infty$ for the surface temperature distribution along the sample, we obtain

$$T(x)=T_0+\frac{Ad\gamma\varrho}{4\alpha}+\frac{B}{\left(\frac{4\pi a}{\Lambda}\right)^2+\frac{4\alpha}{d\gamma\varrho}}\cos\frac{4\pi x}{\Lambda}\,. \tag{19}$$

The formula (19) is important because it connects $T(x,t)$ and the heat transfer coefficient. Let us compare this result with the temperature distribution in the case of the sample being heat isolated from its environment. Then we can write [6]

$$T(x,t)=At+B\left(\frac{\Lambda}{4\pi a}\right)^2\cos\frac{4\pi x}{\Lambda}\,. \tag{20}$$

The first term of Eq. (19) is determined by the temperature of the environment and does not appear in relation (20) because in the process of derivation we assume that the environmental temperature is zero. The second term in Rel. (19) corresponds to the first term in Rel. (20). While for the heat isolated sample this term expresses a uniform increase of temperature in time along the whole sample, in the case of heat transfer into the environment this term acquires in the steady state a time independent value. The third term in Rel. (19) corresponds to the second term in Rel. (20), while both these terms have the same period. Owing to

this fact the environment has no influence on the position of peaks of the surface temperature along the sample.

III. MEASUREMENT OF THERMAL QUANTITIES

The methods of thermophysical quantity measurements are well described in [7]. The basis of all these methods is the knowledge of the surface temperature distribution in the sample depending mainly on the effect and shape of the heat source. There is no known case in which the energy of the mechanical waves absorbed in the sample through which it propagates is the source of heat. We shall point out the possibilities resulting from relations (19) and (20) which we have arrived at.

Assuming the drain of heat into the environment we have to start from relation (19). Only the third term of the sum on the right-hand side of this relation is important for the following consideration:

$$T(x)=\frac{B}{\left(\frac{4\pi a}{\Lambda}\right)^2+\frac{4\alpha}{d\gamma\varrho}}\cos\frac{4\pi x}{\Lambda}\,, \tag{19a}$$

where the meaning of all quantities is clear from what has already been mentioned. If we plot the functional dependence $T(x)$ determined by equation (18) for the third term of the sum and for the temperature maximum (Fig. 1), we obtain the curve asymptotically approaching the value

$$T_1=\frac{B}{\left(\frac{4\pi a}{\Lambda}\right)^2+\frac{4\alpha}{d\gamma\varrho}}\,. \tag{21}$$

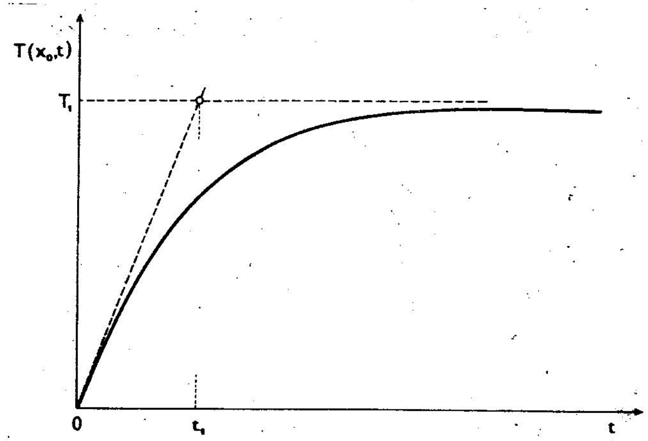

Fig. 1. Relation between temperature T and time t at the temperature maximum.

We are able to measure the temperature T_1 directly and we can determine the wavelength of the ultrasound, e.g. by the method described in [5], but there are three unknown quantities B, α and a in equation (21).

