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A CONTRIBUTION TO THE THERMAL DEFOCUSING OF THE INTENSIVE LIGHT BEAM IN A NONLINEAR MEDIUM

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This paper presents a theoretical and experimental study of thermal self-defocusing in a nonlinear medium neglecting aberrations and convention currents caused by the local heating of the medium along the beam path. To observe this phenomenon, acetone and the He-Ne laser with a relatively small output power were used.

I. INTRODUCTION

The first paper on the experimental observations of the thermal defocusing phenomenon in a nonlinear medium was published in 1967 [1]. Since then the thermal self-action phenomenon of the intensive light wave has been investigated both theoretically and experimentally in many other works.

The theoretical part of this paper deals with the solution of the problem of the intensive light beam thermal self-action in a nonlinear medium. This solution is based on both the Fermat principle and the thermal conductivity equation.

In the experimental part we have verified the theoretical results and investigated the thermal defocusing phenomenon in a liquid. To observe and study thermal defocusing a He-Ne laser was used, the output power of which was about 8 mW.

II. THEORETICAL PART

Let us suppose that an intensive light beam with a divergence of $2\Theta_0$ transverses a medium characterized by the refractive index n_0, the absorption coefficient α, the thermal conductivity k and the density ϱ. For the theoretical calculation we have chosen a coordinate system with the origin in the plane,

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bordering the medium through which the beam passed. The x axis was identical with the beam axis (in the direction of the light propagation), the y axis was in the radial direction.

Owing to the absorption of the beam energy the medium becomes warm. At the time t and in a place determined by the coordinates (x, y) the temperature of the medium increases from the original value T_0 to the value $T_0 + \Delta T(x, y, t)$ and therefore the refractive index of the medium also increases to the value $n_0 + \Delta n(x, y, t)$. Then we can write the refractive index of the medium at the point (x, y) in the form

$$n(x, y, t) = n_0 + \Delta n(x, y, t) = n_0 + \frac{\partial n}{\partial T} \Delta T(x, y, t) . \tag{1}$$

The distribution of the refractive index in the medium is determined by the intensity distribution in the cross-section of the light beam. Let us suppose that we have a light beam with the radius r_0 and with the Gaussian distribution of amplitude $A(y)$ in the cross-section given by

$$A(y) = A_0 \exp\{-y^2/r_0^2\} . \tag{2}$$

Then the change of the temperature $\Delta T(x, y, t)$ may be written as [2]:

$$\Delta T(x, y, t) = 2\pi \int_{y'=0}^{\infty} \int_{t'=0}^{t} y' Q(y') G(y, y', t') \, dy' \, dt' , \tag{3}$$

where

$$Q(y') = \frac{2P\alpha(1 - \alpha x)}{\pi r_0^2} \exp\left\{-\frac{2y^2}{r_0^2}\right\} \tag{4}$$

is the heat developed by the absorption of the beam energy per unit time and per unit volume, and the temperature change due to a unit instantenous cylindrical surface source at $y = y'$ and $t = 0$ is determined by means of the Green function

$$G(y, y', t') = \frac{1}{4\pi k t'} \exp\left\{-\frac{y^2 + y'^2}{4Dt'}\right\} I_0\left(\frac{yy'}{2Dt'}\right) \tag{5}$$

P is the light beam power [W], $D = k/\varrho c$ is the thermal diffusivity [m^2/s], k is the thermal conductivity [W/m grad], c is the specific heat [J/kg grad] and $I_0(x) = J_0(ix)$, where J_0 is the Bessel function.

From the relations (1), (2) and (3) the refractive index of the medium is given by [2, 5]:

$$n(x, y, t) = n_0 + C_1(1 - \alpha x)(C_2 - C_3 y^2) , \tag{6}$$

where

$$C_1 = \frac{\partial n}{\partial T} \frac{\alpha P}{4\pi k} ; \quad C_2 = ln\left(1 + \frac{2t}{t_c}\right); \quad C_3 = \frac{2}{r_0^2(1 + t_c/2t)}$$

and $t_c = r_0^2/4D$ is a characteristic time for the given medium and beam.

