EFFECTS OF INERTIA OF THE MEASURING APPARATUS AND THE FINITE PULSE-TIME IN MEASUREMENTS OF THERMAL DIFFUSIVITY BY THE FLASH METHOD

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The present paper discusses the effect of inertia of the measuring apparatus acting simultaneously with the finite pusle time effect for the case of the first order transfer characteristic and the exponential pulse shape. The results of the analysis are also useful for separate treatment of these effects and allow to judge their influence upon the accuracy of the thermal diffusivity measurement. It will be shown that the above mentioned effects are to some extent additive, allowing thus the construcion of a simple relation for thermal diffusivity.

I. INTRODUCTION

The flash method of the thermal diffusivity measurement, first proposed by Parker et al. [1], has been worked out considerable detail. It concerns mainly the judgement of the influence of some distrurbing factors such as heat losses, finite pulse duration, etc., which are analysed separately. The review paper [2] summarizes the already treated effects and points to effect of inertia of the measuring apparatus and the temperature dependence. The aim of this paper is to examine the effect of inertia of the measuring apparatus together with the finite pulse-time effect.

In contrast to other pulse techniques, the flash method uses a noncontact and noninertial heat source and samples of finite dimensions. The principle of this method is as follows (see Fig. 1): a pulse source of radiant energy 1 irradiates the front surface of the sample 2. Part of this energy is absorbed in a thin surface layer and is converted into heat, which is diffused through the sample. The resulting temperature history on the rear surface is picked up by a thermocouple and recorded by a measuring apparatus. The experimental

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realization of this method must fulfil the following conditions: the heat flow is onedimensional, the material properties are temperature independent, the heat pulse is uniform over the sample surface, planar and instantenous, the temperature sensor is ideal, the measuring device noninertial and there are

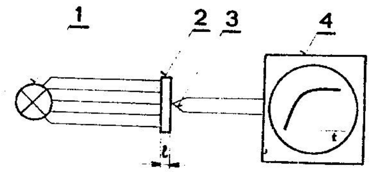

Fig. 1. Schematic plot of the flash method.

no heat losses from surfaces. Then the temperature history of the back surface of the sample is given by the equation [1]:

$$T(t) = T_M(1 + 2\sum_{n=1} -(1)^n \exp(-n^2\pi^2 t/\vartheta), \qquad (1)$$

where T_M is the final temperature of the sample $T_M = q_0/c\varrho l$, q_0 is the total amount of heat absorbed by unit of surface area. This may be expressed through the time dependent pulse heat flow $q(t)$. In the case of the instantaneous pulse $q(t) = q_0\delta(t)$, where $\delta(t)$ is the Dirac celta function, we have

$$q_0 = \int q\,dt = \int_{-\infty}^{+\infty} q_0\delta(t)\,dt. \qquad (2)$$

The other parameters used are: c the specific heat, ϱ the density, L the sample thickness, $\vartheta = L^2/k$ is the characteristic time of heat diffusion in the sample and k is the thermal diffusivity.

The thermal diffusivity is determined from (1), when $T(t) = 0.5\,T_M$. The time $t_{1/2}$, when this ratio is reached, is determined by the equation

$$t_{1/2} = 0.1388\,L^2/k. \qquad (3)$$

II. GENERAL SOLUTION

Let us examine the finite pulse time effect and the effect of inertia of the measuring apparatus. Let us assume that we know the analytic form of the pulse $q(t)$ and that the total amount of the heat absorbed by unit of area is the same as in the case of the instantaneous pulse $q_0 = q(t)dt$. Then the temperature response on the rear surface is obtained by the evaluation of the integral

$$T_1(t) = \int_0^t \frac{q(t-t')}{q_0} T(t')\,dt', \qquad (4)$$

where $T(t')$ is given by (1).