The quantity B is determined by the relatively complicated relation [5] and its solution is not easy, but we can solve it graphically. As the derivative of the third term of the sum on the right-hand side of Rel. (18) at the point (0.0) equals B, the tangent of the function (18) at this point (0.0) has the equation $T = Bt$. This tangent crosses the asymptote $T = T_1$ at the point $(t_1 \,.\, T_1)$ and it is obvious that

$$t_1 = \frac{1}{\left(\frac{4\pi a}{\Lambda}\right)^2 + \frac{4\alpha}{d\gamma\varrho}} \tag{22}$$

from which we have for α and λ

$$\alpha = \frac{d\gamma\varrho}{4}\left[\frac{1}{t_1} - \left(\frac{4\pi}{\Lambda}\right)^2 \cdot \frac{\lambda}{\gamma\varrho}\right] \tag{23}$$

$$\lambda = \left(\frac{\Lambda}{4\pi}\right)^2 \left(\frac{\gamma\varrho}{t_1} - \frac{4\alpha}{d}\right). \tag{24}$$

We can substitute the determination of B by the determination of the time t_1, which can be measured directly. When we measure the time dependence of temperature at the temperature maximum on the sample surface $T(t)$ and determine the temperature T_1, the tangent at the point (0.0) crosses the straight line $T = T_1$ at the point $(t_1 \,.\, T_1)$.

To exclude one of the two remaining unknown quantities α and λ in relations (23) and (24), respectively, we have to be satisfied with a certain approximation. The value of the term $(4\pi a/\Lambda)^2$ is for common materials as much as 100 times smaller than the value of the term $4\alpha/d\gamma\varrho$. With regard to that we can neglect the first term of the sum in the denominator of Rel. (22) and instead of the relation (23) we obtain

$$\alpha = \frac{d\gamma\varrho}{4t_1}. \tag{25}$$

If the sample with stationary ultrasound waves is heat isolated from its environment along the axis of the sample, we can write [6]

$$T(x, t) = At + B\left(\frac{\Lambda}{4\pi a}\right)^2 [1 - e^{-(4\pi a/\Lambda)^2 t}] \cos\frac{4\pi x}{\Lambda}. \tag{26}$$

If this function is derived with respect to time at the temperature maximum, we obtain (for $x = x_0$)

$$\partial T_M/\partial t = A + B \tag{27}$$

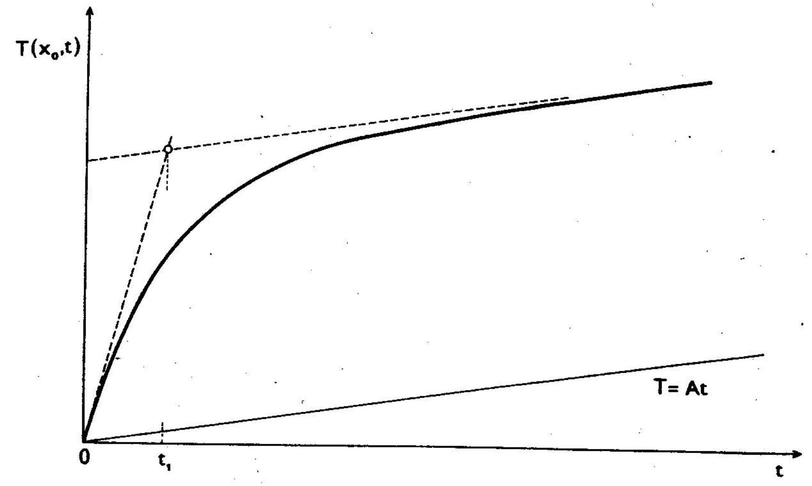

Fig. 2. Determination of time t_1.