The relation (6) shows that if an intensive light beam propagates in a medium, then its refractive index is changed, the medium becomes nonlinear and re-acts on the propagation of the individual rays of the beam. The path of an arbitrary ray in the light beam may be determined by using both the relation (6) and the Fermat principle according to which light propagates between two points M, N in the medium characterized by the refractive index $n(x, y, t)$ along the optical path given by

$$\delta \int_M^N n(x, y, t) \, ds = 0 \quad \text{or} \quad \delta \int_M^N F\left(x, y, \frac{dy}{dx}, t\right) dx = 0 . \tag{7}$$

In our case

$$F(x, y, dy/dx, t) = [n_0 + C_1(1 - \alpha x)(C_2 - C_3 y^2)] \sqrt{1 + (dy/dx)^2} .$$

From the calculus of variations it follows that (7) is valid if the function $y(x, t)$, which determines the path of an arbitrary ray in the beam, is the solution of the Euler equation:

$$\frac{\partial F}{\partial y} - \frac{d}{dx} \frac{\partial F}{\partial y'} = 0 . \tag{8}$$

The equation (8) can be solved by an iterative method. Supposing $n = n_0$ in the first approximation, i.e. there is no nonlinearity of the medium, the line

$$y(x, t) = a_1 x + a_0 , \tag{9a}$$

where

$$a_1 = \text{tg}\, \Theta_0; \quad a_0 = y(0) , \tag{9b}$$

describing the path of the light ray, is the solution of the equation (8). A distance between the ray and the beam axis in the entering plane is denoted as $y(0)$.

For the second approximation let us suppose that the inhomogenous distribution of the refractive index in the medium causes only a small path deviation of the light ray from the line and it can be expressed by a quadratic term in the path equation, i.e.

$$y(x, t) = a_2 x^2 + a_1 x + a_0 , \tag{10}$$

where $a_2x^2 \ll a_1x + a_0$. Taking into account the above mentioned assumptions, the coefficient a_2 is obtained by solving the equation (8) in the form [5]:

$$a_2 = -\frac{\partial n}{\partial T} \frac{\alpha P}{2\pi n_0^2 r_0 k(1 + t_c/2t)} \frac{y(0)}{r_0}. \tag{11}$$

It can be seen from (10) that the self-action of the light beam is dependen on the sign of the coefficient a_2, i.e. on the fact if the index of refraction is an increasing or a decreasing function of the temperature.

In the case of $\partial n/\partial T < 0$, i.e. $a_2 > 0$, the deviation of the rays from both the initial direction and the direction of the beam axis increases, i.e. a thermal defocusing of the light beam appears.

If $\partial n/\partial T > 0$, i.e. $a_2 < 0$, the light rays approach the beam axis after passing through the medium, i.e. there appears thermal self-focusing.

The relationship for the light beam divergence Θ in the nonlinear medium (i.e. the deviation of the different rays from the initial direction) can be obtained by differentiation of (10) in the form

$$\operatorname{tg} \Theta(x, y, t) = -\frac{\partial n}{\partial T} \frac{P\alpha}{\pi n_0^2 r_0 k(1 + t_c/2t)} \frac{y(0)}{r_0} x + \operatorname{tg} \Theta_0 , \tag{12}$$

where the coefficients a_0, a_1, a_2 are given by (9b) and (10). From (12) it may be seen that the divergence Θ increases with an increase of both the laser output power P and the path x passed by a beam in the nonlinear medium, as well. The divergence Θ decreases with both the increase of the beam radius r_0 and the decrease of the distance $y(0)$ of the ray entering the medium.

In paper [3] the divergence relationship for the light beam during thermal self-focusing was derived in the form

$$\Theta_A = \Theta_{OA} + \frac{\partial n}{\partial T} \frac{P[1 - \exp(-\alpha x)]}{\pi n_0^2 r_0 k(1 + \tau/t)}, \tag{13}$$

where $\tau = r_0^2/8D$ and the border rays of the beam only were considered, i.e. $y(0) = r_0$. The relation (13) was obtained by a simultaneous solution of both the wave equation in a geometrical approximation and the thermal conductivity equation.

It can be seen that (12) and (13) are identical by following the assumptions: $\operatorname{tg} \Theta \sim \Theta$; $1 - \exp(-\alpha x) \sim \alpha x$. The relation $t_c = 2\tau$ is valid for the characteristic times t_c and τ. The different signs of the changes $\Delta\Theta = \Theta - \Theta_0$ are due to the fact that the relation (12) is universal while the one in (13) was derived in the case of thermal defocusing where $\partial n/\partial T < 0$ [3].

