

## ANALYSIS OF THE $^{41}\text{K}(\alpha, n)$ REACTION IN THE PRE-EQUILIBRIUM MODEL OF NUCLEAR REACTIONS

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The excitation function of the reaction  $^{41}\text{K}(\alpha, n)$  in the  $\alpha$ -particle energy range from 8 up to 40 MeV has been studied within the frame of the exciton model of nuclear reactions. The main attention has been devoted to the shell structure influence on the pairing-energy corrections, as the nuclei are in the neighbourhood of the doubly magic nuclei. The analysis has been performed using the master-equation approach to the model, and the validity of the approximations, which must be performed in deducing the closed-form formulation of the pre-equilibrium decay, has been investigated for the reaction studied. The influence of the pairing-energy corrections, both the conventional and the Rosenzweig ones, on the lower energy region of the excitation curve of the  $^{41}\text{K}(\alpha, n)$  reaction is remarkable. Finally, the values of the transition matrix element have been obtained in this analysis and compared with the formula given by Kalbach-Cline.

### 1 INTRODUCTION

The exciton model [1] is now successfully used for the study of nuclear reactions. Its original version was in the course of time improved by various authors [2-7] and also extended to different approaches and formulations [8]. The main attention in the frame of the exciton model was devoted to intermediate and high energies, because for the excitation energies up to approximately 15-20 MeV the earlier theory of the equilibrium decay [9], which is a special case of the pre-equilibrium decay [6], was sufficient for the agreement to the experiment. As the pre-equilibrium calculations yield for lower energies results close to the equilibrium ones, we can find use for the lower excitation energy region to study phenomena, usually omitted for the pre-equilibrium decay, using the same formalism for both the pre-equilibrium and the equilibrium parts. Some of these questions are connected with the shell structure effects on the pre-equilibrium decay. The shell structure influence has been studied in the sense of the Rosenzweig effect [10]. It has been investigated in a paper of Cline [11], but in her work the equilibrium and

pre-equilibrium parts of the emission were not taken consistently, but by using the closed-form formulation of the decay. Some further analyses of this kind done at Rochester [12] yield different results for various reactions and it is difficult to make general conclusions only on the basis of them.

For the analysis we have chosen the excitation function of the  $^{41}\text{K}(\alpha, n)$  reaction, which was measured by various authors [13] in the  $\alpha$ -particle energy range from 8 up to 40 MeV. The nuclei, which are involved in this reaction, are in the neighbourhood of the doubly magic nuclei. In this way one can expect strong shell effects, which will permit a detailed analysis and distinguish between different ways of taking the pairing-energy corrections. The other regions of long living doubly magic nuclei are not so suitable, because in the lead region the shell effects are comparably weaker and vice versa the statistical assumptions are not well acceptable for light doubly magic nuclei. The fact that this reaction has been analysed recently [14] using the closed-form formulation also influenced the choice of the reaction, as we can use some of the results from the previous work in this more detailed analysis. Thus we have performed calculations in three ways: without any pairing and including the pairing-energy corrections, both conventional and the Rosenzweig-corrected ones. In the present reaction the pre-equilibrium fraction is near unity [14], therefore the theory of the pre-equilibrium decay is well applicable also to the lower-energy region. In contradistinction to the preceding analyses the master-equation approach should be preferred to the closed form one, because the former removes the discrepancy between various formalisms used for the pre-equilibrium and equilibrium parts of the emission and improves the calculations of the latter. In this connection a test of correspondence between these two approaches is worthwhile to perform. Similarly a test has been done in Ref. [6], but only for low pre-equilibrium fraction reactions. From the comparison of these two approaches an estimate can be obtained, in which region the simpler closed-form formulation can be used.

A value of an average transition matrix element, treated in this formulation of the pre-equilibrium decay as a parameter, enters the theoretical expressions. Fitting the theoretical results to the experimental ones, one can extract this very important quantity. Recently, an extensive study by Kalbach-Cline [15] on transition matrix elements has appeared. A predicted mass and energy dependence of the transition matrix element offers a possibility to compare our results with this formula. Though this formula is obviously valid for more reactions, it has not been compared with calculations in such a neighbourhood of the doubly magic nuclei as in the present reaction. If there appears a deviation between the Kalbach-Cline formula and our value obtained from the agreement between theory and experiment, it may be an indication of some further shell effects on the decay.