The further solution of this problem may be easily carried out by use of the Laplace transform. Since the transform of $T(t)$ is

$$T(p) = \frac{q_0\sqrt{k}}{\lambda\sqrt{p}\sinh\sqrt{p\vartheta}} \qquad (5)$$

then the transform of Duhamel's integral (4) is

$$T_1(p) = \frac{q(p)\sqrt{k}}{\lambda\sqrt{p}\sinh\sqrt{p\vartheta}}. \qquad (6)$$

The effect of inertia of the measuring apparatus is easy to solve if we know the transfer characteristic of the measuring apparatus

$$A(p) = u_2(p)/u_1(p), \qquad (7)$$

where $u_2(p)$ and $u_1(p)$ are Laplace transforms of the output and input signals. The input signal from the ideal temperature sensor is proportional to the temperature $T_1(t)$, so that $u_1(p) = KT_1(p)$, where K is the sensitivity of the sensor. Then, with the use of (7), the output signal has the form

$$u_2(p) = \frac{KA(p)q(p)\sqrt{k}}{\lambda\sqrt{p}\sinh\sqrt{p\vartheta}} \qquad (8)$$

and this is already the final solution of our problem.

If the heat pulse is instantaneous, then $q(p) = q_0$ and the response of the measuring device is

$$u_2(p) = \frac{KA(p)q_0\sqrt{k}}{\lambda\sqrt{p}\sinh\sqrt{p\vartheta}}. \qquad (9)$$

Comparing (9) with (6) it may be seen that the effect of inertia is the same as the finite pulse-time effect if $A(p) = q(p)$. On the other hand, the solution of (8) gives also the solution of the finite pulse-time effect with $q_v(p) = KA(p)q(p)$, or of the effect of inertia if the transfer characteristic has the form $A_v(p) = Kq_0A(p)q(p)$.

For a quantitative analysis we must know the analytical form of the transfer characteristic $A(p)$ and the pulse shape $q(p)$. It is common to characterize the measuring apparatus by a simple transfer characteristic of the first order in the form

$$A(p) = A_0/\tau_2(p + \tau_2^{-1}), \tag{10}$$

where τ_2 is the time constant of the apparatus. The shape of the pulse is dependent on the source of radiant energy used for heating the sample. So far pulses of a triangular and rectangular shape [3] have been analysed, of sawtooth shape [4] and the special case of the partly exponential pulse [5], which is a good approximation to a xenon flash lamp.

In our case we shall examine the exponential pulse with the time constant τ_1. The analytical form of this pulse is

$$q(t) = (q_0/\tau_1)\exp(-t/\tau_1) \tag{11}$$

and its Laplace transform

$$q(p) = \frac{q_0}{\tau_1(p + \tau_1^{-1})}. \tag{12}$$

Then the transform of the output signal is

$$u_2(p) = \frac{KA_0T_M}{\tau_1\tau_2}\,\frac{\sqrt{\vartheta}}{(p + \tau_1^{-1})(p + \tau_2^{-1})\sqrt{p}\sinh\sqrt{p\vartheta}}. \tag{13}$$

The original of this may be found by the inverse formula

$$u_2(t) = \frac{1}{2\pi_i}\int_c \exp(pt)u_2(p)\mathrm{d}p = \sum \mathrm{Res}\,[\mathrm{e}^{pt}u_2(p)], \tag{14}$$

where the residua shall be computed for the poles of the function $\mathrm{e}^{pt}u_2(p)$. These poles are: simple poles $p = 0$, $p = \tau_1^{-1}$, $p = \tau_2^{-1}$ and the infinite number of simple poles $p_n = -n^2\pi^2/\vartheta$. After evaluation and rearranging we obtain

$$u_2(t) = u_{2M}\left\{1 + 2\gamma_1\gamma_2\sum_{n=1}(-1)^n\frac{\exp(-n^2\pi^2t/\vartheta)}{(\gamma_1 - n^2\pi^2)(\gamma_2 - n^2\pi^2)} + \right.$$
$$\left. + \frac{\sqrt{(\gamma_1\gamma_2)}}{\gamma_2 - \gamma_1}\left[\sqrt{\gamma_1}\frac{\exp(-\gamma_2t/\vartheta)}{\sin\sqrt{\gamma_2}} - \sqrt{\gamma_2}\frac{\exp(-\gamma_1t/\vartheta)}{\sin\sqrt{\gamma_1}}\right]\right\} \tag{15}$$

where $u_{2M} = KA_0T_M$ and $\gamma_{1,2} = \vartheta/\tau_{1,2}$.

