Letters to the Editor

DENSITY MATRIX APPROACH TO THE NUCLEAR EQUILIBRATION PROCESS[1]

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The non-equilibrium statistical operator of Zubarev is proposed to treat the nuclear equilibration process in the density matrix formalism.

The density matrix formalism developed in non-equilibrium statistical mechanics to describe the time evolution of open quantal systems can be used to derive the results of the statistical model of nuclear reactions [1]. This approach starts from the Liouville equation $i\dot{\varrho}(t) = L\varrho(t)$ for the reduced density matrix $\varrho(t)$ with a non-hermitean Liouville operator $L = L_c + L_d$, the dissipative part of which results from a (weak) coupling of the system (incoming and outgoing particles, target and residual nuclei) to the environment. For the stationary state $\varrho(t \to \infty)$ (flux equilibrium) an integral equation can be derived without referring to the density matrix at transient times. An approximate solution of this equation leads to expressions for the cross sections of compound reactions (eavporation from equilibrium states) parametrized according to the pumping and damping constants of L_d.

In order to include pre-compound reactions we will construct the non-equilibrium statistical operator $\varrho(t)$ of Zubarev [2] at transient times. We assume that after a few scattering events all the impact energy will be distributed within the „hot gas of excited nucleons". Due to the interaction of this hot nucleon gas with the rest of the nucleus an equilibration (relaxation) process occurs, both temperatures equilibrate to the final temperature of the compound nucleus (hydrodynamic stage [3]).

To be more exact, we start from the Fermigas model and divide the total system into four subsystems (see Fig. 1): (1)-particles occupying states below the Fermi surface ε_F, (2)-particles in states between ε_F and the nucleon binding energy E_B, (3)-particles inside the normalization volume with energies above E_B, (4)-particles emitted from the (decaying) composite system. The total Hamiltonian H contains coupling terms between these subsystems ((4) is coupled to (1) and (2) via (3), only). The relaxation proceeds as an exchange of energy and particles between the subsystems, the flow of which has to be calculated. The main assumption is that there exists approximately a quasiequilibrium

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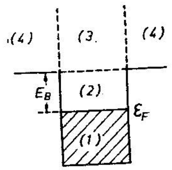

Fig. 1. Subsystems introduced to describe the nuclear equilibration process.

within the subsystems; the mean values of observable P_m (e. g. energy H_i and particle number N_i, $i = 1, 2, 3$) characterize the non-equilibrium state.

According to the method of Zubarev we construct the non-equilibrium statistical operator $\varrho(t)$

$$\varrho(t) = \frac{1}{Q} \exp{(A + B)}, \quad Q = Tr \exp A + B,$$

$$A = - \sum_m F_m(t) P_m$$

$$B = \int_{-\infty}^{0} e^{\varepsilon t'} \sum_m \frac{d}{dt'} F_m(t + t') P_m(t')\, dt',$$

which depends only on the observables P_m and fulfils the Liouville equation for $\varepsilon \to +0$ (infinitesimal contact with the environment).

If the conditions

$$Tr \varrho P_m = Tr \varrho_q P_m$$

with the quasiequilibrium statistical operator

$$\varrho_q = \frac{1}{Q_q} e^A, \quad Q_q = Tr\, e^A$$

are fulfilled, the macroscopic parameters F_m have the physical meaning of the inverse temperature and the chemical potential $\mu_i \beta_i$.

With $\varrho(t)$, which contains memory effects in the term B, we obtain average particle and energy fluxes $\langle \dot{N}_i \rangle$, $\langle \dot{H}_i \rangle$. By the expansion of $\varrho(t)$ for a small B, these fluxes are expressed by correlation functions, and in a linear approximation we can introduce the kinetic coefficients τ^{-1}. Neglecting the term dF_m/dtP_m in B, a system of coupled equations for $\dot{\beta}_i$, $\dot{\mu}_i$ may be set up, e. g.

$$\dot{\beta}_1 = \frac{1}{\tau_{11}} (\beta_2 - \beta_1) + \frac{1}{\tau_{12}} (\beta_2 - \beta_3),$$

$$\dot{\beta}_2 = \frac{1}{\tau_{21}} (\beta_2 - \beta_1) + \frac{1}{\tau_{22}} (\beta_2 - \beta_3),$$

$$\dot{\beta}_3 = \frac{1}{\tau_{31}} (\beta_2 - \beta_1) + \frac{1}{\tau_{32}} (\beta_2 - \beta_3).$$

The kinetic coefficients contain combinations of occupation numbers like

$$\langle n_\alpha^{(i)}(\varepsilon_\alpha, t) \rangle = (1 + e^{\beta_i(t)[\varepsilon_\alpha - \mu_i(t)]})^{-1}, \quad i = 1, 2, 3$$

and squared matrix elements between energy conserving two-body scattering states

$$|\langle \alpha_1' \alpha_2' | V | \alpha_1 \alpha_2 \rangle|^2 \delta(\varepsilon_{\alpha_1} + \varepsilon_{\alpha_2} - \varepsilon_{\alpha_1'} - \varepsilon_{\alpha_2'}).$$

In the simplest approximation the kinetic coefficients are calculated with any constant values of β_i, μ_i (final values). Then, with physical assumptions on the initial conditions $\beta_i(t = 0)$, $\mu_i(t = 0)$ after the first step of the reaction, the coupled equations can be solved immediately by exponential functions, and the inverse temperature and the chemical potential are obtained in dependence on time. Especially, the distribution of the nucleons in the subsystem (3) is described by a time dependent temperature and a time dependent chemical potential, which determines the energy distribution and intensity of the emitted particles (subsystem (4)) in dependence on time.

Dividing the subsystems further we obtain in the limit a kinetic equation for occupation numbers (no temperature). Further the set of observables P_m can be extended by adding the momentum of subsystems P_i to treat the time dependence of the angular distribution of particles emitted during the equilibration process.

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