and thus the tangent at zero point has the equation

$$T = (A + B)t\,. \tag{28}$$

In a steady state

$$T_M(x_0, t) = At + B\left(\frac{\Lambda}{4\pi a}\right)^2 \tag{29}$$

from which $\partial T_M/\partial t = A$, hence the asymptote has the equation

$$T = At + B\left(\frac{\Lambda}{4\pi a}\right)^2. \tag{30}$$

Solving the system of equations (28) and (30) we obtain for the time t_1 (Figure 2)

$$t_1 = \left(\frac{\Lambda}{4\pi a}\right)^2 \tag{31}$$

and from that there follows for the thermal conductivity

$$\lambda = \frac{\varrho\gamma\Lambda^2}{16\pi^2 t_1}. \tag{32}$$

From this we can see that the thermal ultrasound methods enable to measure both the heat drain coefficient and the thermal conductivity. For the determination

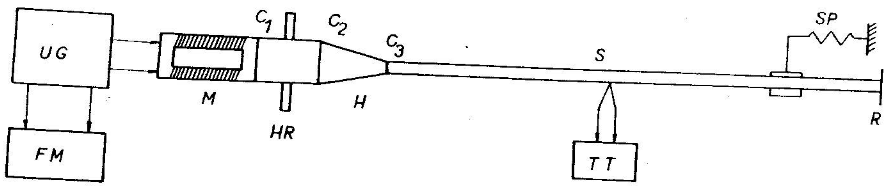

Fig. 3. Block diagram of the experimental device. UG — ultrasonic generator; FM — frequency meter; M — magnetostrictive transducer; HR — half wave resonator; H — horn; SP — spring; TT — thermistor thermomet; S — sample; R — reflector; C_1, C_2, C_3 — acoustic couplers.

of both these quantities it is enough to known the time dependence of temperature at the temperature maximum on the sample surface. The time t_1 can be determined by construction from this time dependence and from the value t_1 and on the basis of relations (25) and (32), respectively, both quantities mentioned above.

IV. EXPERIMENTAL RESULTS

The possibility to measure some thermal quantities by ultrasonic methods was verified on the experimental device in Fig. 3. The measurement was done at room temperature.

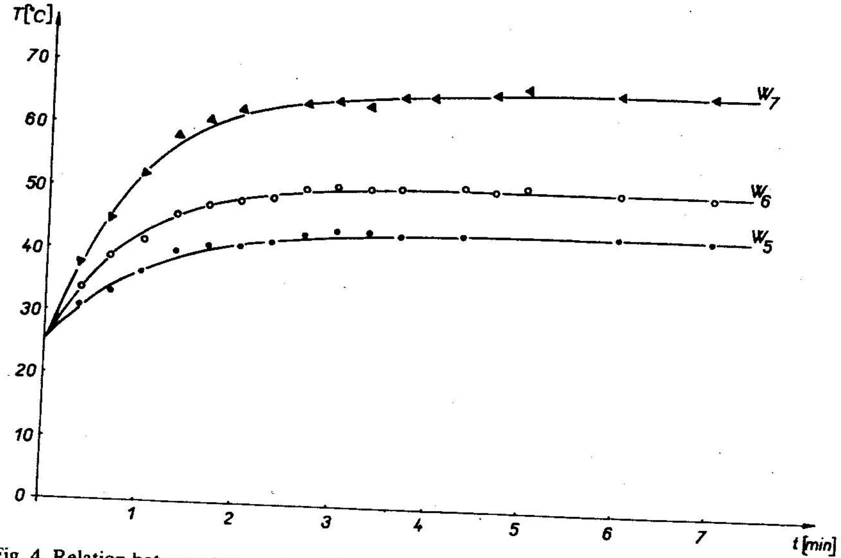

Fig. 4. Relation between temperature T and time t for slowly moving air and different acoustic outputs ($W_5 < W_6 < W_7$).