III. EXPERIMENTAL PART

The thermal defocusing phenomenon of the intensive light beam in a nonlinear medium was investigated. The experimental arrangement is shown in Fig. 1.

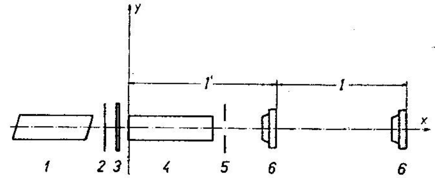

Fig. 1. Experimental arrangement for the measurement of thermal defocusing in a nonlinear medium. 1 — He-Ne laser; 2, 3 — filters; 4 — cell containing a nonlinear medium-acetone; 5 — diaphragm; 6 — camera.

A He-Ne laser with an output power of about $P_0 \sim 8$ mW at $\lambda = 632.8$ nm was used as the source of the light beam. The radiation energy of the laser beam was changed by a neutral polarization filter 3. The laser beam passed through a glass cylindrical cell of a diameter $d = 0.03$ m containing a nonlinear medium. In our case acetone was used. Its parameters were: $n_0 = 1.36$; $-\partial n/\partial T = 4.9 \times 10^{-4}$ grad^{-1}; $k = 0.168$ W/m grad; $\alpha = (3.28 \pm 0.03) \times 10^{-3}$ cm^{-1}. After passing the cell the beam was photographed in two different places of a given distance. The beam divergence was determined by photometering the two photorgaphed beam cross-sections. The output power stability of the laser was checked by a photometer.

During the investigation of the thermal defocusing phenomenon we have determined the dependence of the laser beam divergence on the relative laser power P/P_0, where $P/P_0 = e/e_0$. The values e and e_0 are the photometer read-

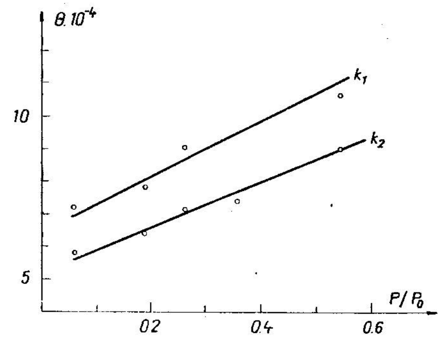

Fig. 2. The dependence of the laser beam divergence on the laser power ($t \gg t_c \sim 3s$, $r_0 = y_1(0) = (10.59 \pm 1.39) \times 10^{-4}$ m, $y_2(0) = (9.16 \pm 1.37) \times 10^{-4}$ m).

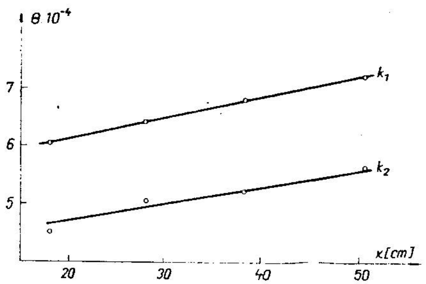

Fig. 3. The dependence of the laser beam divergence on the distance x passed by the beam in a nonlinear medium ($t \gg t_c \sim 3s$, $r_0 = y_1(0) = (10.22 \pm 1.22) \times 10^{-4}$ m, $y_2(0) = (8.87 \pm 1.27) \times 10^{-4}$ m).

ings with or without the filter 3 (Fig. 2). The length of the cell was 50 cm. The divergence Θ was determined for two differently defined beam radii. One of them was defined by the decrease of the maximum intensity of the beam I_0 to the value $I = I_0/e^2$ and the other was given by the decrease of the intensity I_0 to the value $I = I_0/e$ in the same beam cross-section. Let us denote the slopes of the dependences $\Theta = f(P/P_0)$ belonging to two different radius definitions as k_1 and k_2. Then it follows from (12)

$$k_2/k_1 = y_2(0)/y_1(0) . \quad (14)$$

The obtained values are summarized in Tab. 1.