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## II. THEORY

The pre-equilibrium model of nuclear reactions [1] assumes that some configuration with a low number  $n_0$  of excitons, i. e. particles above and holes below the Fermi level, is created at the very beginning of the reaction. From this initial state the nucleus develops towards the equilibrium via residual energy-conserving two-particle interactions. At each stage of this equilibration process, there is some probability of particle emission. In the present paper we follow the master-equation approach [4—6], where the equilibration process of a composite nucleus at the excitation energy  $E$  can be described by the set of master equations

$$\begin{aligned} \frac{dP(n, t)}{dt} = & P(n+2, t)\lambda^-(n+2, E) + P(n-2, t)\lambda^+(n-2, E) - \\ & - P(n, t)[\lambda^+(n, E) + \lambda^-(n, E) + L(n, E)], \end{aligned} \quad (1)$$

where  $P(n, t)$  is the occupation probability,  $\lambda^\pm(n, E)$  are the transition rates for the exciton number change by  $\pm 2$  and  $L(n, E)$  is the emission rate for a state with  $n$  excitons,

$$L(n, E) = \sum_n^{E-B} W_r(n, \epsilon) d\epsilon. \quad (2)$$

Here,  $B$  is the binding energy,  $W_r(n, \epsilon)$  is the probability of the emission of a particle  $r$  with the energy  $\epsilon$  from an  $n$ -exciton state of a composite nucleus. In agreement with the earlier papers [2—4, 6, 14] we have expressed this quantity as

$$W_r(n, \epsilon) d\epsilon = \frac{2s+1}{\pi^2 \hbar^3} \mu \epsilon \sigma(\epsilon) \frac{\omega(n-a_r, U)}{\omega(n, E)} d\epsilon. \quad (3)$$

In Eq. (3),  $\omega(n-a_r, U)$  and  $\omega(n, E)$  are the state densities of residual and composite nuclei in states with  $(n-a_r)$  or  $n$  excitons,  $a_r$  being the mass number of the emitted particle,  $\mu$  its reduced mass,  $s$  the spin of the particle and  $\sigma(\epsilon)$  the inverse cross section.

The transition rates  $\lambda^*(n, E)$ , where \* stands for all three possible exciton number changes, 0 and  $\pm 2$ , are

$$\lambda^*(n, E) = \frac{2\pi}{\hbar} |M|^2 \omega^*(n, E), \quad (4)$$

where  $\omega^*(n, E)$  denotes the density of the final states and  $|M|^2$  is the square of an averaged transition matrix element.

Let us denote the time spent by the nucleus in an  $n$ -exciton state by

$$\tau_n = \int_0^\infty P(n, t) dt. \quad (5)$$

If we remember that  $L(n, E)$  is the emission rate from the  $n$ -exciton state, then the product

$$a_n = L(n, E) \tau_n, \quad (6)$$

is the relative contribution to the decay of a nucleus from the state with  $n$ -excitons. Indeed, the normalization condition

$$\sum_n a_n = 1, \quad (7)$$

is valid for them [6].

Let us suppose for a while that the emission proceeds only from a state with one specified exciton number, say  $n$ . The energy spectrum in this case would be

$$I_r(n, \epsilon) d\epsilon = \alpha(\epsilon) W_r(n, \epsilon) \tau_n d\epsilon = \alpha(\epsilon) W_r(n, \epsilon) \frac{d\epsilon}{L(n, E)}. \quad (8)$$

Here,  $\alpha(\epsilon)$  is the cross section of the forming of the composite nucleus. In this derivation, two previous relations were used.

But in real cases the emission occurs from more states. The spectrum from an  $n$ -exciton state is no longer  $I_r(n, \epsilon) d\epsilon$ , but  $a_n I_r(n, \epsilon) d\epsilon$  and the total spectrum is the linear combination of previous ones,

$$I_r(\epsilon) d\epsilon = \sum_{\substack{n=n_0 \\ \Delta n=2}} a_n I_r(n, \epsilon) d\epsilon. \quad (9)$$

The same holds for the cross sections

$$\sigma(b, \nu) = \sum_{\substack{n=n_0 \\ \Delta n=2}} a_n \sigma_n(b, \nu), \quad (9a)$$

where

$$\sigma_n(b, \nu) = \int_{E-B_2}^{E-B} I_r(n, \epsilon) d\epsilon. \quad (10)$$

Here,  $B_2$  is the minimum effective binding energy of the secondary particles. In the calculations, some correction  $S$  may be added to redefine this secondary binding energy

$$B_2' = B_2 + S, \quad (11)$$

for the approximate accounting of the  $\gamma$ -competition in the residual nucleus [16].