Let us now examine some special cases. If $\tau_2 \to 0$, what means that the apparatus is noninertial, Eq. (15) yields

$$u_2(t)/u_{2M} = 1 + 2\gamma_1\sum_{n=1}(-1)^n\frac{\exp(-n^2\pi^2t/\vartheta)}{\gamma_1 - n^2\pi^2} - \sqrt{\gamma_1}\frac{\exp(-\gamma_1t/\vartheta)}{\sin\sqrt{\gamma_1}} \tag{16}$$

and we obtain the temperature response of the exponential pulse. If $\tau_1 \to 0$, the pulse is instantaneous and the output signal from the recording device with the time constant τ_2 is the same as in the foregoing case, only γ_1 is to be substituted by γ_2.

A further case is the transfer characteristic in the form

$$A_v(p) = A(p)q(p) = A_0q_0/\tau_1\tau_2(p + \tau_1^{-1})(p + \tau_2^{-1}), \tag{17}$$

that is a second order transfer characteristic with the time constants τ_1 and τ_2. The response of the measuring apparatus to the instantaneous pulse is then given by (15). If $\tau_1 = \tau_2 = \tau_m$, we have the case of the proper second order transfer characteristic and (15) reduces to

$$\frac{u_2(t)}{u_{2M}} = 1 + 2\gamma_m^2\sum_{n=1}(-1)^n\frac{\exp(-n^2\pi^2t/\vartheta)}{\gamma_m - n^2\pi^2} -$$
$$- \frac{\sqrt{\gamma_m}\exp(-\gamma_mt/\vartheta)}{2\sin\sqrt{\gamma_m}}(1 + 2\gamma_mt/\vartheta + \sqrt{\gamma_m}\,\mathrm{cotg}\sqrt{\gamma_m}). \tag{18}$$

If the inertia of the measuring apparatus can be neglected and for the transform of the pulse $q_v(p) = A_v(p)$ is valid, then the results of the analysis are the same. The time depencence of this pulse expressed analytically is

$$q_v(t) = \frac{q_0}{\tau_2 - \tau_1}[\exp(-t/\tau_2) - \exp(-t/\tau_1)]. \tag{19}$$

For $\tau_1 = \tau_2 = \tau_m$

$$q_v(t) = \frac{q_0}{\tau_m^2}t\exp(-t/\tau_m), \tag{20}$$

which is the case already analysed by Larson and Koyama [5]. The result of their analysis is Eq. (18).

III. RESULTS

In this section the results of the numerical analyses of Eqs. (15), (16), (18) and (19) are given. For simplicity, we introduce new variables

$$\tau = \tau_1 + \tau_2, \qquad z = \tau_1/\tau_2. \tag{21}$$

Then for the chosen values z and $\gamma = \vartheta/\tau$ and the ratio $u_2(t_{1/2})/u_{2M} = 0.5$, the corresponding values of $\alpha = t_{1/2}/\vartheta$ were found. Since ϑ is not an experimental quantity, they were transformed into a new value $\tau/t_{1/2} = (\alpha\gamma)^{-1}$, which can be determined experimentally. The thermal diffusivity is then found from the relation

$$k = \alpha \frac{L^2}{t_{1/2}} = \frac{\alpha}{0.1388} k_{id} = f_k k_{id}, \qquad (22)$$

where f_k is the correction factor and k_{id} is the uncorrected diffusivity obtained using $t_{1/2}$ and Eq. (3). The values of the correction factor f_k are given in Table 1 as functions of the experimental quantities $\tau/t_{1/2}$ and z. For the values $f_k \leqslant 1.45$, this function can be expressed analytically

$$f_k = 1/(1 - 1.04\tau/t_{1/2}). \qquad (23)$$

The time constant τ_1 and τ_2 can be determined from the mathematical analysis of Eq. (19). This has a maximum at the time t_m

$$t_m = \tau_2 \frac{z}{z-1} \ln z = \tau_1 \frac{\ln z}{z-1} = \frac{z \ln z}{(z-1)^2} \tau. \qquad (24)$$

This equation contains two unknown variables z and τ. In order to determine these, we express Eq. (19) in the nondimensional form

$$v(x) = \frac{q_v(t)}{q_v(t_m)} = \frac{z^x - 1}{z^x} \frac{z}{z-1} (z)^{(1-x)/z-1}; \quad x = t/t_m$$

and this relation is depicted in Fig. 2.

