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ELECTROPRODUCTION DATA AND THE TRANSVERSE MOMENTUM CUT-OFF IN THE PARTON MODEL

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A transverse momentum cut-off in the parton model is discussed on a phenomenological level. Electroproduction data are shown to indicate correlations between longitudinal and transverse momenta of partons. Modifications of the Drell-Yan relation due to various forms of transverse momentum cut-offs are investigated.

I. INTRODUCTION

The parton model has been quite successfully used in describing deep inelastic lepton scattering data. Recent neutrino experiments seem to agree with the predictions of the quark-parton model based on results of electron scattering data [1, 2, 3]. Both electron and neutrino seem to see the same parton distribution functions.

However, if transverse momenta of partons within a hadron are limited, only the longitudinal momentum distribution functions of partons play an important role in deep inelastic lepton scattering. Therefore the longitudinal momentum distribution functions are known, at least qualitatively [4, 5, 6], much better than the distribution functions of the transverse momentum. The existence of a cut-off of transverse momenta of partons within a hadron is suggested by small mean transverse momenta of hadrons produced in hadronic collisions. However, the mechanism of this cut-off has so far not been explained [7].

In this paper we want to discuss the problem of the transverse momenta of partons on a phenomenological level. In Sec. II we try to obtain some information on the transverse momenta of partons by a simple analysis of the electroproduction data. In Sec. III we discuss the possible modifications of the Drell-Yan relation due to various types of transverse momentum cut-offs. The used normalizations and conventions on parton wave functions are presented in the Appendix.

II. ELECTROPRODUCTION DATA AND THE TRANSVERSE MOMENTUM CUT-OFF

In the parton model calculation it is usually assumed that the transverse momentum cut-off is independent of the parton longitudinal momenta. In this

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section we discuss the consistency of this assumption with the electroproduction data.

Since nobody so far has been fortunate enough to see free partons, the information about parton distribution functions has to be deduced from the observed outgoing hadron distribution. Of course, a model for the hadron production from partons is necessary for such an analysis. In this paper we assume that hadrons are produced by a parton fragmentation via a chain decay.

According to the usual assumptions of the parton fragmentation, if one wants to learn something about the parton momenta one has to look for hadrons produced in the first decay of the parton chain decay. Hadrons produced during the final stages of the chain decay are supposed to be produced completely statistically, "forgetting" all the information about their parent parton.

The electroproduction seems to be the most suitable process for discussing problems connected with the parton distribution functions. In hadron-hadron collisions, there exists an additional problem of "elementary" parton-parton interaction.

We are interested in the inclusive electroproduction:

$$e+p \rightarrow e+h+\text{anything}$$

The interaction mechanism can be sketched as in Fig. 1.

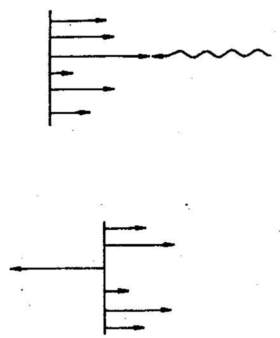

Fig. 1. Virtual photon absorption in the Breit frame.

The virtual photon is absorbed by a parton. The parton is kicked out of the proton.

We define the variables as follows

ν, $-Q^2$: energy and four momenta squared of the virtual photon in the laboratory frame

$x, p_\perp$: the Feynman variable and the transverse momentum of the hadron h in the CMS of the virtual photon and the proton.

If the initial proton momentum in the Breit frame is P, then the longitudinal momentum of the interacting parton is $x_{BJ}P$, where $x_{BJ}=Q^2/2M\nu$, is the Bjorken

scaling variable. It means that if one knows ν and Q^2, one knows the longitudinal momentum fraction of the kicked out parton* $\eta = x_{BJ}$

The transverse momentum dependence of the inclusive cross section can be parametrized as

$$\frac{d\sigma}{d p_\perp^2} \sim \exp(-Bp_\perp^2).$$

The slope parameter B seems to be, however, Q^2 dependent [8, 9]. The Q^2 dependence, for a fixed value of ν, may be interpreted within the framework of the parton model, as a consequence of the dependence on the variable x_{BJ}. After a simple calculation one can present the data [8, 9] in the form of Fig. 2.