Solving the set of master equations we can get all the  $\tau_n$ , or, which is equivalent, coefficients  $a_n$  in Eqs. (6—9).

These time-consuming calculations are not necessary every time, since for many problems certain approximations may be used and the closed-form expressions can be obtained. Let us divide the time interval of the reaction into two parts, the first from the composite system creation up to some time  $T$ , when the nucleus reaches a dynamical equilibrium [6], and the second from this moment on. After reaching the equilibrium we can express [6] the emission similarly as that from the compound nucleus [9]. If we denote the relative amount of particles emitted before reaching the equilibrium, i. e. the pre-equilibrium fraction as  $\beta$ , the equilibrium (compound-nucleus) spectrum as  $I_p^{EQ}(\epsilon) d\epsilon$ , and the part of the coefficients  $a_n$ , which corresponds only to the emission before equilibrium, as  $a_n^{PEQ}$ , we can write

$$I_p(\epsilon) d\epsilon = \sum_{\substack{n=10 \\ J_{n=2}}^{\infty} a_n^{PEQ} I_n(n, \epsilon) d\epsilon + (1 - \beta) I_p^{EQ}(\epsilon) d\epsilon, \quad (12)$$

and similarly for the cross sections. Here, the pre-equilibrium fraction  $\beta$  is

$$\beta = \sum_{\substack{n=10 \\ J_{n=2}}^{\infty} a_n^{PEQ} \quad (13)$$

and the calculation is reduced to the proper evaluation of  $a_n^{PEQ}$ , s. In Ref. [6] they were estimated by

$$a_n^{PEQ} \approx \frac{I(n, E)}{\lambda^+(n, E) + \lambda^-(n, E) + \lambda^0(n, E) + L(n, E)} \quad (14)$$

This relation was found to be good in reactions with a low pre-equilibrium fraction.

Since for the present reaction the pre-equilibrium fraction is near unity [14], the usage of the closed form approach may be connected with some problems for our analysis. The master-equation approach will be therefore more suitable for the reaction investigated. This description also includes both pre-equilibrium and equilibrium emissions of the particles in the same way. Thus, irrespective of the real value of the pre-equilibrium fraction, the master-equation approach is useful for the pre-equilibrium decays including both extreme cases.

In our analysis we have investigated the influence of pairing, which was taken both conventionally and taking into account shell corrections according to the Rosenzweig model [10]

$$E_{pair} = E_{pair}^0 + \Delta E_{pair}, \quad (15)$$

where

$$\Delta E_{pair} = \frac{d_n}{2\gamma_n} \left( n - \frac{\gamma_n}{2} \right)^2 - \frac{\gamma_n d_n}{12} + \frac{d_p}{2\gamma_p} \left( p - \frac{\gamma_p}{2} \right)^2 - \frac{\gamma_p d_p}{12}. \quad (16)$$

Here,  $E_{pair}^0$  is the conventional pairing-energy correction,  $E_{pair}^0 = 0$  for odd-odd nuclei,  $E_{pair}^0 = \delta_n$  or  $\delta_p$  for odd- $A$  nuclei and  $E_{pair}^0 = \delta_n + \delta_p$  for even-even nuclei,  $\delta_n$  and  $\delta_p$  are the pairing energies for the neutron and proton, respectively,  $\Delta E_{pair}$  is the pairing-energy correction due to the shell structure,  $\gamma_n$  and  $\gamma_p$  are the average degeneracies of the neutron and proton orbitals near the Fermi surface, respectively,  $d_n$  and  $d_p$  are the average spacing between orbitals and  $n$  and  $p$  are the numbers of neutrons and protons in the last occupied orbital.

### III. CALCULATIONS

In agreement with the preceding analysis of the reaction [14] we assumed the relation  $p = h + 1$  to hold for the composite nucleus and the initial numbers of excitons  $n_0$  to be equal to 3, 5, and 7.