Comparing the experimental pulse curve, plotted in the nondimensional form, with Fig. 2, the value of z can be found. The time constants are easily found from (24), knowing t_m.

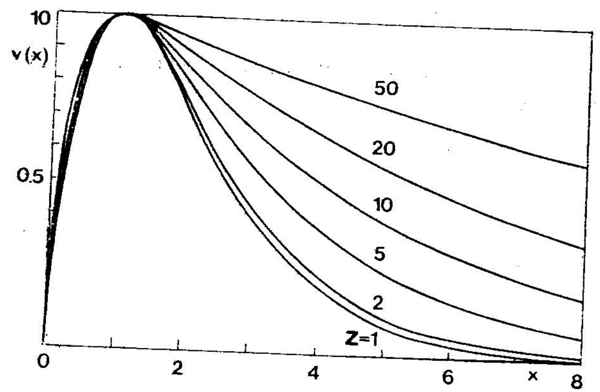

Fig. 2. Nondimensional pulse shape as a function of the parameter $z = \tau_1/\tau_2$.

Table 1

The correction factor f_K for the coupled finite pulse-time effect and the effect of inertia of the recording apparatus as functions of the nondimensional parameters z and $\tau/t_{1/2}$

$\tau/t_{1/2}$	$z = 1$	2	5	10	∞
0.00	1.000	1.000	1.000	1.000	1.000
0.05	1.055	1.055	1.055	1.055	1.055
0.10	1.115	1.115	1.115	1.115	1.115
0.15	1.19	1.19	1.19	1.19	1.19
0.20	1.27	1.27	1.27	1.27	1.27
0.25	1.36	1.36	1.36	1.36	1.36
0.30	1.46	1.46	1.45	1.45	1.45
0.35	1.58	1.575	1.56	1.55	1.545
0.40	1.71	1.70	1.68	1.66	1.645
0.45	1.86	1.84	1.80	1.78	1.755
0.50	2.02	2.00	1.94	1.91	1.785
0.55	2.20	2.18	2.10	2.05	2.00
0.60	2.42	2.39	2.26	2.20	2.14
0.65	2.67	2.62	2.45	2.37	2.30
0.70	2.96	2.90	2.67	2.56	2.47
0.75	3.31	3.21	2.92	2.78	2.66
0.80	3.76	3.60	3.20	3.04	2.87
0.85	4.27	4.09	3.55	3.32	3.11
0.90	5.00	4.73	3.99	3.67	3.40
0,95	6.01	5.58	4.51	4.09	3.73
1.00	7.63	6.90	5.21	4.62	4.13

IV. DISCUSSION

The proposed method of analysis of the simultaneous influence of the finite pulse-time and of the inertia of the measuring apparatus has been worked out for general pulse shape and transfer characteristic of the apparatus. This method can be successfuly used if we know the analytical expression of the function, which approximates the heat source pulse or transfer response of the recording apparatus. The results of this analysis are also valuable, when dealing with these effects individually.

The quantitative analysis of this problem enables to set the condition, when the disturbing factor can be neglected. Thus in our case it may be seen from Eq. (23) that for an accuracy of the thermal diffusivity measurement better than 1 %, the condition $\tau/t_{1/2} \leqslant 0.01$ must be fulfilled. Let us remark that the correction relation (23) expresses the indpenence of the considered distrurbing factors for small values of $\tau_1/t_{1/2}$ and $\tau_2/t_{1/2}$. Then

$$f_k = f_k^{\tau_1} f_k^{\tau_2} = \frac{1}{1 - 1.04\tau_1/t_{1/2}} \frac{1}{1 - 1.04\tau_2/t_{1/2}} \doteq \frac{1}{1 - 1.04\tau/t_{1/2}}$$

where $f_k^{\tau_1}$ and $f_k^{\tau_2}$ are the correction factors for the finite pulse-time effect or of the effect of inertia.

The possibility of a quantitative evaluation of these effects allows to use the flash method for shorter time $t_{1/2}$ and brings a number of advantages: a smaller influence of heat losses, a lower amount of energy needed for heating the sample and the possibility of measuring samples with a higher diffusivity.

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