Table 1

The table presents the comparison of both the theoretical and the experimental results

	k_2/k_1	$y_2(0)/y_1(0)$	P_0 [mW]	P_{0a} [mW]
$\Theta = f(P/P_0)$	0.973	0.865	8.28 ± 3.40	9.86 ± 2.26
$\Theta = f(x)$	0.791	0.868	1.72 ± 0.55	2.04 ± 0.51

Fig. 3 shows the dependences of the light beam divergence Θ on the distance which the beam passed in the nonlinear medium. The distance x was changed using the cells of several lengths $x = 19$ cm; 29 cm; 39 cm; 50 cm. The dependence $\Theta = f(x)$ was determined for the two differently defined radii ($I = I_0/e^2$; I_0/e), too. Our measurements can be verified again according to relation (14) and the obtained values are given in Tab. 1.

The values of $y_1(0)$, $y_2(0)$ were determined by both the light beam divergence Θ_0 in the air and the distances l and l′ (Fig. 1). The radius defined by the decrease of the intensity I_0 to the value $I = I_0/e^2$ was regarded as the radius of the whole beam. It is obvious from this that $r_0 = y_1(0)$.

Fig. 4 presents some records used to determine the beam divergence.

IV. CONCLUSION

From the used iterative method it is obvious that in our experimental conditions the relation (12) is only valid up to distances of $x \ll 5$ m [5]. Since the lengths of our cells were $x \leq 50$ cm, the relation (12) in our case was valid in the first approximation.

From Figs. 2 and 3 it may be seen that the theoretically derived linear character of both dependences $\Theta = f(P)$ and $\Theta = f(x)$ has been verified

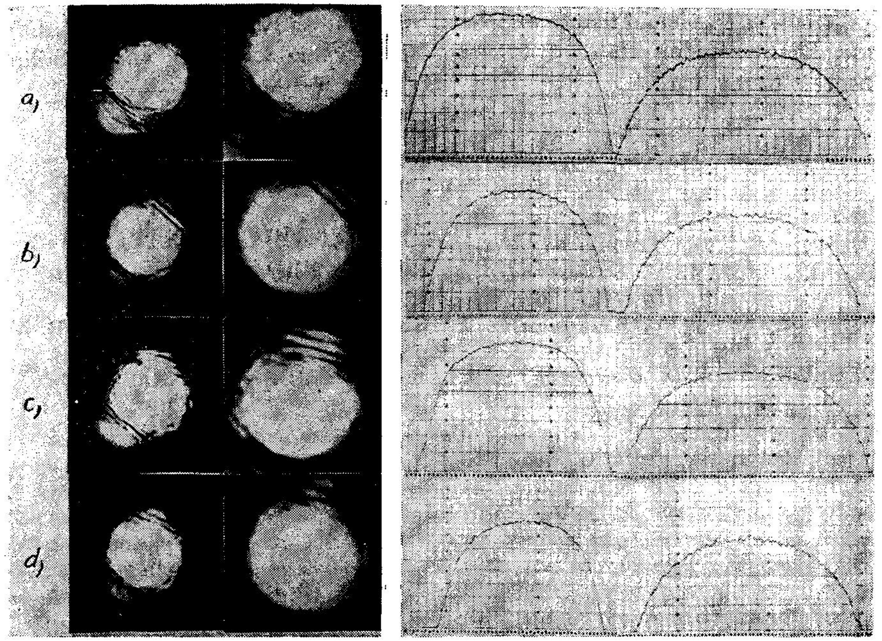

Fig. 4. Some photograph and photometer records used to determine the beam divergence. The distance between the two photographed cross-sections of the laser beam was $l = 1.6$ m. The length of the cell was 50 cm. The relative laser powers of the beam passing through the acetone were $a - P/P_0 = 6.08$, $c - P/P_0 = 26.1$. The relative laser powers of the beam passing air were $b - P/P_0 = 6.08$, $d - P/P_0 = 26.1$.

by the experimental results. The fact that the different parts of the light beam (having the axes identical with the beam axis) diverge differently, i.e. $\Theta = f(y)$, has been verified, too.

From our theory according to (12) it follows that both the fractions k_2/k_1 and $y_2(0)/y_1(0)$ should have the same value. As Tab. 1 shows our experiments confirmed this fact within the experimental errors.

According to (12) the output lasser power P_0 (Tab. 1) has been determined from the slopes of both dependences $\Theta = f(P/P_0)$ and $\Theta = f(x)$ belonging to the whole beam. We could not compare the values P_0 with those measured directly, so we compared them with the experimental results shown in [4] which we recalculated by (12) with respect to our conditions. From Tab. 1 it can be seen that both in this way determined values P_0 and P_{0a} are identical within the accuracy of measurements.

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