Firstly, we have investigated the relation between the squared average transition matrix element  $|M|^2$  and the pre-equilibrium fraction for the present reaction. They are related via Eq. (13) and the estimate (14), as shown in Ref. [6]. As the estimation of this type lies in closed-form approaches, we have calculated the pre-equilibrium fraction in the case of conventional pairing for  $n_0 = 3$  and 5 from the above-mentioned equations and by a direct solving of the set of master equations. In the latter calculations, the value of  $\beta$  presents some uncertainty, as shown in [6]. For many calculations, we obtain in this way the usually adopted relation

$$\beta \sim |M|^{-2}, \quad (17)$$

when the pre-equilibrium fraction is low, i. e. the matrix element is high, so that we can neglect the last term in the denominator of (14) with respect to the preceding ones. Fig. 1 shows the real dependences of  $\beta$  on  $|M|^2$  for three  $\alpha$ -particle energies, as obtained from solving the set of master equations. These curves are far from being straight, as predicted by the simplified condition (17). For a more precise comparison, we present in the same figure also the estimate based on Eqs. (13) and (14) for medium  $\alpha$ -particle energies,  $E_\alpha = 20$  MeV. For a low pre-equilibrium fraction up to approximately 0.4 the corresponding curves differ only a little. The difference is connected with the uncertainty in choosing the value of  $\beta$  from the emission curve as a function of time from the creation of the composite nucleus [6] and remains up to this value of  $\beta$  practically unchanged. For lower values of  $|M|^2$  the curves are

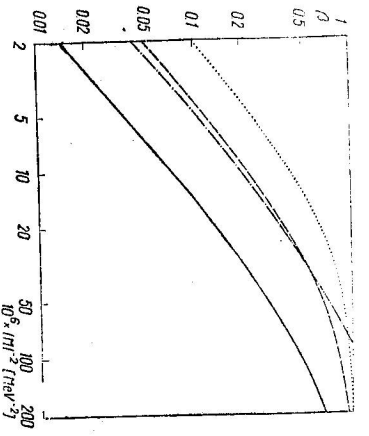


Fig. 1a. Dependence of the pre-equilibrium fraction  $\beta$  on the squared transition matrix element, obtained through master-equation approach for three  $\alpha$ -particle energies: 11 MeV (full line), 20 MeV (dashed line) and 40 MeV (dotted line) for  $n_0 = 3$ . The dotted-dashed curve is calculated using Eqs. (13) and (14). In both calculations the conventional pairing-energy corrections were used.

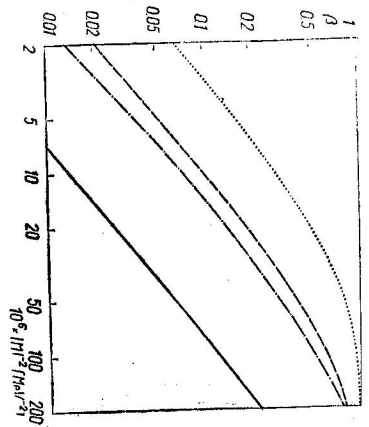
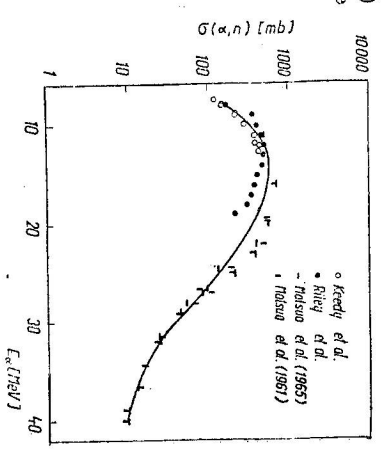


Fig. 1b. The same as 1a, but for  $n_0 = 5$ .

still close, but when crossing the value of  $\beta \sim 0.7$ , the behaviour of relations (13) and (14) is incomparably worse and shortly afterwards it exceeds the maximally possible value  $\beta = 1$ . Hence the value of  $\beta \sim 0.7$  is the farthest one for the present reaction and a very rough estimate in general, where the present closed-form formulation of the problem, similar to ours [6], is acceptable.