The investigated sample was in the shape of a cylinder with the diameter of 5 mm and its length was several tens of centimetres. The sample was long enough so that the keeping of the boundary conditions had practically no effect on the magnitude of heat maxima inside the sample. There was a reflector R at the one end of the sample securing the generation of stationary waves. As a source of ultrasound a magnetostrictive transducer M with the resonance frequency of 16.8 kHz was used

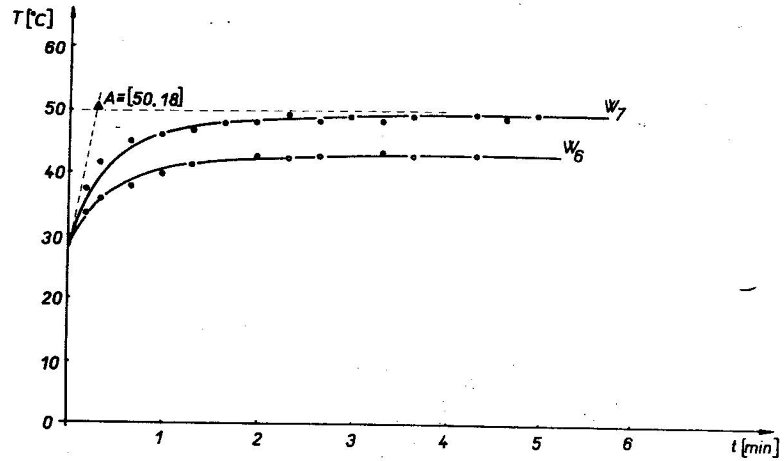

Fig. 5. Relation between temperature T and time t for quicker moving air and two acoustic outputs ($W_6 < W_7$).

and feeded by an ultrasonic generator UG (VUMA-UG-250). The transducer M was complemented by the half wave resonator and catenoidal horn H with the amplification factor 10.6. With regard to lowering the acoustic energy losses on junctions the individual components were silversoldered together. For a better connection with the ultrasonic horn H the sample was pressed to it by the spring SP.

The surface temperature was measured by a thermistor thermometer consisting of the thermistor 14NR15 and a measuring device. The thermistor was in non-current mode.

First the heat transfer coefficient for the air at the temperature of the environment $T_0 = 25$ °C was determined. As a standard an oak wood cylinder was used. The parameter $t_1 = (74 \pm 4)$ s was determined from the temperature time dependence at the temperature peak for slowly moving air (Figure 4). To this value of the t_1 there corresponds the heat transfer coefficient $\alpha_0 = 23.3$ Jm^{-2}K^{-1}s^{-1} (computed per area unit with the relation $\alpha_0 = \alpha d/4$ — for the cylindrical sample). For more rapidly moving air (propelled by a small air compressor), for which the time dependence is in Fig. 5, we have $\alpha_0 = 101.6$ Jm^{-2}K^{-1}s^{-1}. The obtained values are in

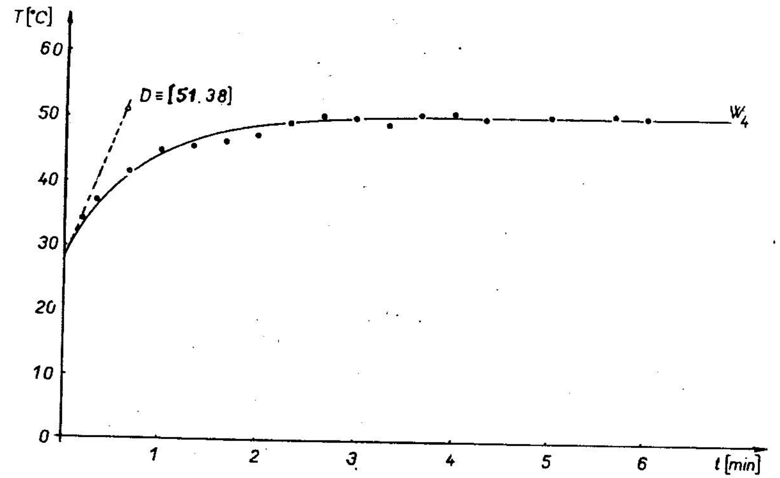

Fig. 6. Relation between temperature T and time t at the temperature maximum for a sample made of oak wood and an output W_4.