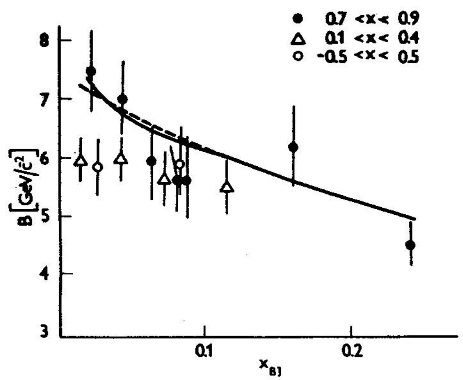

Fig. 2. Value of the slope parameter B plotted against Bjorken scaling variable for different x values, for inclusive π^+ electroproduction. Data are taken from Ref. [9]. The lines represent our fits (1) — the solid line and (2) — the dashed line. ● $0.7<x<0.9$; △ $0.1<x0.4$; ○ $-0.5<x<0.5$.

The data suggest that for large values of the Feynman variable x, the parameter B decreases with an increasing $\eta = x_{BJ}$. For smaller x values B seems to be nearly independent of η.

However, the electroproduction data discussed here come from the region of rather small Q^2 : $0<Q^2<2.5$ $(\text{GeV/c})^2$. This region is still rather far from the region of the expected Bjorken scaling. Therefore in addition to the x_{BJ} dependence, the "pure" Q^2 dependence is surely present in the data. However, we shall investigate the possibility, whether the above mentioned qualitative features of the data can be found for larger Q^2, too.

According to the usual assumptions of the chain decay model of the parton fragmentation, hadrons with large x values are produced during the first stages of the chain decay and therefore they can carry some information about their parent parton.

* In the present paper we denote the longitudinal momentum fraction as η (not by x as it is more common) to avoid the confusion with the Feynman variable.

Thus the data suggest that if the parton transverse momentum cut-off is of the form

$$\exp(-Ap_\perp^2),$$

the parameter A is probably η-dependent and decreases with an increasing η.

However, the transverse momentum of the outgoing hadron is not equal to the transverse momentum of its parent parton because of the fragmentation process.

Since we are interested only in the qualitative analysis, we shall not perform any exact calculation of the fragmentation process. For the sake of simplicity we shall assume, rather arbitrarily, that the outgoing hadron spectrum can be expressed by a "classical" form of the convolution

$$\exp(-B\boldsymbol{p}^2) \sim \int \exp(-A\bar{\boldsymbol{p}}_\perp^2)\exp(-\alpha(\boldsymbol{p}_\perp-\bar{\boldsymbol{p}}_\perp)^2\,\mathrm{d}^2\bar{\boldsymbol{p}})$$

where

$$B=\frac{A\alpha}{A+\alpha}.$$

In the case of the classical theory the expression

$$\exp(-\alpha\bar{p}_\perp^2)$$

would play the role of the probability that the first decay product of a parton fragmentation has the transverse momentum $\bar{p}_\perp$ relatively to the parton direction.

We have tested two parametrizations*)

1) $A=A'\,|\ln\eta|$ (1)

2) $A=A'/\eta$. (2)

With A' and α as free parameters good fits were obtained for both parametrizations for the data from the region $0.7<x<0.9$. The following values for the parameters A' and α were obtained: $A'=6.3$; $\alpha=10.3$ for the parametrization (1) and $A'=3.2$; $\alpha=7.5$ for the parametrization (2). (Fig. 2). For both the fits $\chi^2\approx 0.2$ per degree of freedom. (For the η-independent parametrization ($B=$ const.) a good fit cannot be obtained: $\chi^2\approx 2.3$ p.d.f.).

It is clear that although the data suggest that the transverse momentum cut-off depends on the longitudinal momentum of the partons, it is difficult to find the right form of this dependence by the above mentioned data analysis.

Therefore one has to look for other experimental information which could help to choose between various possible forms of this dependence. We shall show that such an information can be found in the Drell-Yan relation.

*) As we shall show shortly, these two parametrizations give very different results for the form of the Drell-Yan relation.

III. THE DRELL-YAN RELATION

The Drell-Yan relation connects the threshold behaviour of the structure function νW_2 with the asymptotic behaviour of the elastic formfactor [11]. This relation states that if

$$\nu W_2 \sim (1-x_{BJ})^\gamma \qquad \text{for} \qquad x_{BJ}\to 1,$$

then

$$F(Q)\sim 1/Q^{\gamma+1} \qquad \text{for} \qquad Q\to\infty.$$

However, this relation was derived [11] using the assumption of the η-independent transverse momentum cut-off. We shall show here briefly that more general assumptions about a transverse momentum cut-off can lead to the modification of the Drell-Yan relation. It is well known that the Drell-Yan relation is in good agreement with the experiment in its original form with $\gamma=3$. Therefore modifications of the Drell-Yan relation do not seem to be acceptable and in this way one can reject such types of cut-offs which lead to the modifications.