For the calculation single-particle level densities were taken from the paper of Gilbert and Cameron [17], pairing energies according to Nemirovsky and Adamchuk [18], inverse cross sections for neutrons and protons by Mani et al. [19], and for the  $\alpha$ -particle cross sections the paper by Huzinaga and Igo was used [20]. The average degeneracies of neutron and proton orbitals and their spacings were approximated using the tables of Lederer et al. [21]. The integration over the energy was replaced by a summation in a 1 MeV step and in the calculation of the emission rates only neutrons and protons were supposed to be present in the exit channels. The parameter  $S$ , which redefines the secondary binding energy in Eq. (11), and which was introduced due to the  $\gamma$ -competition, was varied from 0 up to 3 MeV. The method of solving the set of master equations was the same as described in Ref. [6]. All the calculations were performed using the GIER computer of the Slovak Academy of Sciences.

Fig. 2. Experimental points of the  $^{41}\text{K}(\alpha, n)$  excitation function and the "reference average" curve.



Since four experimental excitation curves are available [13], we have constructed a "reference average" excitation function, supposing equal weights of all four experimental sets. As such an assumption may be a matter of some discussion, we must keep in mind the errors of the individual points. The uncertainty of our construction can be approximated by these errors and by differences among the measurements of various authors. Our "reference average" curve is presented with all four sets of measurements in Fig. 2. All the values of the transition matrix element, which we refer to, were obtained from the fitting to this "average" excitation curve.

#### IV. RESULTS AND DISCUSSION

In our calculations we have varied the transition matrix element from zero to infinity, and simultaneously the initial number of excitons has been taken equal to 3, 5, and 7. For  $|M|^2 \rightarrow \infty$  we have obtained the equilibrium results, whereas for  $|M|^2 \rightarrow 0$  in the extreme limit the calculated quantities are pure ones with  $n = n_0$ .

Thus, if  $|M|^2$  is extremely low, only the first term in (9) is significant and the spectrum is identical with that obtained for  $n = n_0$ . Of course, in this case the approximations (12—14) lose their validity. The other terms in (9) are for higher  $|M|^2$  also significant, so that the spectrum is their linear combination. As all the  $a_n$  in the linear combination on the right-hand side of Eq. (9) are non-negative, both the particle spectra and the cross sections of the reactions are for arbitrary  $|M|^2$  between the lowest and the highest values of all excitation numbers. The term corresponding to  $n = 7$  in Eq. (12) is very close to the equilibrium one, mainly in the low energy region. That is why we present here only the excitation functions as they were calculated for  $n = 3, 5, 7$ , and the equilibrium one, i. e. the three lowest possible terms and the

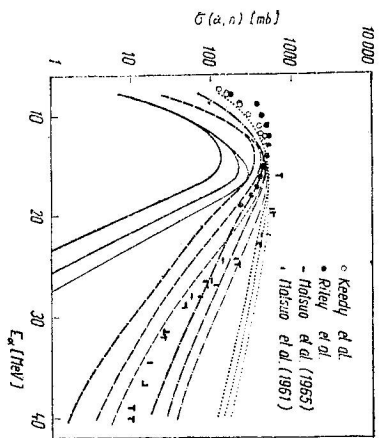


Fig. 3a. Calculated excitation curves and the experimental points. The calculations were performed without any pairing-energy correction. The dotted, dotted-dashed, dashed and full lines correspond to the quantities  $\sigma_n(\theta, \nu)$  for  $n = 3, 5$  and  $7$  and to the equilibrium term, respectively, so that the resulting excitation function must lie among the presented dependences. For each  $n$  as well as for the equilibrium calculations, three curves are presented. The thick, half-thick and thin lines correspond to three values of the parameter  $S$ , viz.  $0, 1$  and  $2$  MeV, respectively.

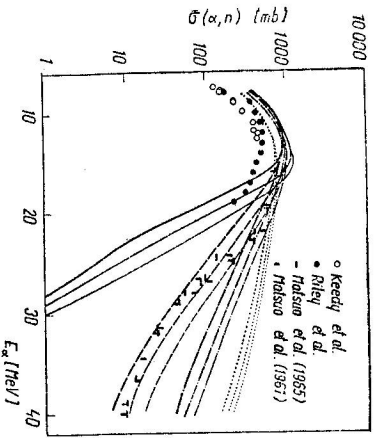


Fig. 3b. The same as 3a., but calculated in the case of conventional pairing-energy corrections.