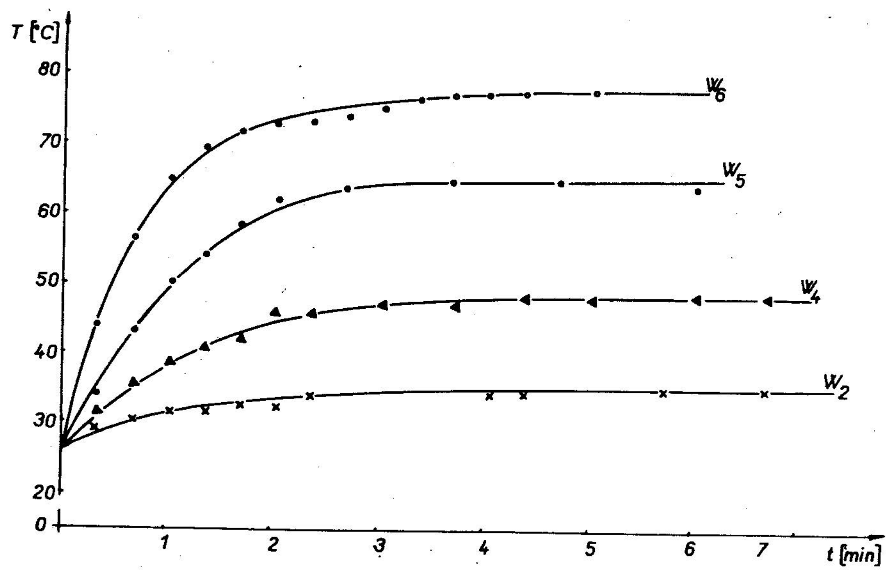

Fig. 7. Relation between temperature T and time t at the temperature maximum for a sample made of texgumoid for different acoustic outputs ($W_6 > W_5 > W_4 > W_2$).

agreement with those described, e.g. in [2], where the heat transfer coefficient for moving air is within the range from 23 to 120 $Jm^{-2}K^{-1}s^{-1}$. Figures 6 and 7 present the time dependence on temperature in maxima for samples made of oak wood and texgumoid in very slowly moving air. For oak wood we obtain after determining t_1 the value of the thermal conductivity $\lambda = 0.21\ WK^{-1}m^{-1}$, which is in good agreement with the value in [7]. A 10 % deviation can easily be explained by the difference in temperature between the heat transfer coefficient measurement (25 °C) and the temperature of the environment during the measurement of the thermal conductivity (28 °C).

We do not mention the results for texgumoid because we cannot compare them with the table values.

V. CONCLUSION

The measurement of some thermal quantities by the ultrasound methods mentioned in this paper means a further extension of thermal ultrasound methods, which were primarily used only for the measurement of basic ultraacoustic quantities [1]. We can not use this method for arbitrary materials, only for those with a relatively great absorption coefficient, which has as a consequence the rise of a sufficient thermal energy. That is why plastics are very suitable. We believe that a wider application of thermal ultrasonic methods makes a definite contribution to the measurement methods for the investigation of the properties of solids and that the thermal ultrasonic methods can help not only in the course of the educational process at the university but in practic as well.

REFERENCES

[1] Štrba A., Fyz. čas. SAV *21* (1971), 2.
[2] Kondratiev G. M., *Teplovyje izmerenija.* Mašgiz, Moskva 1957.
[3] Tichonov,A. N., Samarskij A. A., *Uravnenia matematičeskoj fiziki.* Goz. izd. tech.-teor. lit. Moskva—Leningrad 1951.
[4] Carslaw H. S., Jaeger J. C., *Conduction of heat in solids.* Claredon Press, Oxford 1959.
[5] Štrba A., *Dissertation,* 1971.
[6] Štrba A., Čsl. čas. fys. *A 22* (1972), S.
[7] Krempaský J., *Meranie termofyzikálnych veličín.* Nakl. SAV, Bratislava 1969.

Received July 29th, 1975