In terms of the momentum space wave functions the structure function and the formfactor are given as [10, 11]

$$\nu W_2(x_{BJ})=\sum_n\sum_a x_{BJ}\lambda_a^2\int\prod_{i=1}^n\left(\frac{\mathrm{d}\,\eta_i}{\eta_i}\,\mathrm{d}^2K_i\right)\delta(\Sigma \quad \delta(1-\Sigma\,\eta_i)$$
$$\delta(x_{BJ}-\eta_a)\,|f(\eta_1,\ldots,\eta_n,\,\boldsymbol{K}_1,\ldots,\boldsymbol{K}_n)|^2$$

$$F(Q)=\sum_n\sum_a\lambda_a\int\prod_{i=1}^n\left(\frac{\mathrm{d}\,\eta_i}{\eta_i}\,\mathrm{d}^2K_i\right)\delta(\Sigma\,K_i)\,\delta(1-\Sigma\,\eta_i)\times$$
$$\times f^*(\eta_1,\ldots,\eta_n,\,\boldsymbol{K}_1-\eta_1\boldsymbol{Q},\ldots,\boldsymbol{K}_a+(1-\eta_a)\boldsymbol{Q},\ldots$$
$$\ldots\boldsymbol{K}_n-\eta_n\boldsymbol{Q})\,f(\eta_1,\ldots,\eta_n,\,\boldsymbol{K}_1,\ldots,\boldsymbol{K}_n),$$

where λ_a is the charge of the a-th parton. For formal reasons it is useful to transform these expressions into the transverse position space base [12] (see Appendix):

$$\nu W_2(x_{BJ})=\sum_n\sum_a\lambda_a^2x_{BJ}\int\prod_{i=1}^n\left(\frac{\mathrm{d}\,\eta_i}{\eta_i}\,\mathrm{d}^2\boldsymbol{b}_i\right)\delta(1-\Sigma\,\eta_i)\,\delta(\Sigma\eta_i\boldsymbol{b}_i)\times$$
$$\times\delta(x_{BJ}-\eta_a)\,|\Psi(\eta_1,\ldots,\eta_n,\,\boldsymbol{b}_1,\ldots,\boldsymbol{b}_n)|^2$$

$$F(Q)=\sum_n\sum_a\lambda_a\int\prod_{i=1}^n\left(\frac{\mathrm{d}\,\eta_i}{\eta_i}\,\mathrm{d}^2b_i\right)\delta(1-\Sigma\,\eta_i)\,\delta(\Sigma\eta_i\boldsymbol{b}_i)\times$$
$$\times e^{i\boldsymbol{Q}\boldsymbol{b}_a}\,|\Psi(\eta_1,\ldots,\eta_n,\,\boldsymbol{b}_1,\ldots,\boldsymbol{b}_n)|^2$$

A transverse momentum cut-off has to be introduced into the above expressions. This is in general a subtle question because of the momentum conservation.

However, we are not going to give a general rigorous proof. We want to show that some parametrization of the cut-off can lead to the modification of the Drell-Yan relation. Furthermore we shall see that in the region $Q^2 \to \infty$ only the region $\eta_a \to 1$ contributes. Because η plays the role of the particle mass in transverse motion [10], relation $\eta_a \to 1$ means that one of the partons is very "heavy". Therefore we assume as physically plausible the following Ansatz

$$\Psi(\eta_1, \ldots, \eta_n, \boldsymbol{b}_1, \ldots, \boldsymbol{b}_n) = \Phi(\eta_1, \ldots, \eta_n) \prod_{i=1}^{n} \frac{1}{\sqrt{\pi A(\eta_i)}} \times \exp\left(-\frac{(\boldsymbol{b}_i - \boldsymbol{b}_a)^2}{2A(\eta_i)}\right),$$