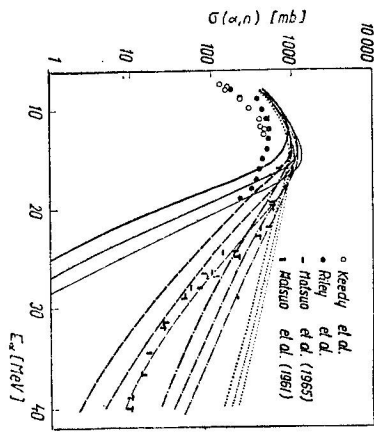


Fig. 3c. The same as 3a., but calculated for the Rosenzweig pairing-energy corrections.

last one in Eq. (12). According to the above discussion the resulting excitation function for  $n_0 = 3$  should be (in dependence on  $|M|^2$ ) a linear combination between the lowest and the highest curves (or their parts) from all possible excitation curves with specified exciton numbers and the equilibrium excitation curve, and e. g., in the case of  $n_0 = 7$  the result is expected to lie between the 'own' term, i. e. the term corresponding to  $n = 7$ , and the equilibrium line, as in this case the coefficients  $a_3^{p20}$  and  $a_7^{p20}$  in Eq. (12) are negligibly small.

The influence of the pairing-energy corrections is easily seen for all the investigated exciton numbers and for equilibrium calculations from Fig. 3. The results obtained without any pairing are rather different with respect to the two other sets. For the low and middle energies they are too much below

the experimental data. Including the conventional pairing corrections improves the agreement a little, but all the possible theoretical excitation curves are now above the experiment in the low energy region. Moreover, the changes of the initial exciton number  $n_0$  and the transition matrix element  $|M|^2$ , which are free parameters in this calculation, cause only negligible variations in the region of interest (e. g. for the  $\alpha$ -particle energy of 8 MeV the theoretical values can range from 389 mb up to 421 mb, and the experimental data are about 160 mb).

The best fit, though not complete, to the experiment can be achieved for the Rosenzweig-corrected pairing. The region of possible cross section values, mainly in the low energy part, spreads towards the experimental points (for the  $\alpha$ -particle energy of 8 MeV from 290 mb up to 410 mb) when compared with the case of the conventional pairing. In this case the agreement is not complete either. It may be due to the uncertainty of choosing the way of averaging the quantities in Eq. (16) so that a slight variation of them can improve the fit to the experiment. The second possibility, which can be responsible for these deviations, are the uncertain values of the inverse cross sections for the low energies of the  $\alpha$ -particles. We have taken the best available theoretical cross section values [20], but as the energy is much below the Coulomb barrier, these values may be wrong, and for some nuclei they are wrong, as follows, e. g., from the fact that the experimental cross section of the reaction  $^{51}\text{V}(\alpha, n)$ , measured by Bowman and Blann [2] for the lowest energy is considerably higher than the inverse  $\alpha$ -particle cross section [20]. With this fact in mind, we have not varied the parameters in Eq. (16), since for the energies below the Coulomb barrier the values of the inverse cross sections may not be reliable.

For the pre-equilibrium decay a very important parameter  $|M|^2$  enters the calculation. We have extracted its value from this reaction for the non-zero pairing energies. The results for both possible ways of pairing, conventional and Rosenzweig-corrected ones, are presented for middle and higher energies and for  $n_0 = 3$  and 5 in Fig. 4. The calculation of the transition matrix element for the low energy region has had very little importance, due to many influencing factors, discussed above. The values of the average transition matrix elements for both cases are in a rather good agreement with each other.