where $A(\eta_i)$ is a function of η_i. After a simple calculation one gets

$$\nu W_2(x_{BJ}) = \sum_n \sum_a x_{BJ} \lambda_a^2 \int \prod_{i=1}^{n} \left(\frac{\mathrm{d}\,\eta_i}{\eta_i}\right) \delta(1 - \Sigma\, \eta_a)\, \delta(x_{BJ} - \eta_a) \times |\Phi(\eta_1, \ldots, \eta_n)|^2 \tag{3}$$

$$F(Q) = \sum_n \sum_a \lambda_a \int \prod_{i \neq a} \left(\frac{\mathrm{d}\,\eta_i}{\eta_i} \exp\left(-\frac{1}{4} A(\eta_i) Q^2 \eta_i^2\right) \frac{\mathrm{d}\,\eta_a}{\eta_a} \delta(1 - \Sigma\, \eta_i)\right) \times |\Phi(\eta_1, \ldots, \eta_n)|^2 . \tag{4}$$

If $A(\eta_i) = A = \text{const.}$, then the main contribution to the integral (4) comes from the region

$$0 < \eta_i < 1/Q .$$

Following [11] if one takes

$$\int_{1-1/Q}^{1} \nu W_2(x_{BJ})\, \mathrm{d}x_{BJ} ,$$

one obtains in the r.h.s. of (3) a similar expression as in (4). Thus the η--independent cut-off gives

$$Q \to \infty \qquad F(Q) \sim \int_{1-1/Q}^{1} \nu W_2(x_{BJ})\, \mathrm{d}x_{BJ}$$

and the Drell-Yan relation results.

Assuming now as in (2)

$$A(\eta_i) = A/\eta_i ,$$

one finds that the main contribution to the integral (4) comes from the region

$$0 < \eta_i < 1/Q^2$$

and one gets

$$Q \to \infty : F(Q) \sim \int_{1-1/Q^2}^{1} \nu W_2(x_{BJ})\, \mathrm{d}x_{BJ} .$$

This gives the modification of the Drell-Yan relation of the form

$$\nu W_2 \sim (1 - x_{BJ})^\gamma \;\Rightarrow\; F(Q) \sim \frac{1}{Q^{2(\gamma+1)}} .$$

It seems that such a behaviour is ruled out experimentally.

We see that the expected η dependence of the transverse momentum cut-off should be a "gentle" one, otherwise the Drell-Yan relation would be spoilt.

Assuming as in (1)

$$A(\eta_i) = A\, |\ln \eta_i| ,$$

the cut-off in Eq. (4) has the form

$$\exp(-A\, |\ln \eta_i|\, \eta_i^2 Q^2) .$$

For $Q \to \infty$ the main contribution to the integral (4) comes from the region

$$0 < \eta_i < 1/Q$$

(up to $\sim \ln Q$) and from $\nu W_2 \sim (1 - x_{BJ})^\gamma$ we get

$$F(Q) \sim \frac{1}{Q^{\gamma+1}}$$

up to some logarithmic factor.

IV. CONCLUSION

On the basis of a phenomenological analysis, a "gentle" correlation seems to exist between transverse and longitudinal momenta of partons. The transverse momentum cut-off of the type

$$\exp(-A\, |\ln \eta|\, p_\perp^2)$$

agrees with the electroproduction data and does not spoil the Drell-Yan relation.

Additional experimental checks are certainly needed. One of the possibilities would be an application of the same analysis as in Sec. II to the diffractive dissociation experiment

$$\mathrm{p} + \mathrm{p} \to \pi + \text{anything}$$

in the region $x \to 1$. The ISR experiments are needed, to avoid the kinematical effects due to energy momentum conservation.

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APPENDIX

We repeat here briefly the basic kinematical aspects of the parton wave functions in the infinite momentum frame [10, 12]. Let the initial hadron momentum be

$$P^\mu = \left(P + \frac{M^2}{2P}, 0, 0, P\right) \qquad P \to \infty .$$

The particle states can be specified according to their longitudinal momentum

$$\beta P$$

and the transverse momentum $\boldsymbol{K}$ as

$$|\beta, K\rangle .$$

Denoting the free parton states as

$$|\beta_i, \boldsymbol{K}_i\rangle ,$$

we write the hadron states in terms of the momentum space wave function as

$$|\beta, \boldsymbol{K}\rangle = \sum_n \int \beta\delta(\beta - \Sigma\beta_i)\, \delta(\boldsymbol{K} - \Sigma\boldsymbol{K}_i) \times$$

$$\times f_{\beta K}(\beta_1, \ldots, \beta_n, \boldsymbol{K}_1, \ldots, \boldsymbol{K}_n) \prod_{i=1}^{n} \frac{d\beta_i}{\beta_i} d^2\boldsymbol{K}_i\, |\beta_i, \boldsymbol{K}_i\rangle .$$