Recently, Kalbach-Cline has published a paper [15] on the values of the transition matrix elements. The predicted mass and energy dependence has there the form

$$|M|^2 = KA^{-3}E^{-1}, \quad (18)$$

where  $E$  is the composite nucleus energy,  $A$  its mass number and  $K$  is the constant, which depends on the incident particle. As the transition rates in

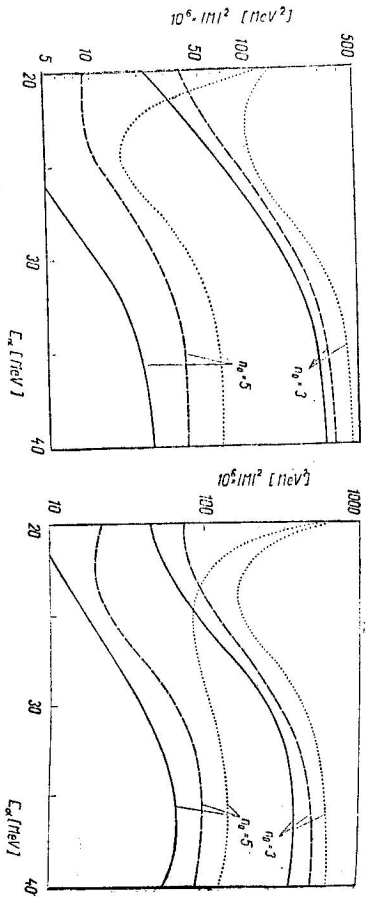


Fig. 4a. Values of the transition matrix elements as obtained from the fit to the experiment in the case of the conventional pairing-energy corrections for  $n_0 = 3$  and 5. Full, dashed and dotted lines correspond to the values of  $S = 0, 1$  and 2 MeV, respectively.

Fig. 4b. The same as 4a., but for the Rosenzweig pairing-energy corrections.

the work of Kalbach-Cline do not contain the factor  $\frac{1}{2}$ , introduced with respect to the indistinguishability of the excitons [6], her values of  $|M|^2$  and  $K$  must be multiplied by a factor of 2. With this correction, for the  $\alpha$ -particle induced reaction the value  $K = 1450$  MeV<sup>3</sup> should be taken. We have compared the predicted energy dependence with ours calculated in the case of the non-zero pairing corrections. The calculated energy dependence is rather different when compared with our results; whereas our values of  $|M|^2$  are increasing with the energy in the investigated regions, according to Eq. (18) the predicted values should decrease approximately by the factor  $\frac{1}{2}$ . The values given by the formula (18) are equal to ours for  $n_0 = 3$  and  $E = 40$  MeV, but for the lower energies and/or  $n_0 = 5$  our transition matrix elements are considerably below those given by Eq. (18). Though not much reliability can be expected as regards the lower energy behaviour [6, 15], the discrepancies seem to be remarkable and they may be some further expression of the shell structure effects.

### V. CONCLUSIONS

We have analysed the excitation function of the  $^{41}\text{K}(\alpha, n)$  reaction. As the pre-equilibrium fraction is very close to unity in this reaction, we have solved the set of master equations. For an estimate of goodness of the closed-form

approach, we have tested in the present reaction the validity of approximations, used in obtaining the closed-form formulation through the master-equation approach. These approximations were found to be fairly good up to a value of the pre-equilibrium fraction  $\beta \sim 0.7$ . All the present analysis has been performed using the master-equation approach. Moreover, this way of calculations is more suitable while investigating the low energy region.

We have studied shell effects in the sense of the Rosenzweig effect in this reaction, where all the nuclei are rather near to the doubly magic nucleus. These effects are pronounced mainly in the low energy region, which is only weakly sensitive to such parameters of the pre-equilibrium decay calculations as the average transition matrix element and the initial number of excitons, though these parameters are usually supposed to be dominant in the master-equation approach to the pre-equilibrium calculations of nuclear reactions.

Though the agreement between the theory and the experiment is not remarkable, the results of our analysis are strongly in favour of pairing and mainly of the Rosenzweig-corrected pairing corrections. The incorporation of the pairing-energy corrections improved the fit to the experiment, and the Rosenzweig effect showed a further improvement of this fit. Even when all the corrections have been performed, a slight disagreement remains in the low energy region. It can be caused by the uncertainties in the averaging shell properties near the Fermi level and by not exact  $\alpha$ -particle inverse cross section values below the Coulomb barrier.

Finally, we have obtained the transition matrix element from the fit to the experiment. The values obtained in this way have been compared with the Kalbach-Cline formula. There are discrepancies between them and the values are close only for  $n_0 = 3$  and the highest energy; whereas for lower energies we have obtained considerably lower transition matrix elements than those corresponding to the paper of Kalbach-Cline. The reason for this may be another effect of the shell structure.

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