The Lorentz and rotational invariances imply

$$f_{\beta K}(\beta_1, \ldots, \beta_n, K_1, \ldots, K_n) = f(\eta_1, \ldots, \eta_n, \boldsymbol{K}_1 - \eta_1\boldsymbol{K}, \ldots, \boldsymbol{K}_n - \eta_n\boldsymbol{K}),$$

where

$$\eta_i = \beta_i/\beta .$$

For some calculations it is convenient to introduce the transverse position space base

$$|\beta_i, \boldsymbol{b}_i\rangle = \frac{1}{2\pi} \int d^2\boldsymbol{K}_i\, e^{-i\boldsymbol{K}_i\boldsymbol{b}_i}\, |\beta_i, \boldsymbol{K}_i\rangle$$

and transverse position space wave functions $\Psi_{\beta K}(\beta_1, \ldots, \beta_n, \boldsymbol{b}_1, \ldots, \boldsymbol{b}_n)$:

$$|\beta, K\rangle = \sum_n \int \beta\, \delta(\beta - \Sigma\beta_i)\, \Psi_{\beta K}(\beta_1, \ldots, \beta_n, \boldsymbol{b}_1, \ldots, \boldsymbol{b}_n) \prod_{i=1}^{n} \frac{d\beta_i}{\beta_i} d^2\boldsymbol{b}_i\, |\beta_i, \boldsymbol{b}_i\rangle .$$

The translational invariances imply

$$\Psi_{\beta K}(\beta_1, \ldots, \beta_n, \boldsymbol{b}_1, \ldots, \boldsymbol{b}_n) = \frac{1}{2\pi} e^{i\boldsymbol{K}\boldsymbol{b}}\, \Psi(\eta_1, \ldots, \eta_n, \boldsymbol{b}'_1, \ldots, \boldsymbol{b}'_n),$$

where $\boldsymbol{b} = \sum_i \eta_i \boldsymbol{b}_i$; $\quad \boldsymbol{b}'_i = \boldsymbol{b}_i - \boldsymbol{b}$

The transformation relations are

$$\Psi(\eta_1, \ldots, \eta_n, \boldsymbol{b}_1, \ldots, \boldsymbol{b}_n) = \frac{1}{(2\pi)^{n-1}} \int \delta(\Sigma\, \boldsymbol{P}_i)\, f(\eta_1, \ldots, \eta_n, \boldsymbol{P}_1, \ldots, \boldsymbol{P}_n) \times$$

$$\times \prod_{i=1}^{n} e^{i\boldsymbol{P}_i\boldsymbol{b}_i}\, d^2\boldsymbol{P}_i$$

$$f(\eta_1, \ldots, \eta_n, \boldsymbol{P}_1, \ldots, \boldsymbol{P}_n) = \frac{1}{(2\pi)^{n-1}} \int \delta(\Sigma\, \eta_i\boldsymbol{b}_i)\, \Psi(\eta_1, \ldots, \eta_n, \boldsymbol{b}_1, \ldots, \boldsymbol{b}_n) \times$$

$$\times \prod_{i=1}^{n} e^{-i\boldsymbol{P}_i\boldsymbol{b}_i}\, d^2\boldsymbol{b}_i .$$

The wave functions are normalized as follows

$$\sum_n \int \prod_{i=1}^{n} \left(\frac{d\eta_i}{\eta_i} d^2\boldsymbol{P}_i\right) \delta(1 - \Sigma\, \eta_i)\, \delta(\Sigma\boldsymbol{P}_i)\, |f(\eta_1, \ldots, \eta_n, \boldsymbol{P}_1, \ldots, \boldsymbol{P}_n)|^2 = 1$$

$$\sum_n \int \prod_{i=1}^{n} \left(\frac{d\eta_i}{\eta_i} d^2\boldsymbol{b}_i\right) \delta(1 - \Sigma\, \eta_i)\, \delta(\Sigma\boldsymbol{b}_i\eta_i)\, |\Psi(\eta_1, \ldots, \eta_n, \boldsymbol{b}_1, \ldots, \boldsymbol{b}_n)|^2 = 1 .$$

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